A Hands-on Approach to Neural Connectivity Inference Methods

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Program Overview

Morning

- Introduction and Overview - L. A. Baccalá
- Applications of Granger Causality to Neuroscience - Mingzhou Ding
- Statistical and Software Applications - K. Sameshima
- Power User Applications - L. Astolfi

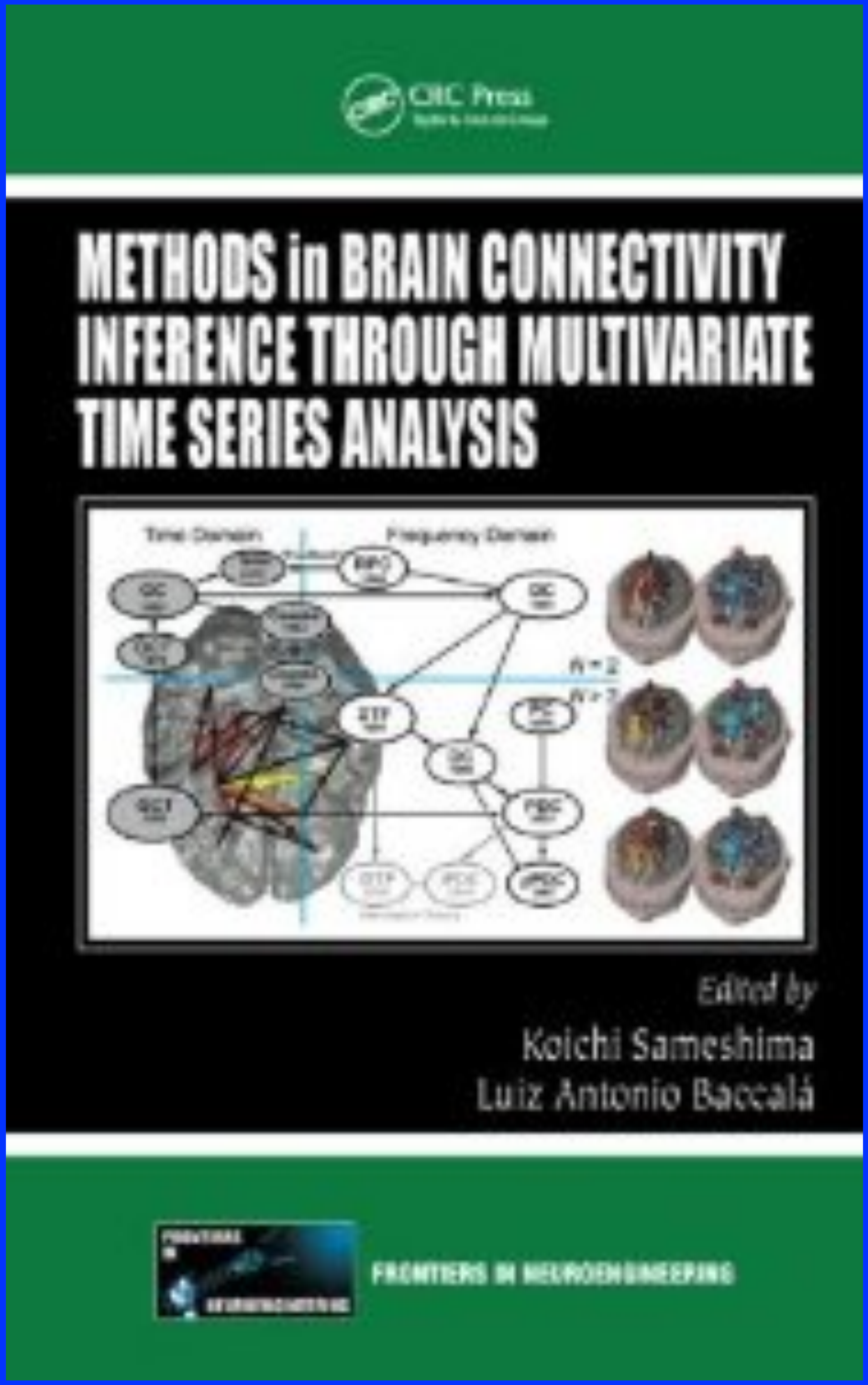
Afternoon

Data Analysis Challenges

Challenge Resolution and Discussion

Overview

- Introduction - Historical Perspective
- Correlation based Connectivity and its Inadequacy
- Granger Causality based Connectivity
- Partial Directed Coherence
- Directed Transfer Function
- Model Diagnostics and Interpretation



Sameshima & Baccalá (ed.)

(2014)

CD inside

http://www.lcs.poli.usp.br/~baccala/pdc/CRCBrainConnectivity/

Key Technology: Multichannel Neural Signal Recording

Modalities

EEG

LFPs

voxel based

MEG

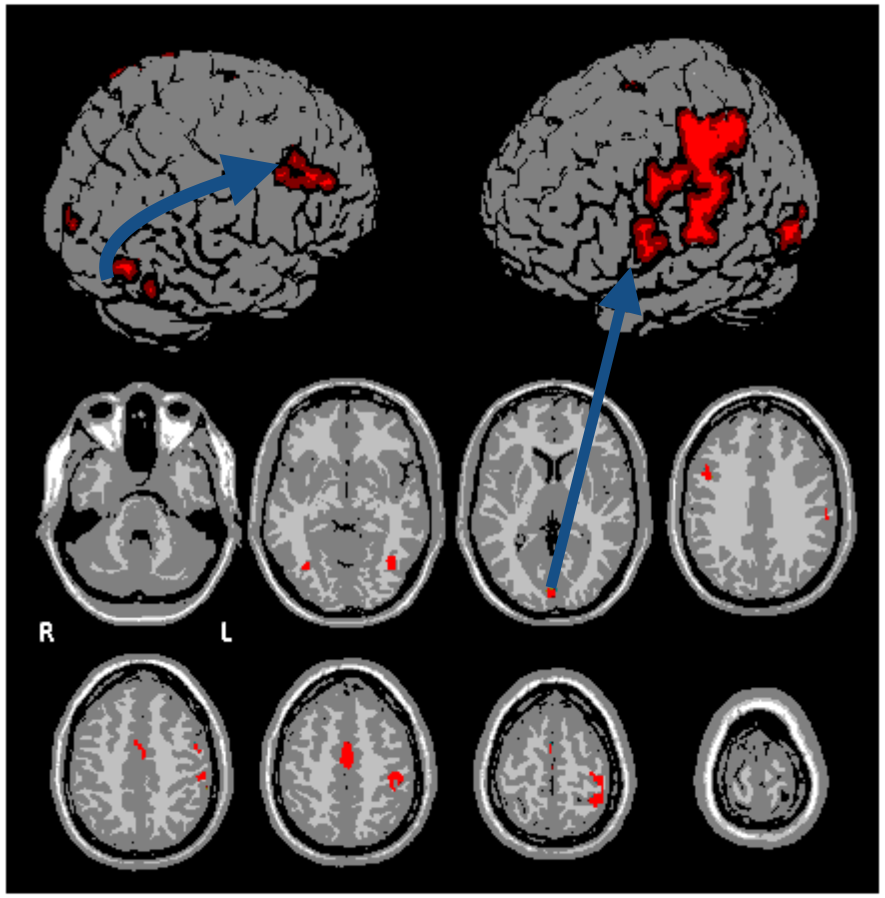
fMRI

Multi-Single Unit Data

Paradigm Change

Active Areas x Interrelations among areas

neo-phrenology ⟶ connectivity

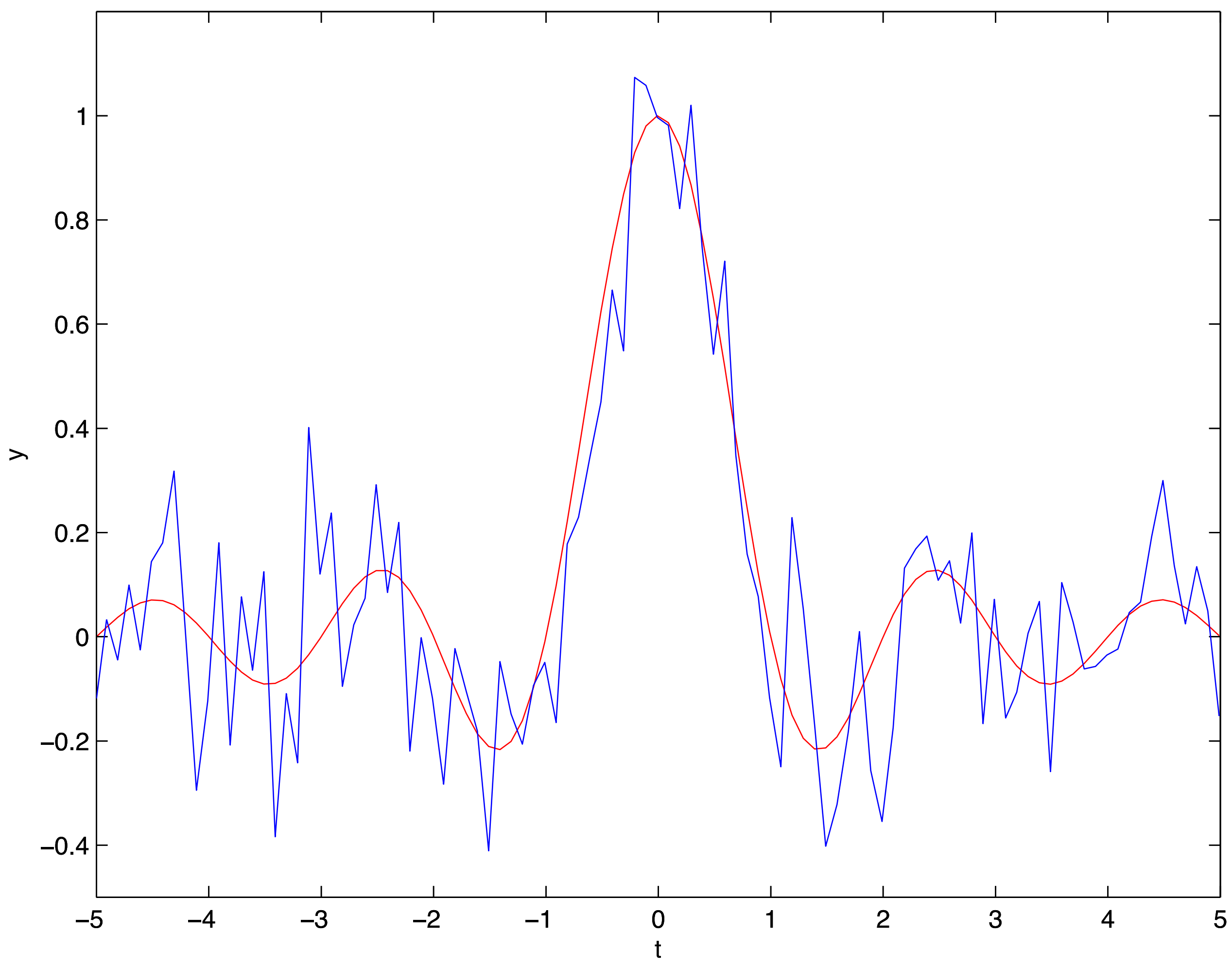


Investigate Information Flow

Goal: Recover the active dependence structure between neural observations

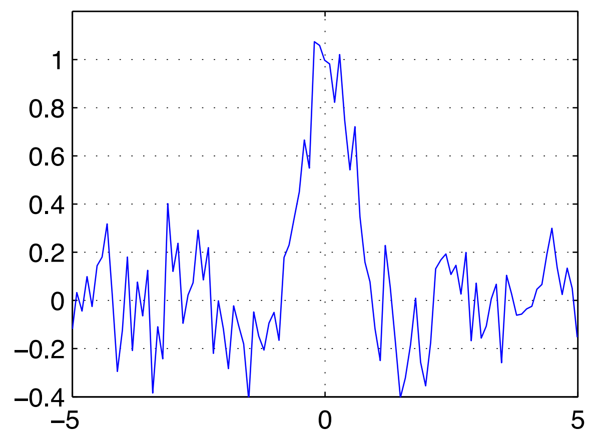
Correlation Ideas

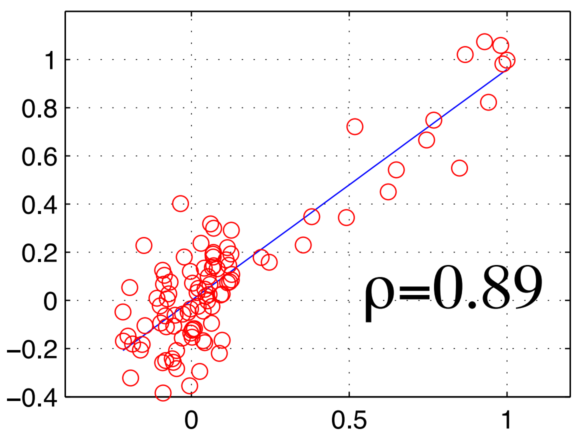
Karl Pearson - dependence between variables



Correlation Ideas

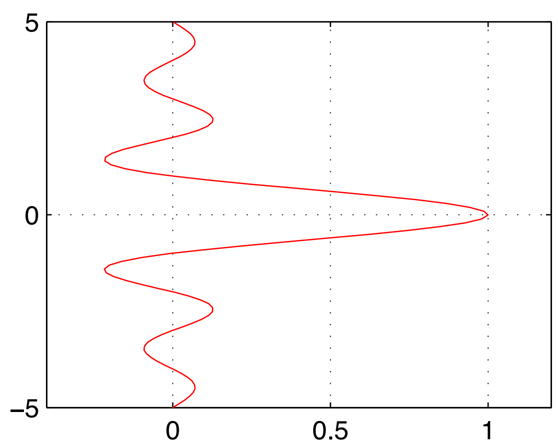
Karl Pearson - dependence between variables (1896)





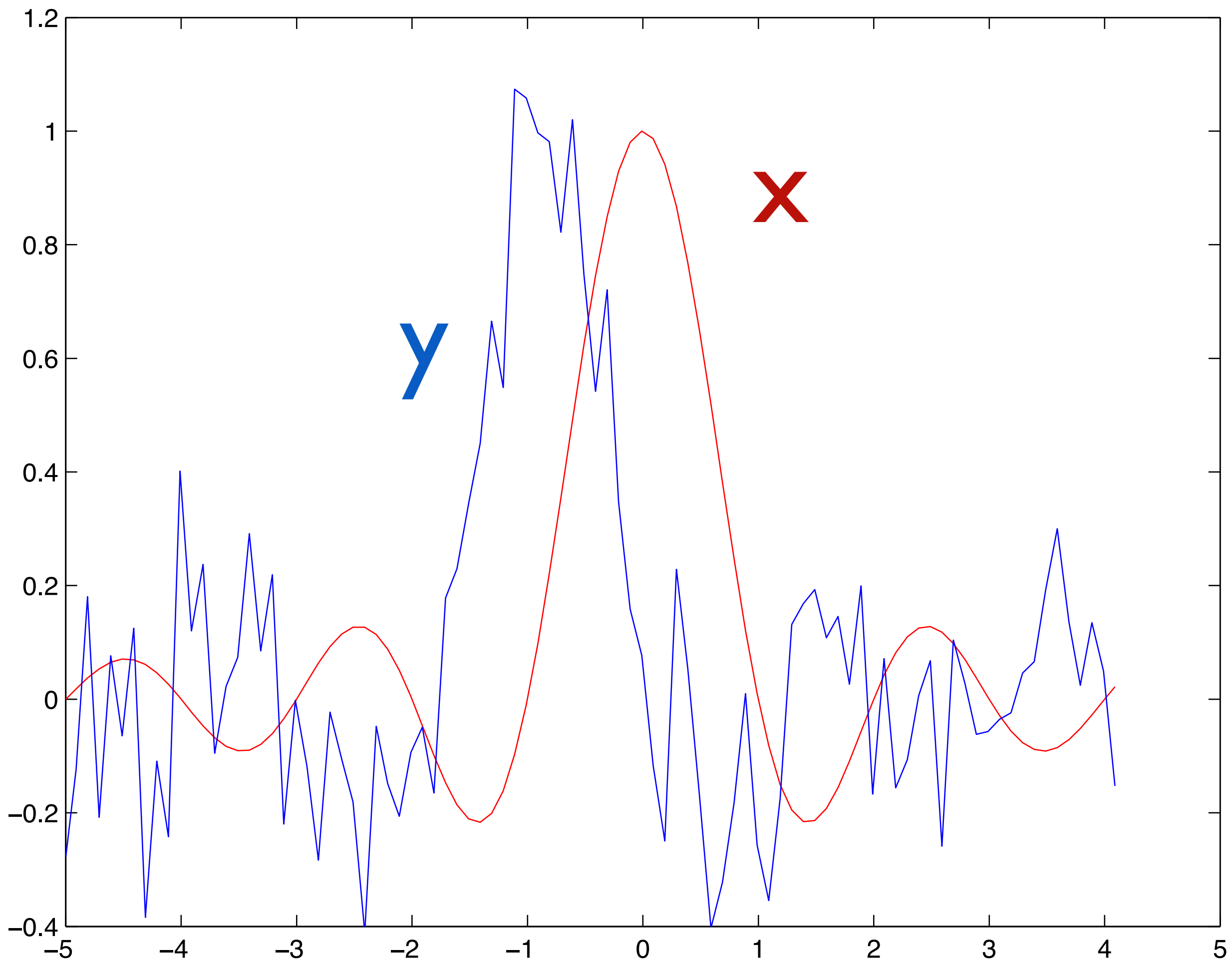
Correlation Coefficient

$$0 \leq |\rho| \leq 1$$

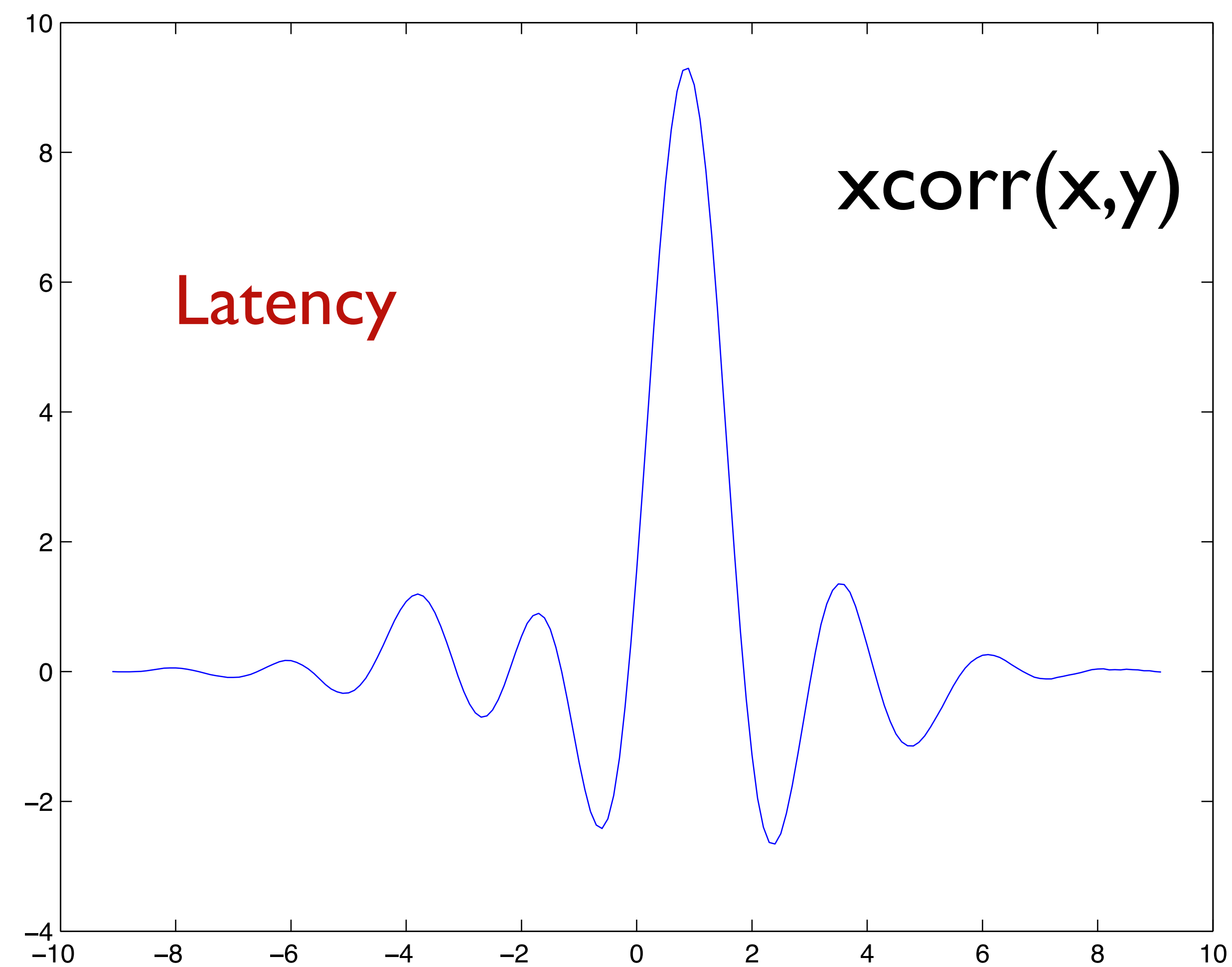


Degree of geometrical similarity

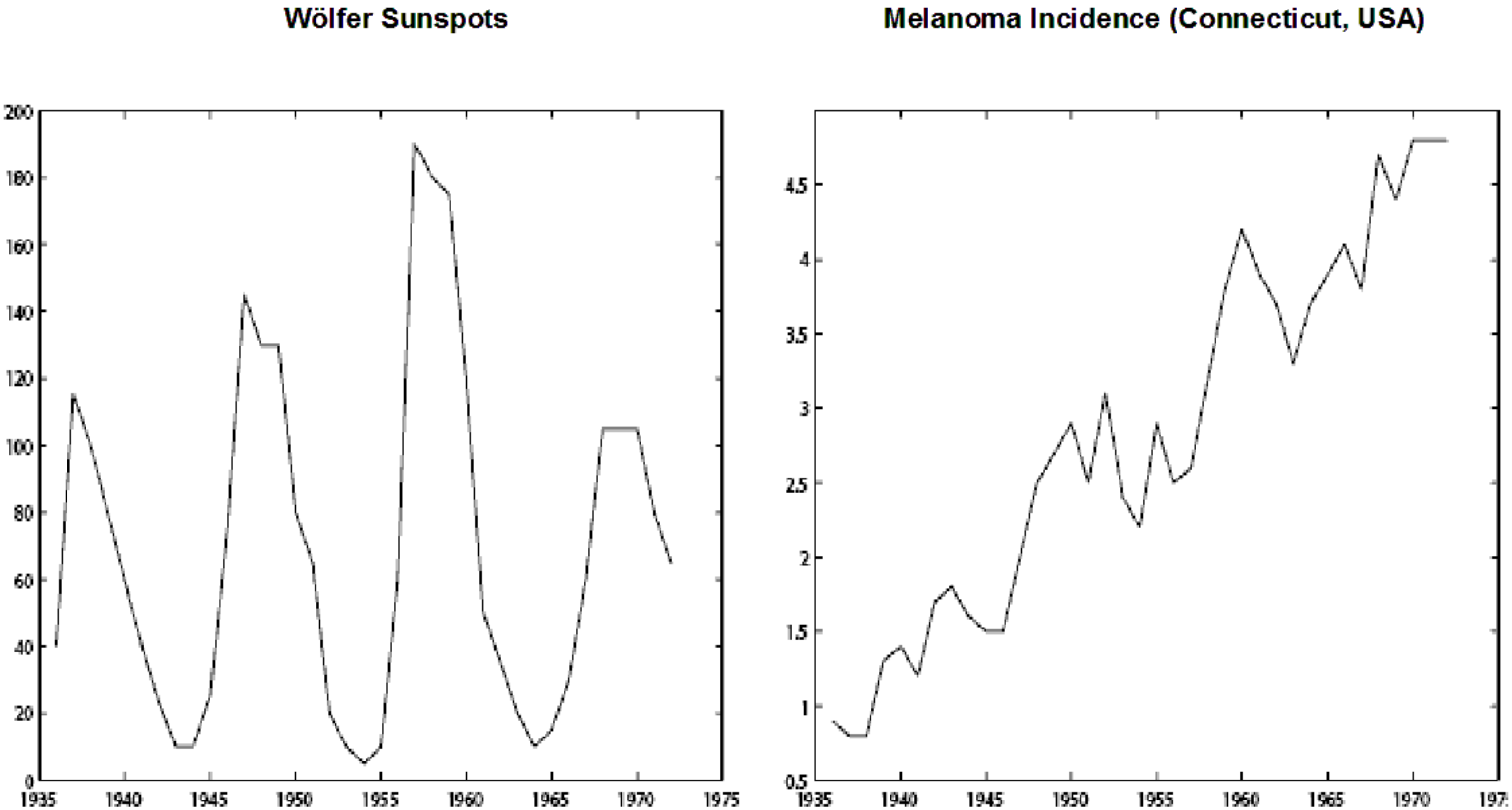
Delayed Waveforms



Cross Correlation Function



Delay of x with respect to y

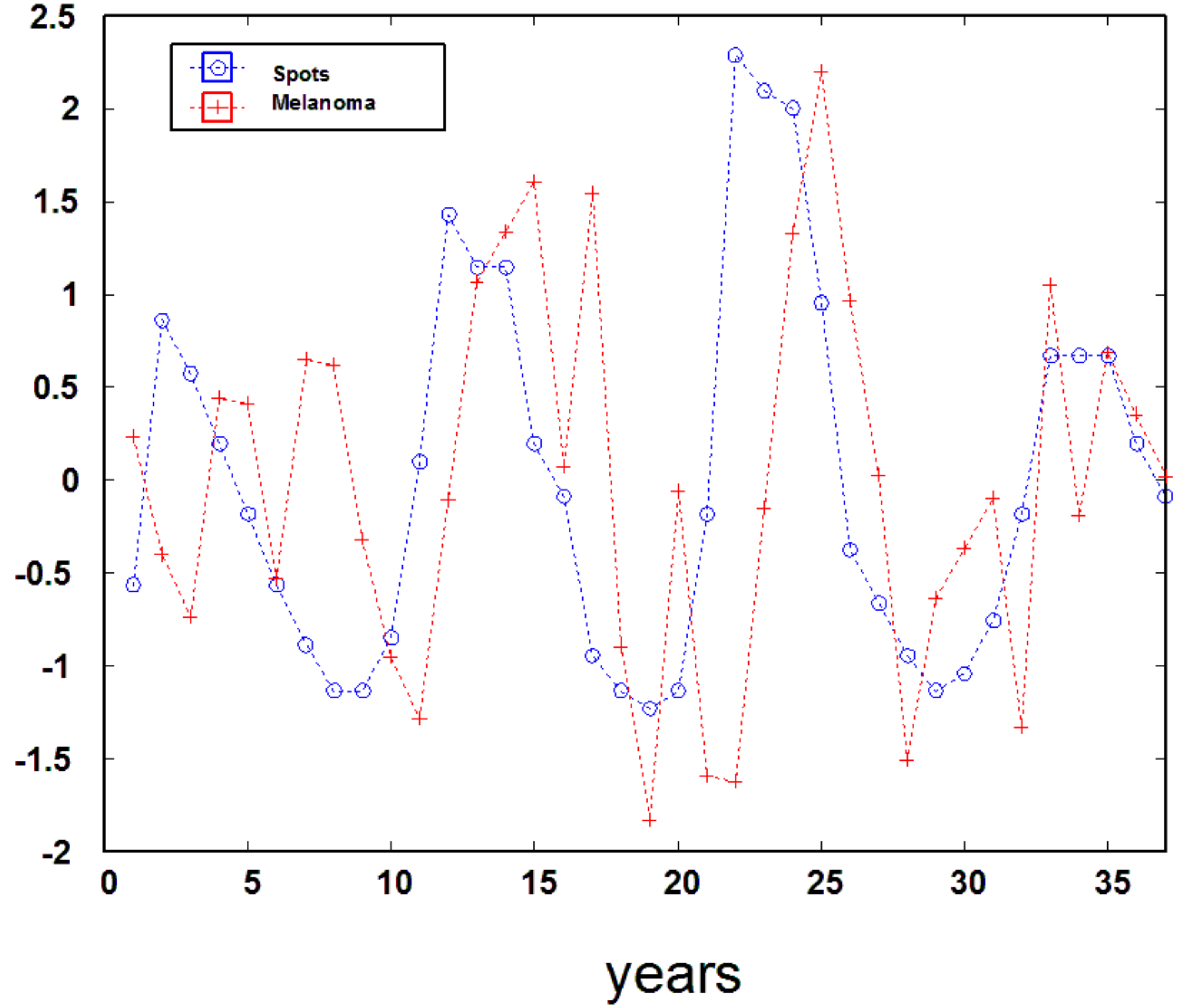


(a)

(b)

Time Series

Are Melanomas and Solar activity connected?

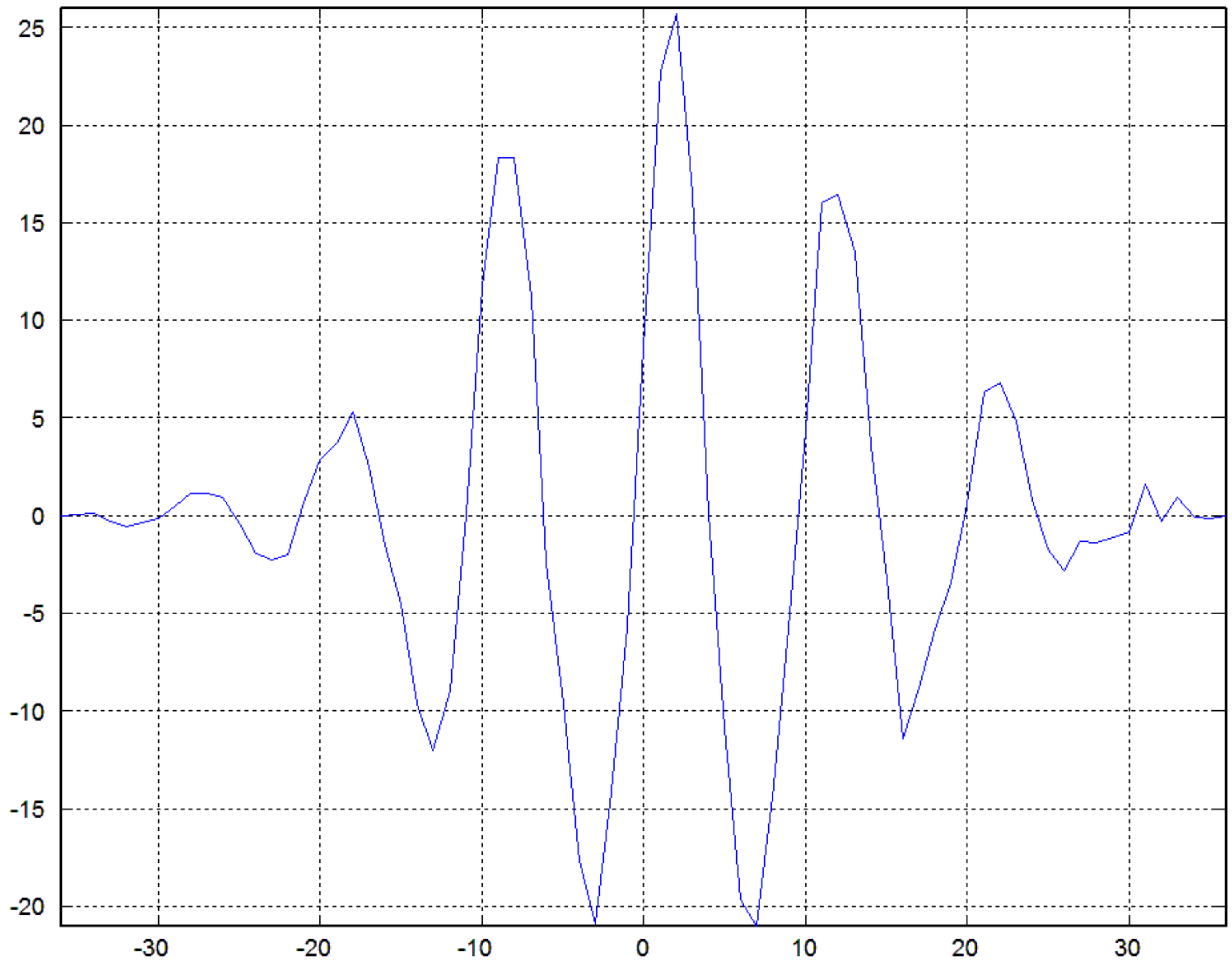


(after extracting means and trends and normalizing with respect to variance)

What is the meaning of cross-correlation?

Latency - when the correlation is maximum

(propagation delay) 3 years



Melanoma peaks are late with respect to solar activity

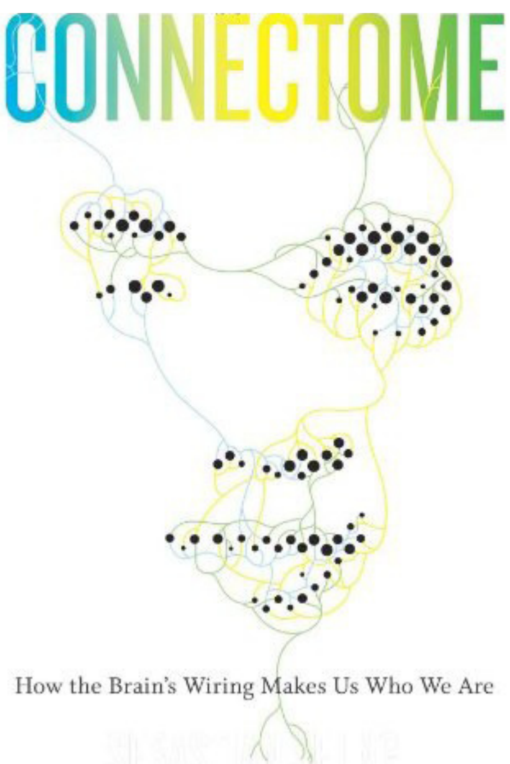
Logic

If 2 wave shapes are similar (accounting for delay) the physical processes are 'connected'

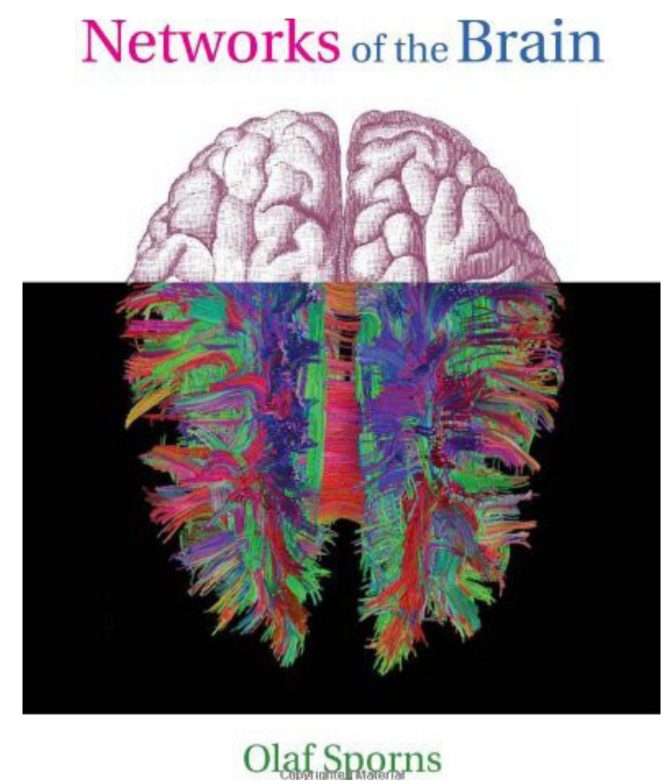
Minimal "Distortion" Implicit

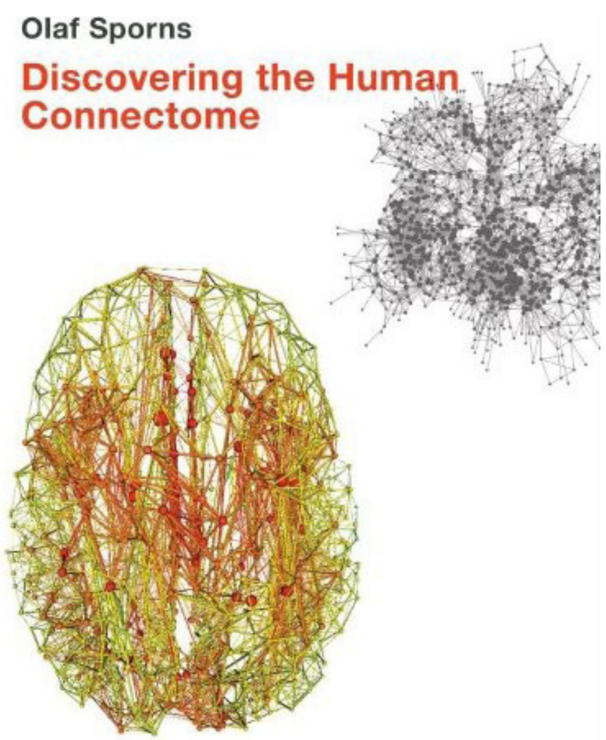
Limitation: Pairwise Analysis

Popular Expositions



Correlation
Centered





This is Your Brain: Mapping the Connectome

It's been 20 years since Francis Crick and Edward Jones, in the midst of the so-called Decade of the Brain, lamented science's lack of even a basic understanding of human neuroanatomy. "Clearly what is needed for a modern human brain anatomy is the introduction of some radically new techniques," the pair wrote in 1993. Clearly, researchers were listening. Today, they are using novel technologies and automation to map neural circuitry with unparalleled resolution and completeness. The NIH has dedicated nearly $40 million to chart the wiring of the human brain, and the Allen Brain Institute has poured in millions more to map the mouse brain. The data will take years to compile, and even longer to understand. But the results may reveal nothing less than the nature of human individuality. As MIT neuroscientist Sebastian Seung writes, "You are more than your genes. You are your connectome."

By Jeffrey M. Perkel

When Seung says in *Connectome: How the Brain's Wiring Makes Us Who We Are*, "You are your connectome," what he means is that neural connectivity is like a fingerprint. Each person has their own unique blend of genetics, environmental influences, and life experience. Those factors influence the detailed circuitry of the brain, such that even identical twins likely differ at the "Genomes are child's play compared with connectomes."

Nevertheless, researchers are making a stab at the problem. From the so-called macroscale of magnetic resonance imaging, to the microscale of electron microscopy, the connectome is slowly coming into focus, one synapse at a time.

The Human Connectome Project

When thinking about the connectome, says Hongkui Zeng, senior director of research science at the **Allen Institute for Brain Science,** think Google Maps. Neuroscientists would like to navigate the brain in

Earlier

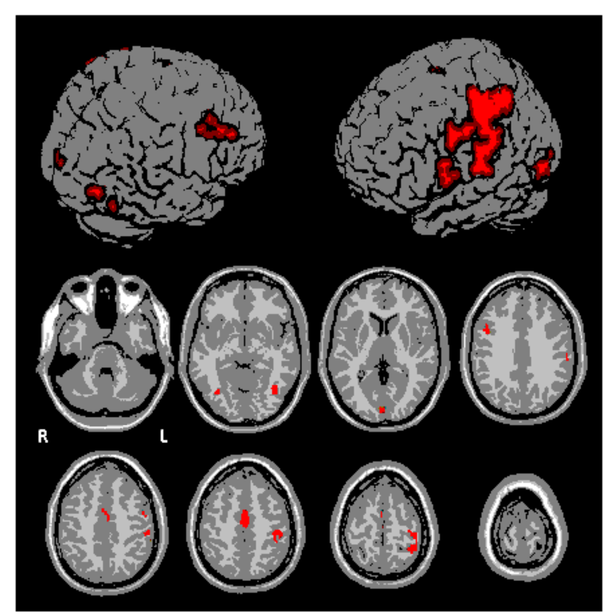
Limited to Comparing the activity of Pairs of Structures

Correlation Analysis

(Coherence Analysis - Fourier Representation)

Search for Structures of Correlated/Coherent Activity

neo-phrenology



Frequency Domain

- Describe Phenomena important over 'bands'

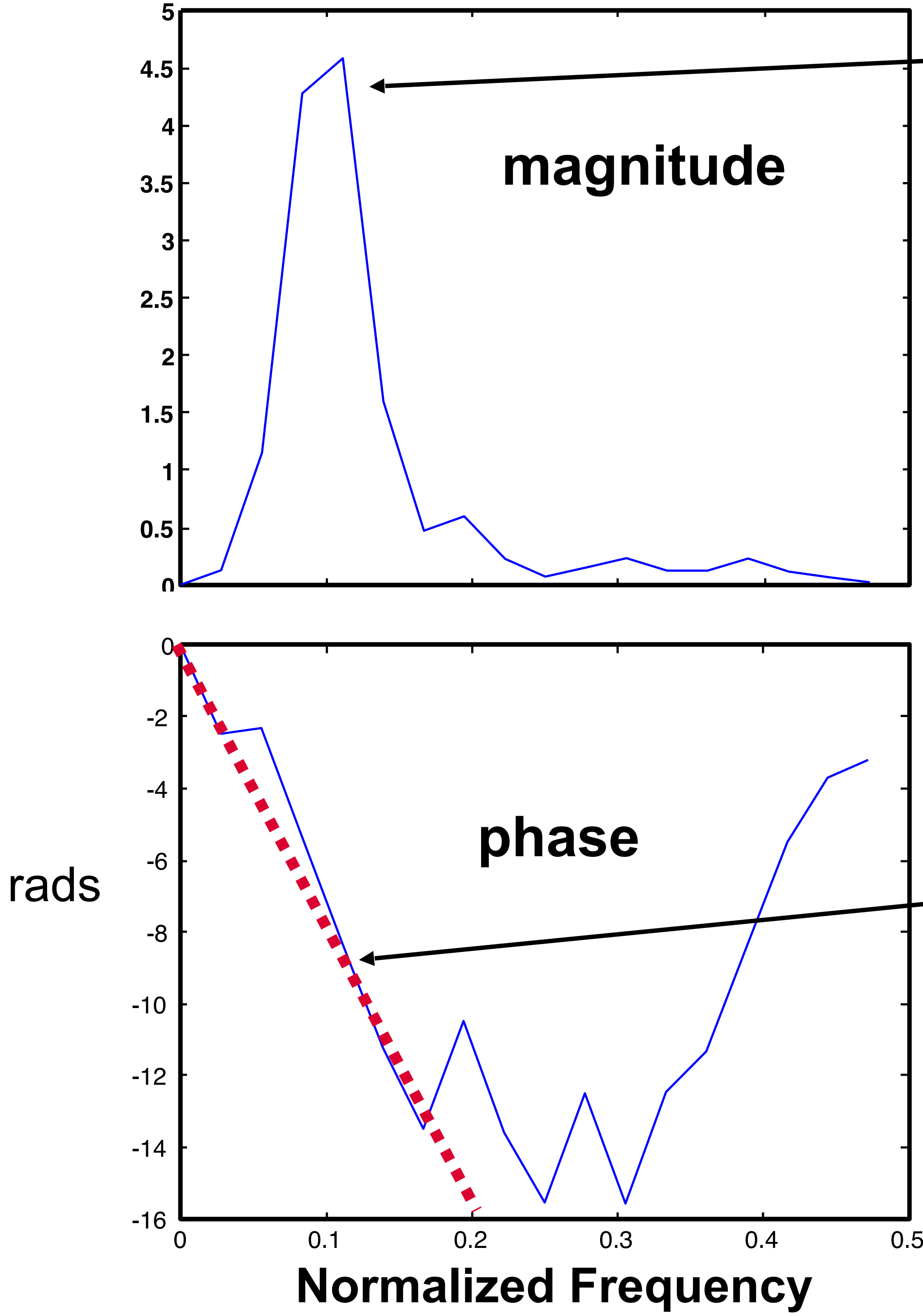
EEG α, β, γ, δ bands

- Robust against Linear Distortion

when different bands propagate at different speeds

- Fourier Transform of Cross Correlation Function

Cross Spectrum



Common frequency

Same information as cross-correlation

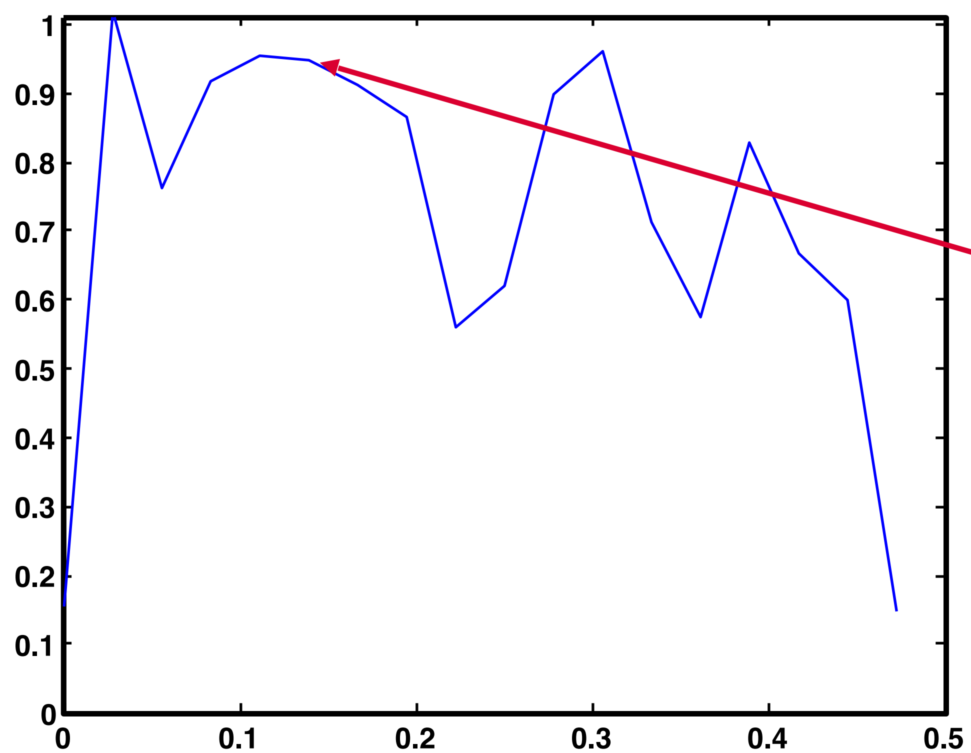
Phase is linear over the region of common variance

(Nonparametric Estimates)

<u>Coherence Function</u>: normalized cross-spectrum

- Linear relationships
- Synchronization between structures

Coherence

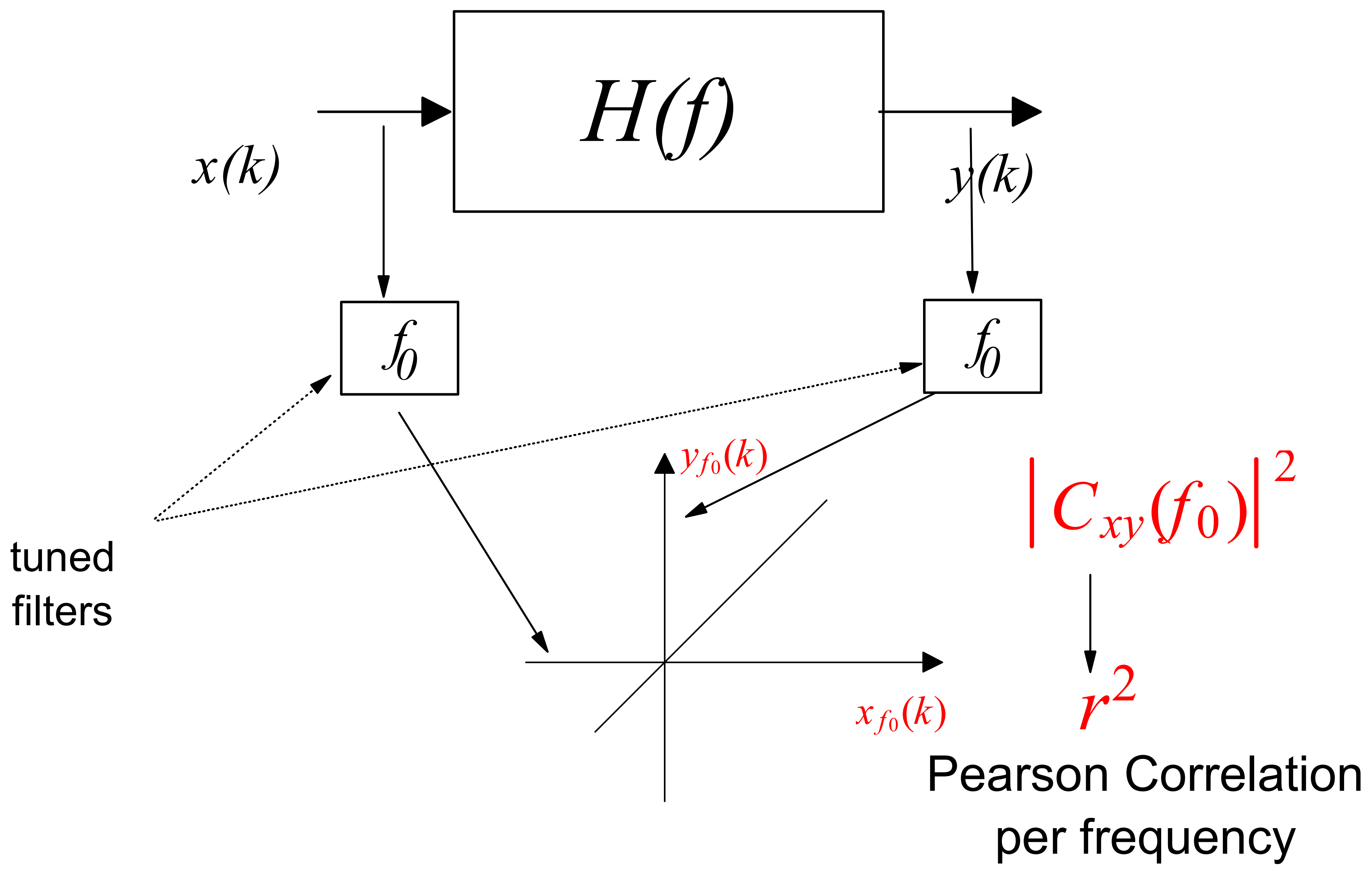


Normalized Frequency

High Coherence
points to link
in those frequencies

Nonparametric Estimate

Interpretation:

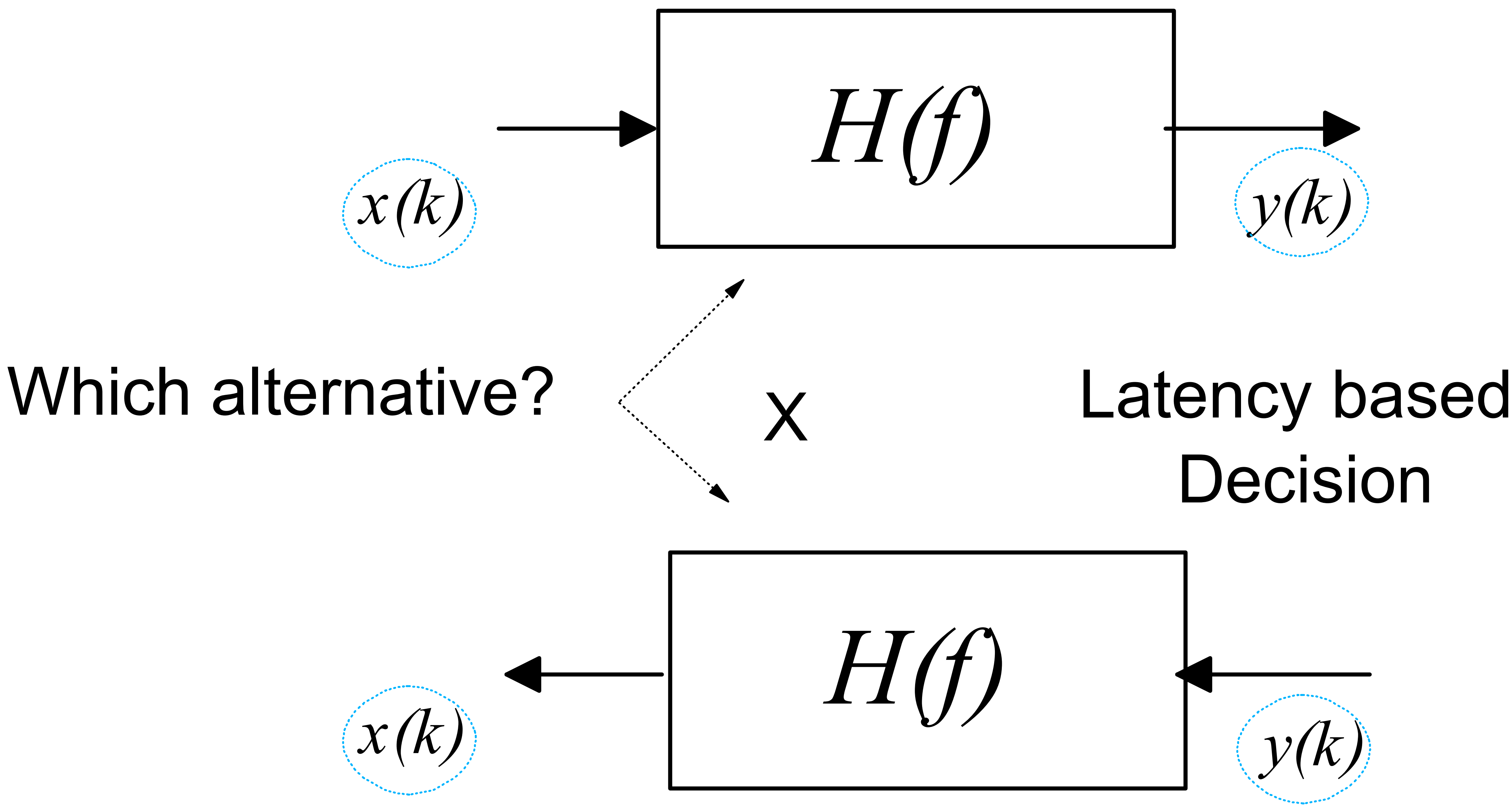


$$\left|C_{xy}(f)\right|^2 < 1$$

Remarks:

- nonlinear link
- uncorrelated additive noise

"Internal" *H(f) Structure*



M.A.L. Nicolelis (Ed.)
Progress in Brain Research, Vol. 130

CHAPTER 3

Overcoming the limitations of correlation analysis for many simultaneously processed neural structures

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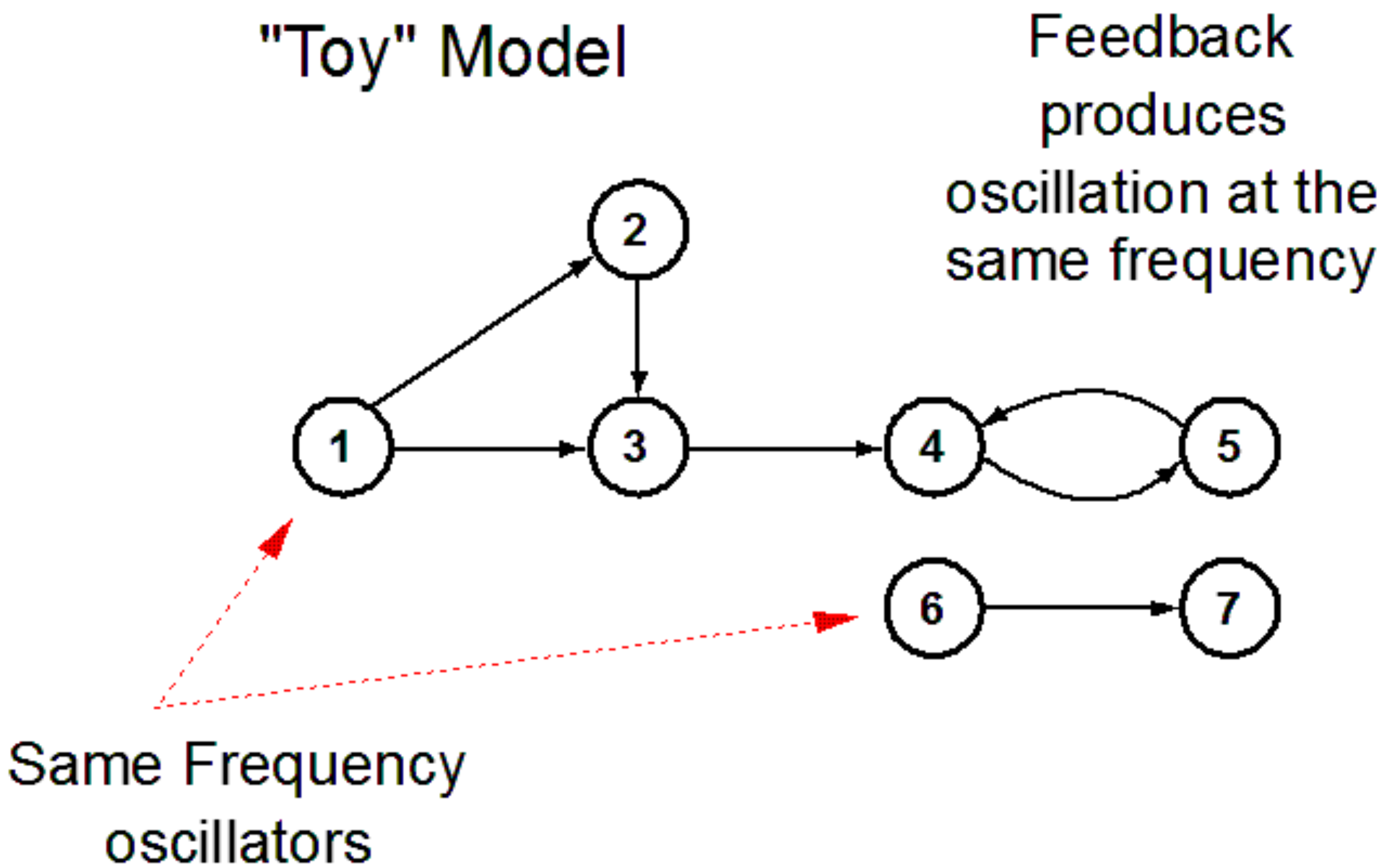
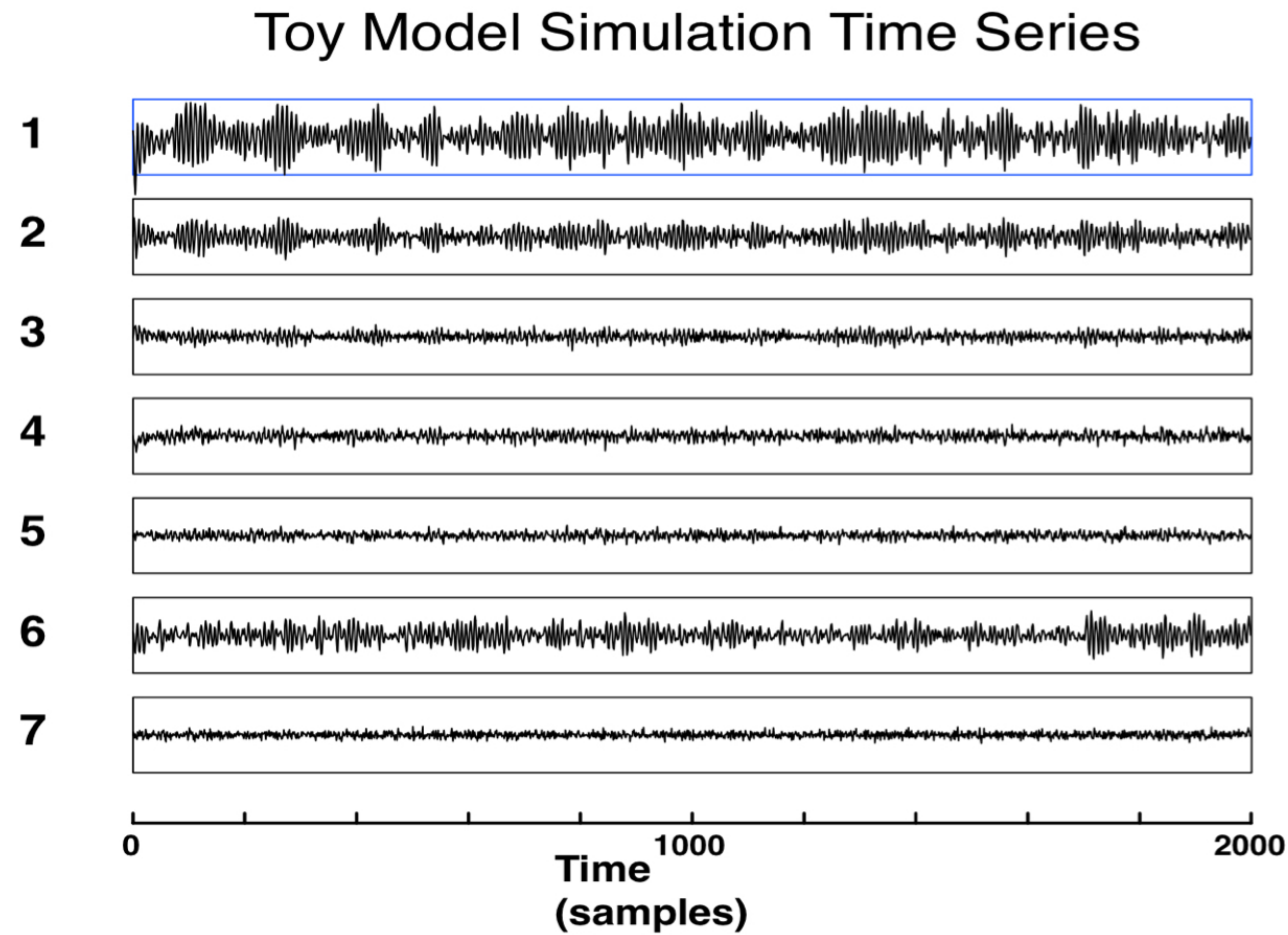
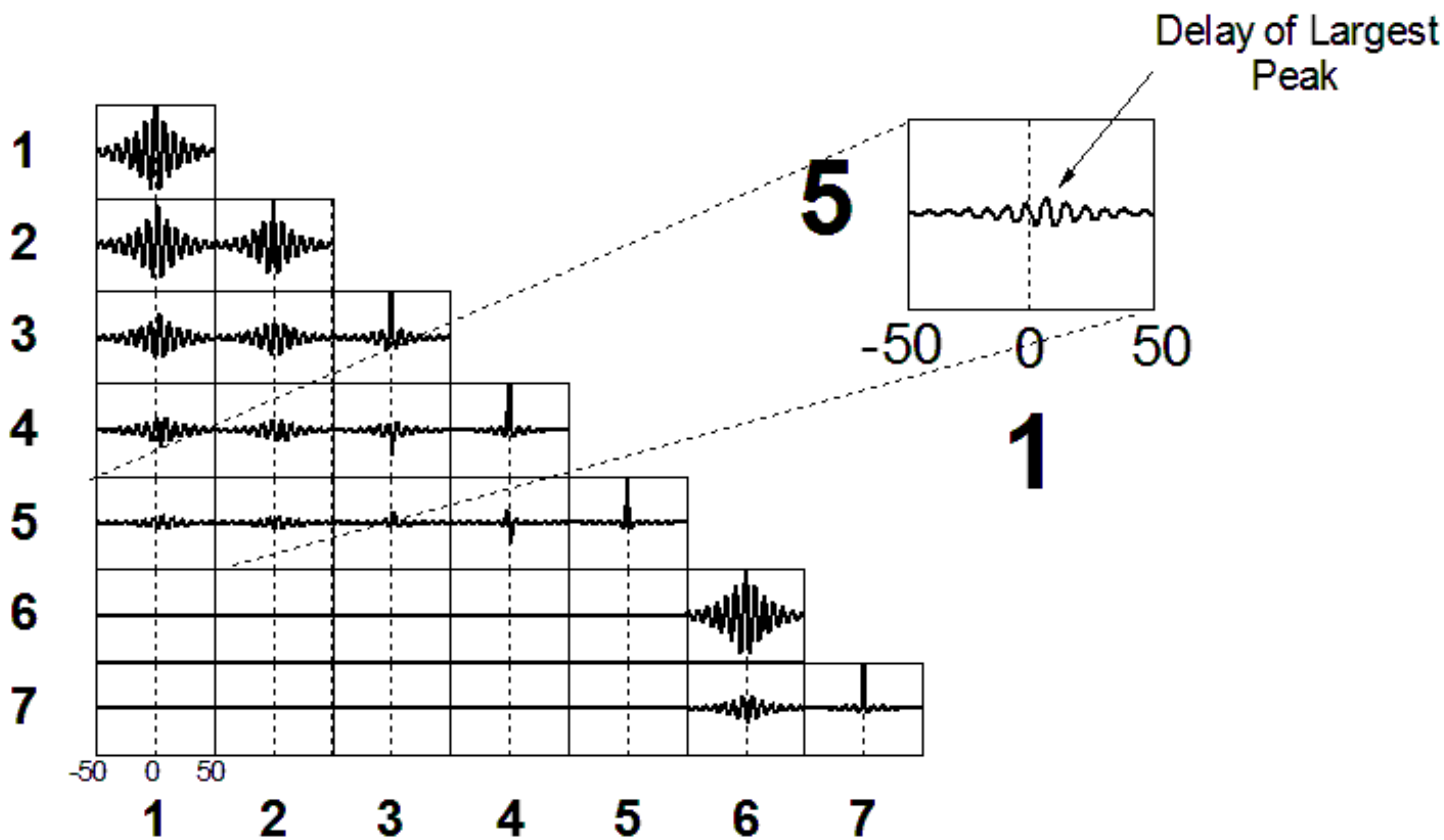
"Toy" Model


Feedback produces oscillation at the same frequency
2
1
3
4
5
6
7
Same Frequency oscillators

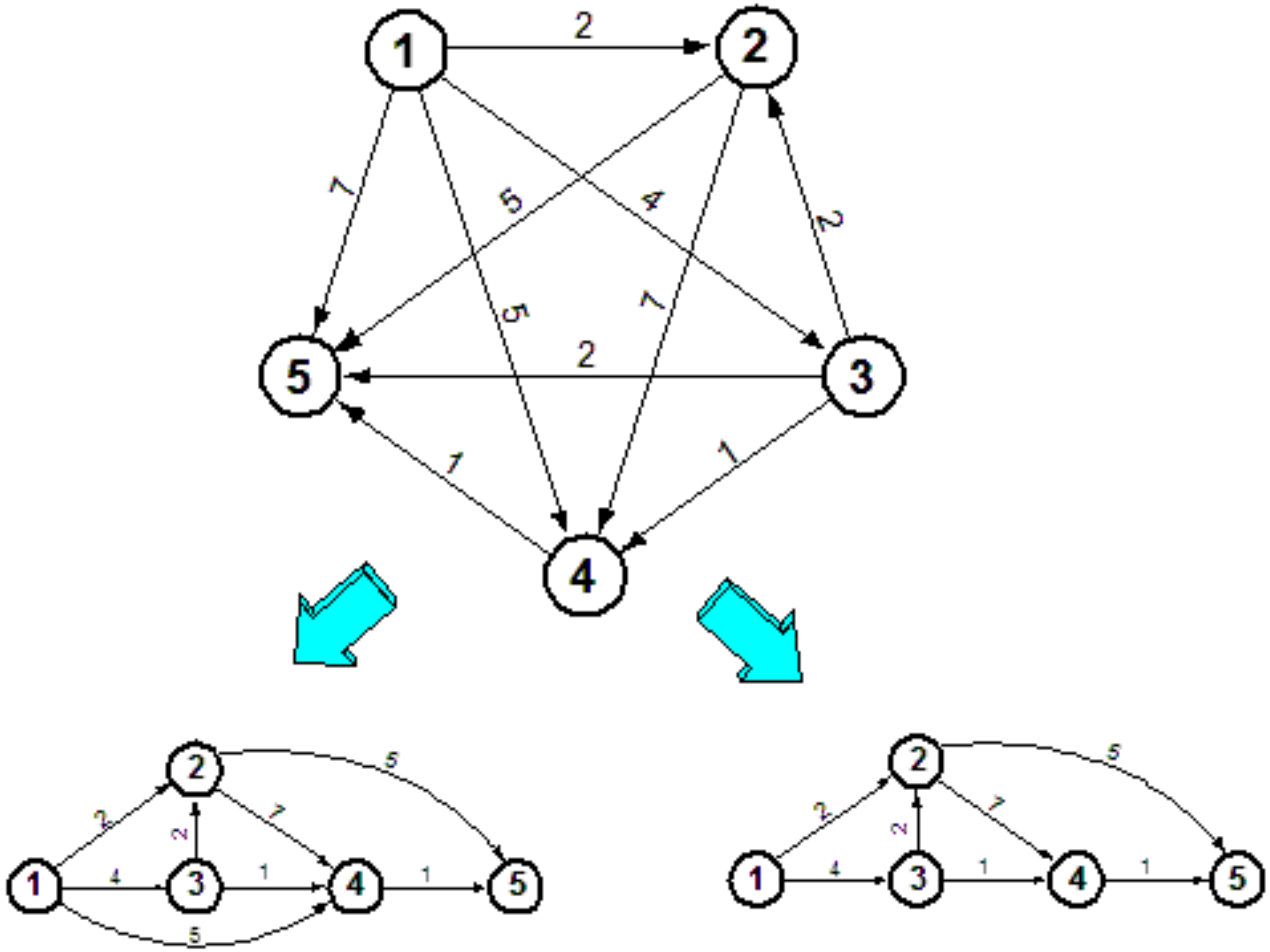

Toy Model Simulation Time Series
1
2
3
4
5
6
7
0
1000
2000
Time
(samples)

Latency Measurement

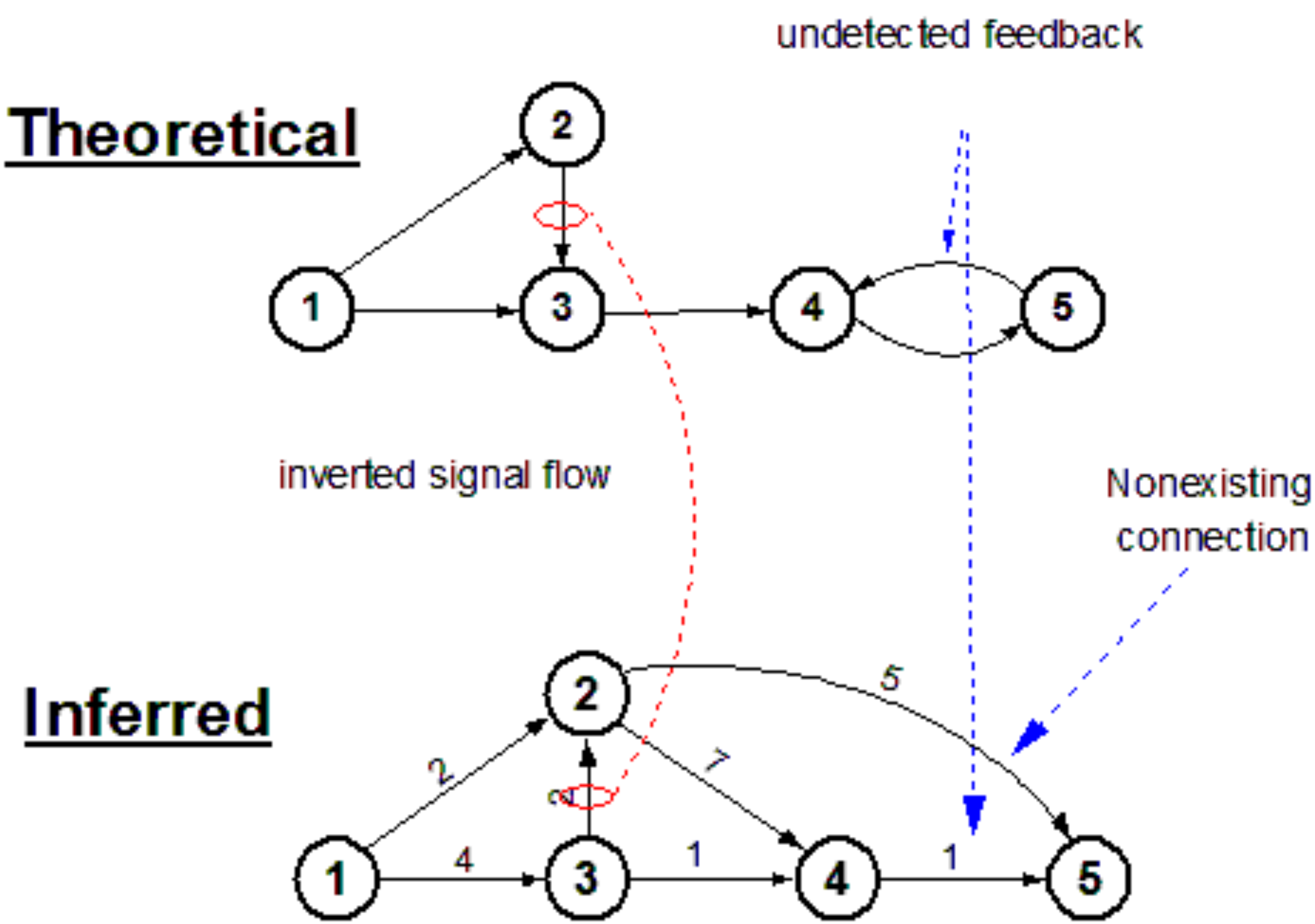


Cross-Correlation

Toy Model Latency Measurements

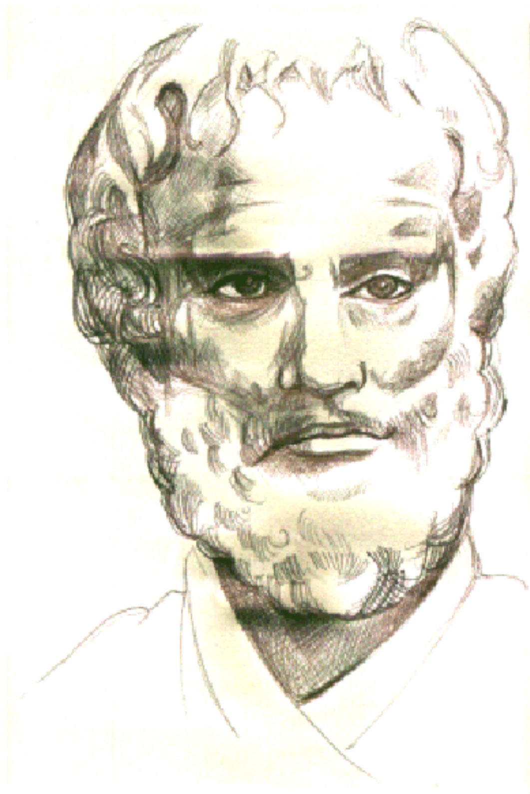


Comparison between theoretical and latency-based structural inference



Granger-Causality Based Connectivity

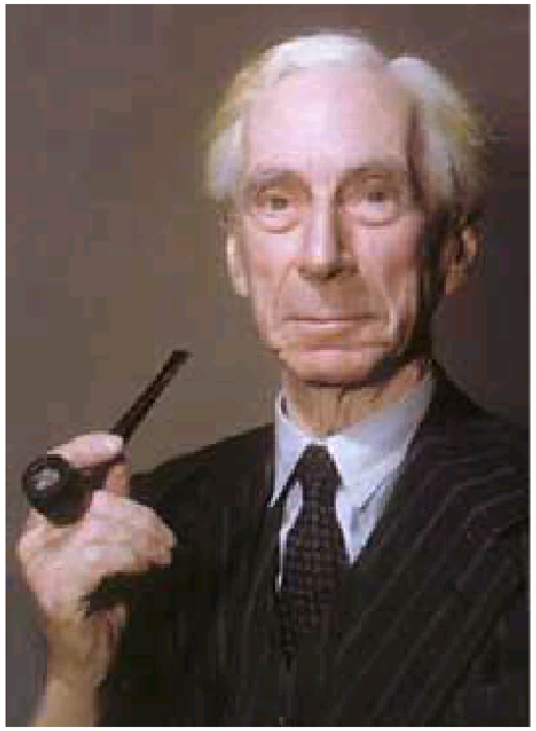
Causality



Aristotle
Physics II 3 and Metaphysics V 2.
~ 350 BC

Predecessor is responsible for (generates) successor

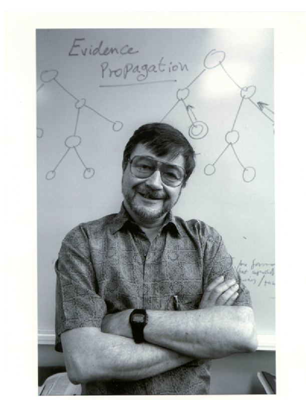
Hume
An Enquiry Concerning Human Understanding (1758)



Bertrand Russell
On the Notion of Cause
(1913)

Causality

Bayes ~1763



Pearl, J., **Causality** - 1999

Smoke and Cancer?
Proabilistic Relationship

Bayesian Networks ~ 1988

Physical Systems have no response prior to excitation

Kramers-Kronig ~ 1920

Clive Granger (~1969)
Nobel Econ. 2003

Consistent Temporal Precedence

Prediction Improvement

Causality

Bayesian

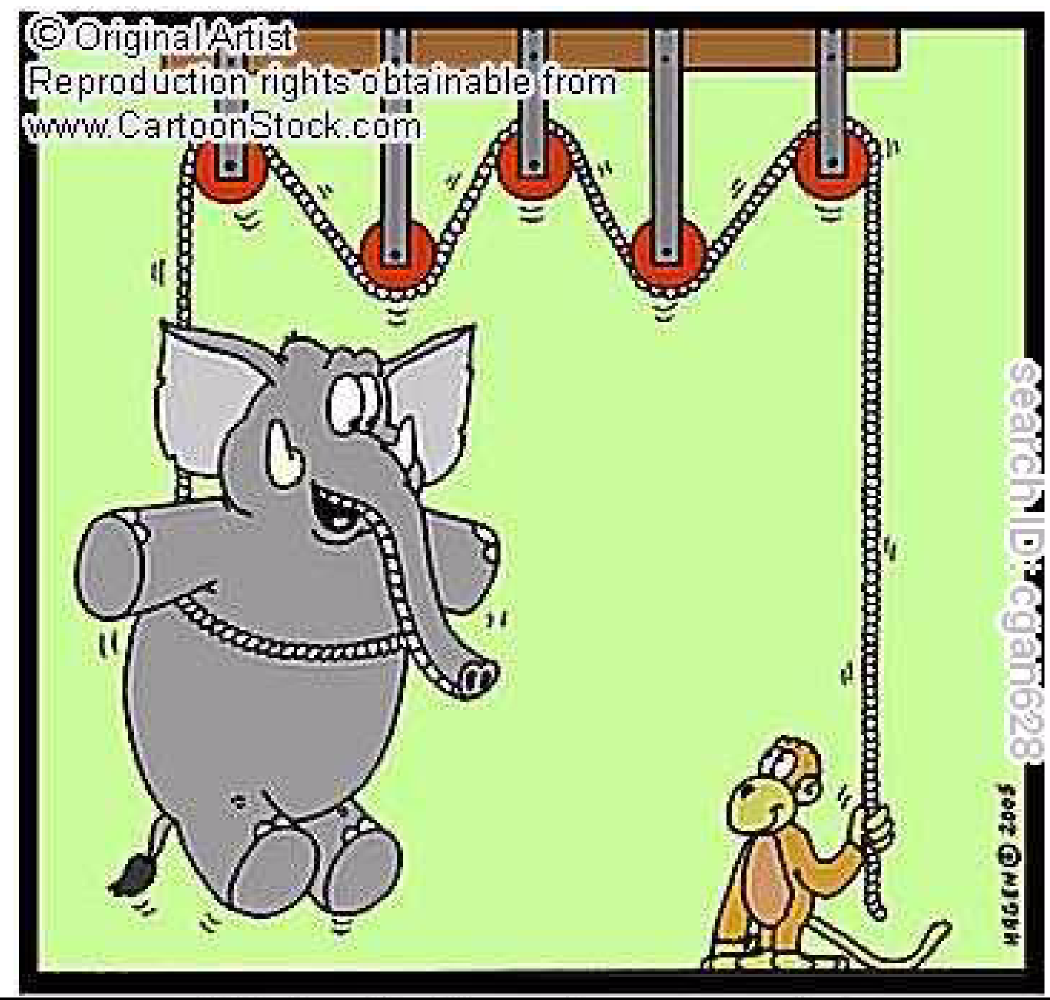
Not necessarily temporal
Many "samples" - population based

Physical

Intervention based

Granger

Observational
Temporal



Alright, alright you´ve won your bet:
You can lift me with one hand.

Causality

Sublata causa, tollitur effectus

Do away with cause, suppress its the effect

requires manipulation by the experimenter

a binary relation - who is the culprit?

Granger Causality

1969 (in econometrics)

Saito & Harashima (1981) (in neuroscience)

- Observational only - non-interventional
- Originally uses bivariate time series analysis
  - based on prediction improvement
- Coupling Directionality

$a_{xy}(k) = 0$ No G-cause condition for all k

$$x(n) = \underbrace{\sum_{k=1}^{p} a_{xx}(k)x(n-k)}_{x\text{'s past}} + \underbrace{\sum_{k=1}^{p} a_{xy}(k)y(n-k)}_{y\text{'s past}} + \underbrace{\varepsilon(n)}_{\text{prediction error}}$$

Consistent Temporal Precedence

2 Time Series

$$a_{xy}(k) = 0$$

means lack of connection

$$x(n) = \underbrace{\sum_{k=1}^{p} a_{xx}(k)x(n-k)}_{\text{past of } x} + \underbrace{\sum_{k=1}^{p} a_{xy}(k)y(n-k)}_{\text{past of } y} + \underbrace{\varepsilon(n)}_{\text{error}}$$

Y is "causal" to X only if its past significantly influences the present of X

(Granger causality)

Granger Causality is <u>unreciprocal</u>

Y is "causal" to X

does not mean

X is "causal" to Y

Unlike correlation which is reciprocal

Two Time Series

Least Squares Adjustment

prediction error

Error Covariance Matrix

$$\begin{bmatrix} x(k) \\ y(k) \end{bmatrix} = \sum_{i=1}^{p} \begin{bmatrix} a_{xx}(i) & a_{xy}(i) \\ a_{yx}(i) & a_{yy}(i) \end{bmatrix} \begin{bmatrix} x(k-i) \\ y(k-i) \end{bmatrix} + \begin{bmatrix} e_x(k) \\ e_y(k) \end{bmatrix}$$

$$\begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{yx} & \sigma_y^2 \end{bmatrix}$$

Data

Granger causality = testing if prediction error reduction for the other time series when both are modelled compared to separate models

$$x(k) = \sum_{i=1}^{p} \beta_x(i)x(k-i) + \varepsilon_x(k)$$

Comparing

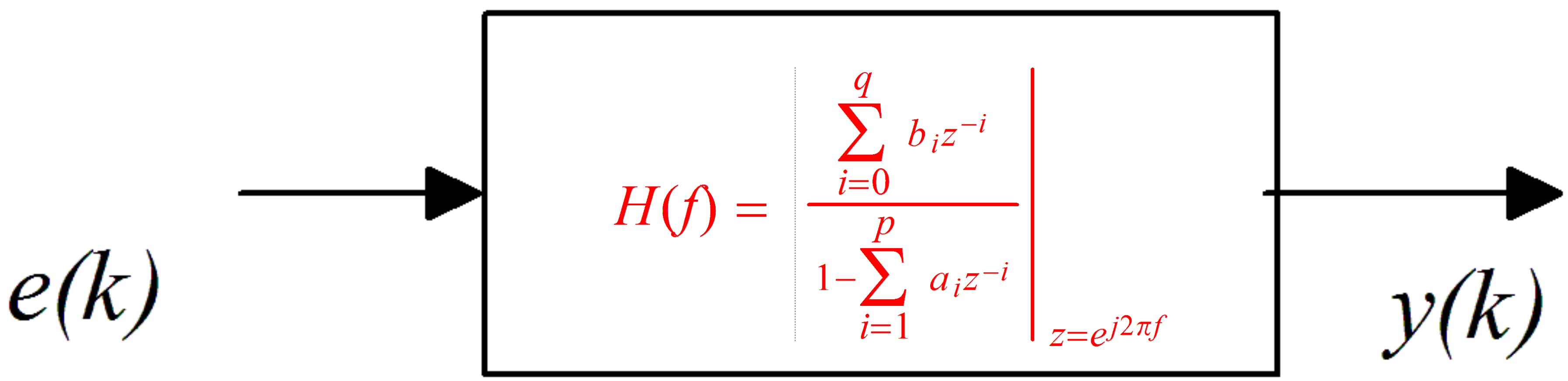
$$VAR(e_x) << VAR(\varepsilon_x) \Leftrightarrow a_{xy}(k) \neq 0$$

Frequency Domain Interpretation

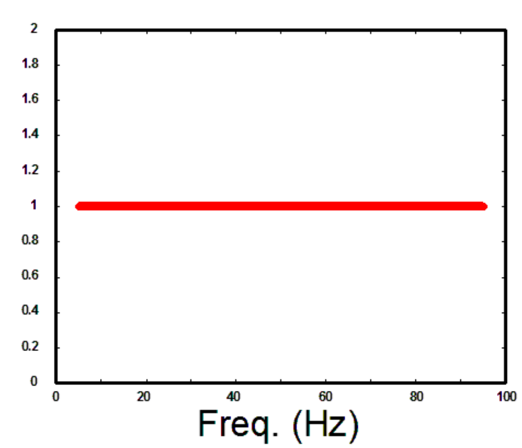
Univariate Parametric Representation

white noise (innovations)

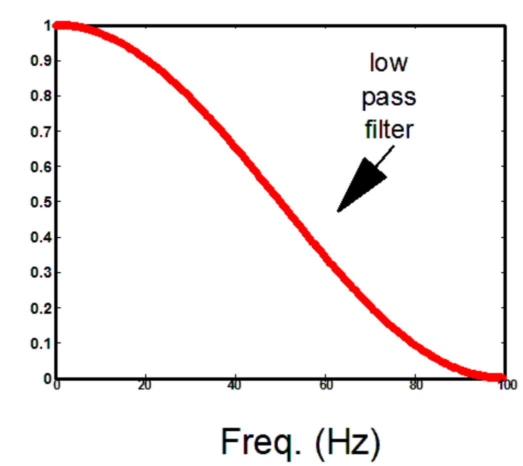
$$y(k) = \underbrace{\sum_{i=1}^{p} a_i y(k-i)}_{} + \underbrace{\sum_{i=0}^{q} b_i e(k-i)}_{}$$



$$H(f) = \left. \frac{\sum_{i=0}^{q} b_i z^{-i}}{1 - \sum_{i=1}^{p} a_i z^{-i}} \right|_{z=e^{j2\pi f}}$$



White Noise



Filtered White Noise

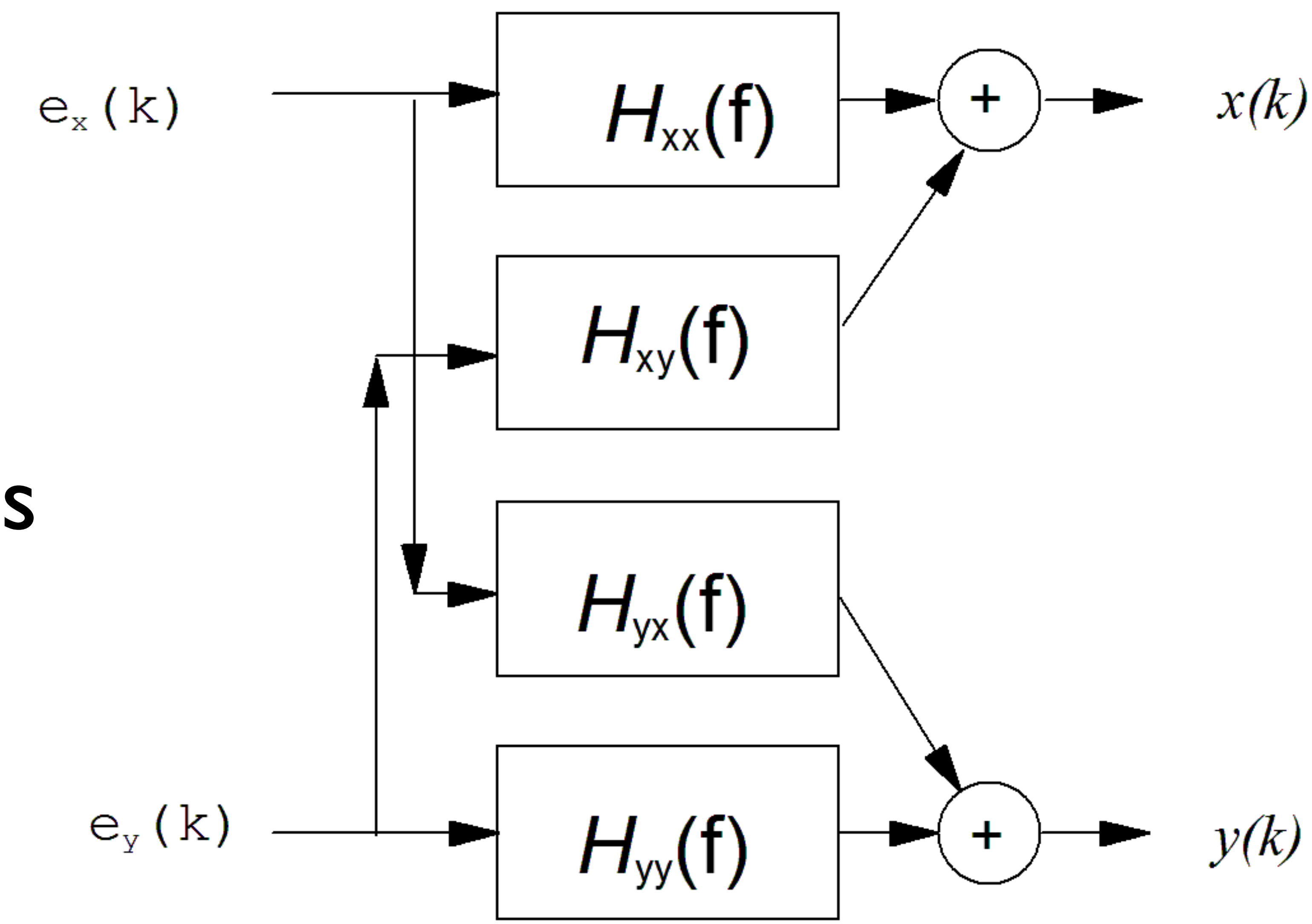
$$S_y(f) = |H(f)|^2 \sigma_e^2$$

Canonical Decomposition

Spectral Density Matrix

$$\begin{bmatrix} S_x(f) & S_{yx}(f) \\ S_{xy}(f) & S_y(f) \end{bmatrix} = \begin{bmatrix} H_{xx}(f) & H_{xy}(f) \\ H_{yx}(f) & H_{yy}(f) \end{bmatrix} \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{yx} & \sigma_y^2 \end{bmatrix} \begin{bmatrix} H_{xx}(f) & H_{xy}(f) \\ H_{yx}(f) & H_{yy}(f) \end{bmatrix}^H$$

Innovations



Spectral Density Matrix Factorization

Least squares fit

$$\begin{bmatrix} x(k) \\ y(k) \end{bmatrix} = \sum_{i=1}^{p} \begin{bmatrix} a_{xx}(i) & a_{xy}(i) \\ a_{yx}(i) & a_{yy}(i) \end{bmatrix} \begin{bmatrix} x(k-i) \\ y(k-i) \end{bmatrix} + \begin{bmatrix} e_x(k) \\ e_y(k) \end{bmatrix}$$

Residue covariance

$$\begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{yx} & \sigma_y^2 \end{bmatrix}$$

Data

$$\begin{bmatrix} H_{xx}(f) & H_{xy}(f) \\ H_{yx}(f) & H_{yy}(f) \end{bmatrix} = \left(\begin{bmatrix} 1 - \sum_{i=1}^{p} a_{xx}(i) e^{-j2\pi f i} & -\sum_{i=1}^{p} a_{xy}(i) e^{-j2\pi f i} \\ -\sum_{i=1}^{p} a_{yx}(i) e^{-j2\pi f i} & 1 - \sum_{i=1}^{p} a_{yy}(i) e^{-j2\pi f i} \end{bmatrix} \right)^{-1}$$

$$|f| < 1/2$$

Parametric Coherence Representation

(special case)

$$\begin{bmatrix} \sigma_x^2 & \sigma_{xy} = 0 \\ \sigma_{yx} = 0 & \sigma_y^2 \end{bmatrix}$$

$S_{xy}(f) \longrightarrow$ **Depends on the interaction in both directions**

$$C_{xy}(f) = \frac{\sigma_x H_{xx}(f) H^*_{yx}(f) + \sigma_{yy} H_{xy}(f) H^*_{yy}(f)}{\sqrt{S_x(f)}\,\sqrt{S_y(f)}}$$

Coerência Direcionada

(Saito & Harashima 1981)

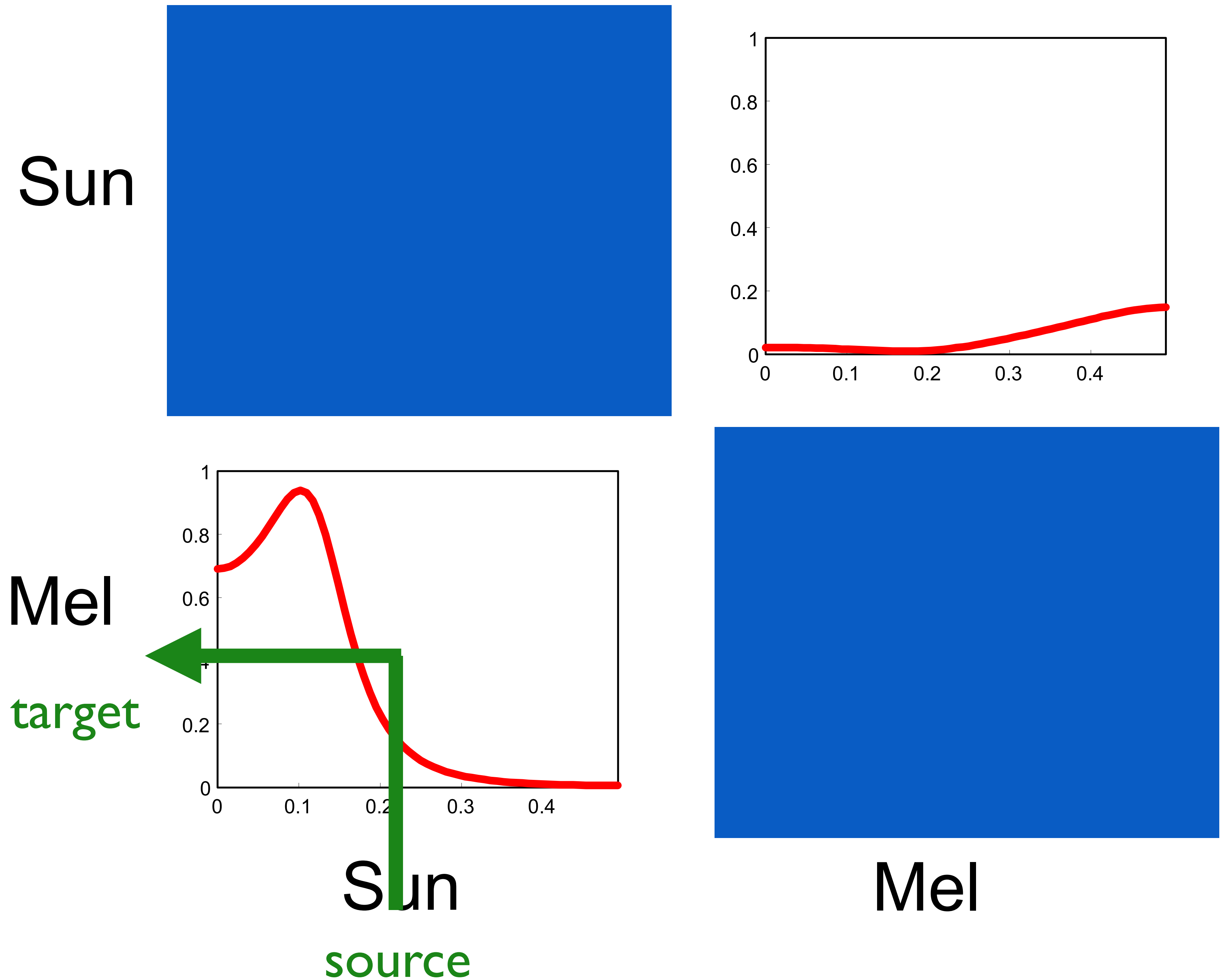
$$\gamma_{xy}(f) = \frac{\sigma_{yy} H_{xy}(f)}{\sqrt{\sigma_{yy}^2 \left|H_{xy}(f)\right|^2 + \sigma_{xx}^2 \left|H_{xx}(f)\right|^2}} = \frac{\sigma_{yy} H_{xy}(f)}{\sqrt{S_x(f)}}$$

Reason for the name: a factor into which coherence can be decomposed

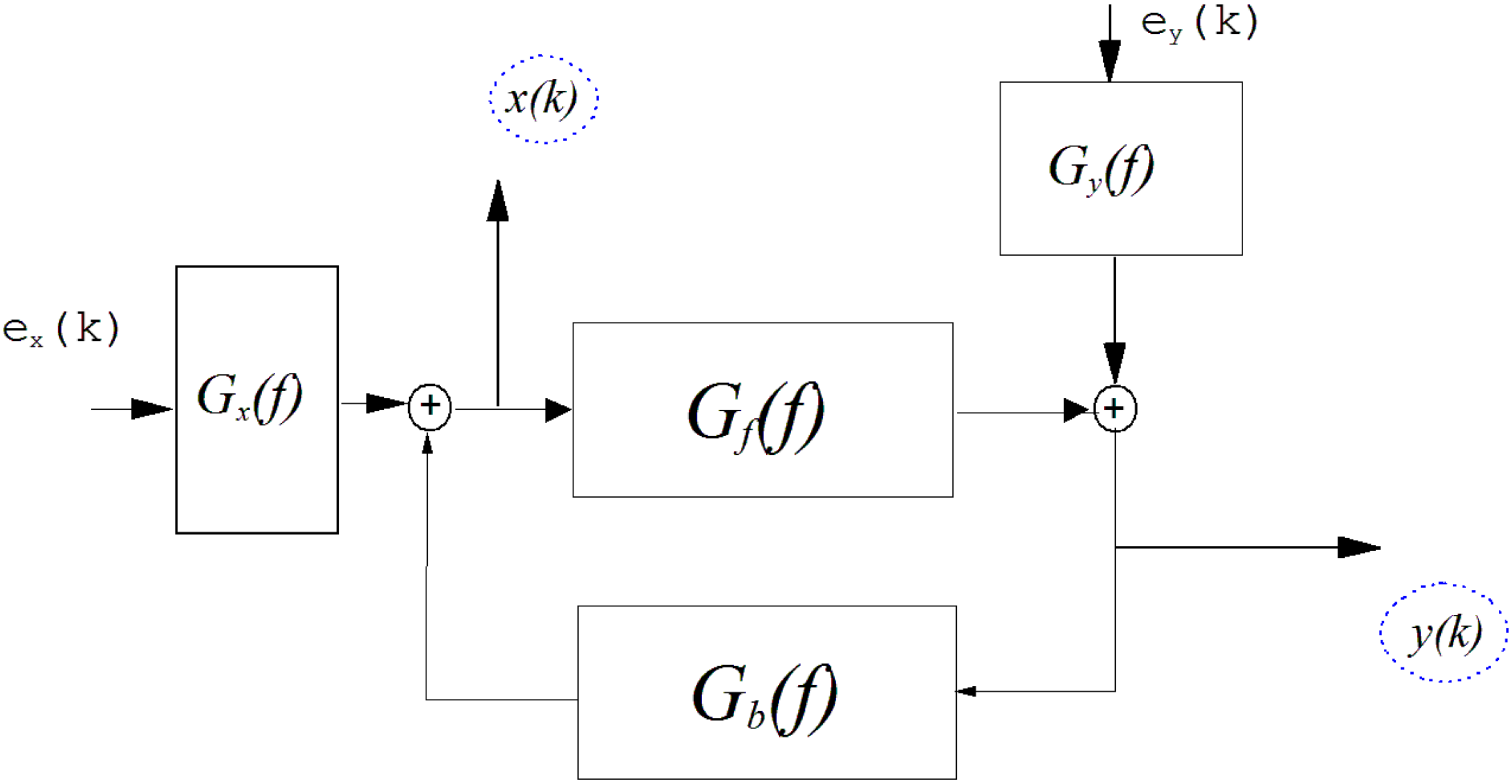
DTF: Directed Transfer Function (Franaczuc et. al 1985)

$$DTF_{xy}(f) = \frac{H_{xy}(f)}{\sqrt{\left|H_{xy}(f)\right|^2 + \left|H_{yy}(f)\right|^2}}$$

Causal Relations using directed coherence $\gamma_{ij}(f)$



Equivalent Model



$$a_{xy}(r) = 0 \Leftrightarrow H_{xy}(f) = 0 \Leftrightarrow G_b(f) = 0$$ (Sims 1982)

Key Technology: Multichannel Neural Signal Recording

Modalities

EEG

LFPs

voxel based

MEG

fMRI

Multi-Single Unit Data

<u>In the Multivariate Case</u>

How does Granger causality generalize?

$N>2$

The same data set has

2 <u>different</u>

possible representations

Time Series Representations

Autoregressive (AR) $\xrightarrow{\text{frequency domain}}$ PDC

$$\begin{bmatrix} x_1(k) \\ \vdots \\ x_N(k) \end{bmatrix} = \sum_{r=1}^{+\infty} \begin{bmatrix} a_{11}(r) & a_{12}(r) & \dots & \dots & a_{1N}(r) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & a_{ij}(r) & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{N1}(r) & \dots & \dots & \dots & a_{NN}(r) \end{bmatrix} \begin{bmatrix} x_1(k-r) \\ \vdots \\ x_N(k-r) \end{bmatrix} + \begin{bmatrix} w_1(k) \\ \vdots \\ w_N(k) \end{bmatrix}$$

Moving Average (MA) $\xrightarrow{\text{frequency domain}}$ DTF

$$\begin{bmatrix} x_1(k) \\ \vdots \\ x_N(k) \end{bmatrix} = \sum_{r=0}^{+\infty} \begin{bmatrix} h_{11}(r) & h_{12}(r) & \dots & \dots & h_{1N}(r) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & h_{ij}(r) & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{N1}(r) & \dots & \dots & \dots & h_{NN}(r) \end{bmatrix} \begin{bmatrix} w_1(k) \\ \vdots \\ w_N(k) \end{bmatrix}$$

Representation One

Moving Average (MA) $\xrightarrow{\text{frequency domain}}$ DTF

$$\begin{bmatrix} x_1(k) \\ \vdots \\ x_N(k) \end{bmatrix} = \sum_{r=0}^{+\infty} \begin{bmatrix} h_{11}(r) & h_{12}(r) & \dots & \dots & h_{1N}(r) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & h_{ij}(r) & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{N1}(r) & \dots & \dots & \dots & h_{NN}(r) \end{bmatrix} \begin{bmatrix} w_1(k) \\ \vdots \\ w_N(k) \end{bmatrix}$$

Alternative to the multivariate Case

Spectral Density Matrix for more than 2 time series

Spectral Density Matrix

$$S(f) = \boldsymbol{H}(f)\Sigma\boldsymbol{H}^H(f)$$

$$\boldsymbol{H}(f) = \begin{bmatrix} H_{11}(f) & H_{12}(f) & \dots & H_{1N}(f) \\ H_{21}(f) & H_{22}(f) & \dots & H_{2N}(f) \\ \vdots & \vdots & \ddots & \vdots \\ H_{N1}(f) & H_{N2}(f) & \dots & H_{NN}(f) \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12} & \dots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22}^2 & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \dots & \dots & \sigma_{NN}^2 \end{bmatrix}$$

Directed Transfer Function (Kaminski & Blinowska, 1991)

$$DTF_{ij}(f) = \frac{H_{ij}(f)}{\sqrt{\sum_{j=1}^{N} |H_{ij}(f)|^2}}$$

Directed Coherence (Baccala et al, 1998)

$$\gamma_{ij}(f) = \frac{\sigma_{jj} H_{ij}(f)}{\sqrt{\sum_{j=1}^{N} \sigma_{jj}^2 |H_{ij}(f)|^2}}$$

Multivariate Autoregressive Time Series Models

$$\begin{bmatrix} x_1(k) \\ \vdots \\ x_N(k) \end{bmatrix} = \sum_{r=1}^{p} \begin{bmatrix} a_{11}(r) & a_{12}(r) & \cdots & \cdots & a_{1N}(r) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & a_{ij}(r) & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{N1}(r) & \cdots & \cdots & \cdots & a_{NN}(r) \end{bmatrix} \begin{bmatrix} x_1(k-r) \\ \vdots \\ x_N(k-r) \end{bmatrix}$$

$$\bar{A}_{ij}(f) = \begin{cases} 1 - \sum_{r=1}^{p} a_{ij}(r) e^{-j2\pi fr}, & \text{if } i = j \\ -\sum_{r=1}^{p} a_{ij}(r) e^{-j2\pi fr}, & \text{otherwise} \end{cases}$$

Obtain by model fitting

$$\boldsymbol{H}(f) = \begin{bmatrix} H_{11}(f) & H_{12}(f) & \dots & H_{1N}(f) \\ H_{21}(f) & H_{22}(f) & \dots & H_{2N}(f) \\ \vdots & \vdots & \ddots & \vdots \\ H_{N1}(f) & H_{N2}(f) & \dots & H_{NN}(f) \end{bmatrix}$$

Inversion Duality

$$\longleftrightarrow$$

$$A(f) = \begin{bmatrix} A_{11}(f) & A_{12}(f) & \dots & A_{1N}(f) \\ A_{21}(f) & A_{22}(f) & \dots & A_{2N}(f) \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1}(f) & A_{N2}(f) & \dots & A_{NN}(f) \end{bmatrix}$$

$$\boldsymbol{H}(f) = \left[\bar{\boldsymbol{A}}(f)\right]^{-1}$$

Power Spectral Density Matrix

$$\boldsymbol{S}(f) = \boldsymbol{H}(f)\boldsymbol{\Sigma}\boldsymbol{H}^{H}(f)$$

$$\boldsymbol{H}(f) = \begin{bmatrix} H_{11}(f) & H_{12}(f) & \dots & H_{1N}(f) \\ H_{21}(f) & H_{22}(f) & \dots & H_{2N}(f) \\ \vdots & \vdots & \ddots & \vdots \\ H_{N1}(f) & H_{N2}(f) & \dots & H_{NN}(f) \end{bmatrix}$$

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12} & \dots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22}^2 & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \dots & \dots & \sigma_{NN}^2 \end{bmatrix}$$

Representation Two

Autoregressive (AR) $\xrightarrow{\text{frequency domain}}$ PDC

$$\begin{bmatrix} x_1(k) \\ \vdots \\ x_N(k) \end{bmatrix} = \sum_{r=1}^{+\infty} \begin{bmatrix} a_{11}(r) & a_{12}(r) & \dots & \dots & a_{1N}(r) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & a_{ij}(r) & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{N1}(r) & \dots & \dots & \dots & a_{NN}(r) \end{bmatrix} \begin{bmatrix} x_1(k-r) \\ \vdots \\ x_N(k-r) \end{bmatrix} + \begin{bmatrix} w_1(k) \\ \vdots \\ w_N(k) \end{bmatrix}$$

Partial Coherence

$$\left|\kappa_{ij}(f)\right|^2 = \frac{\left|\boldsymbol{a}_i^H(f)\,\Sigma^{-1}\,\boldsymbol{a}_j(f)\right|^2}{(\boldsymbol{a}_i^H(f)\,\Sigma^{-1}\,\boldsymbol{a}_i(f))(\boldsymbol{a}_j^H(f)\,\Sigma^{-1}\,\boldsymbol{a}_j(f))}$$

matrix columns

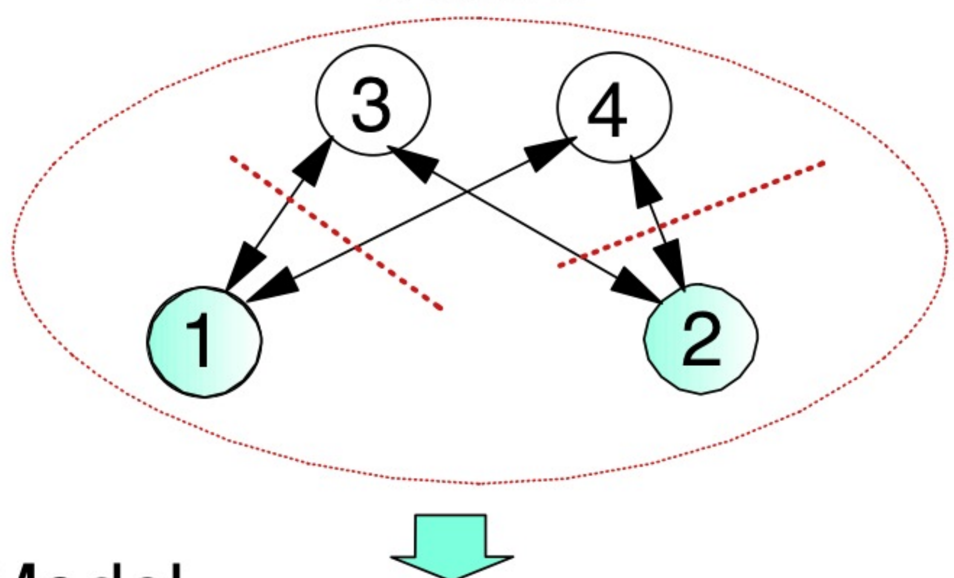
$$\boldsymbol{a}_i(f) = \left[\bar{A}_{ij}(f)\right]$$

$$\bar{A}_{ij}(f) = \begin{cases} 1 - \sum_{r=1}^{p} a_{ij}(r)e^{-j2\pi fr}, & \text{if } i = j \\ -\sum_{r=1}^{p} a_{ij}(r)e^{-j2\pi fr}, & \text{otherwise} \end{cases}$$

can be directly computed from the multivariate autoregressive model (Baccalá 2001)

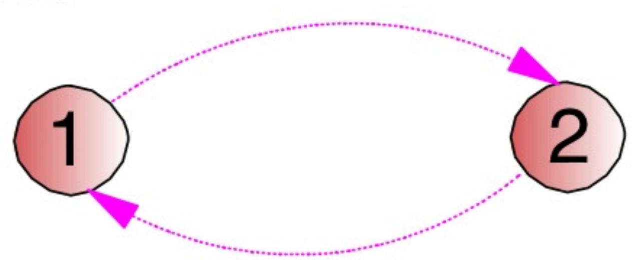
Partial Coherence

Coherence between the resulting series after optimal subtraction of the effect of other N-2 series



Extraction of the effect between the other series

Residual Model



$$\begin{bmatrix} x_1(k) \\ \vdots \\ x_N(k) \end{bmatrix} = \sum_{r=1}^{p} \begin{bmatrix} a_{11}(r) & a_{12}(r) & \dots & \dots & a_{1N}(r) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & a_{ij}(r) & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{N1}(r) & \dots & \dots & \dots & a_{NN}(r) \end{bmatrix} \begin{bmatrix} x_1(k-r) \\ \vdots \\ x_N(k-r) \end{bmatrix} + \begin{bmatrix} w_1(k) \\ \vdots \\ w_N(k) \end{bmatrix}$$

Result:

Given a multivariate autoregressive of the data, the pairwise partial coherence may be written using

$$\bar{A}_{ij}(f) = \begin{cases} 1 - \sum_{r=1}^{p} a_{ij}(r) e^{-j2\pi fr}, & \text{if } i = j \\ - \sum_{r=1}^{p} a_{ij}(r) e^{-j2\pi fr}, & \text{otherwise} \end{cases}$$

PDC

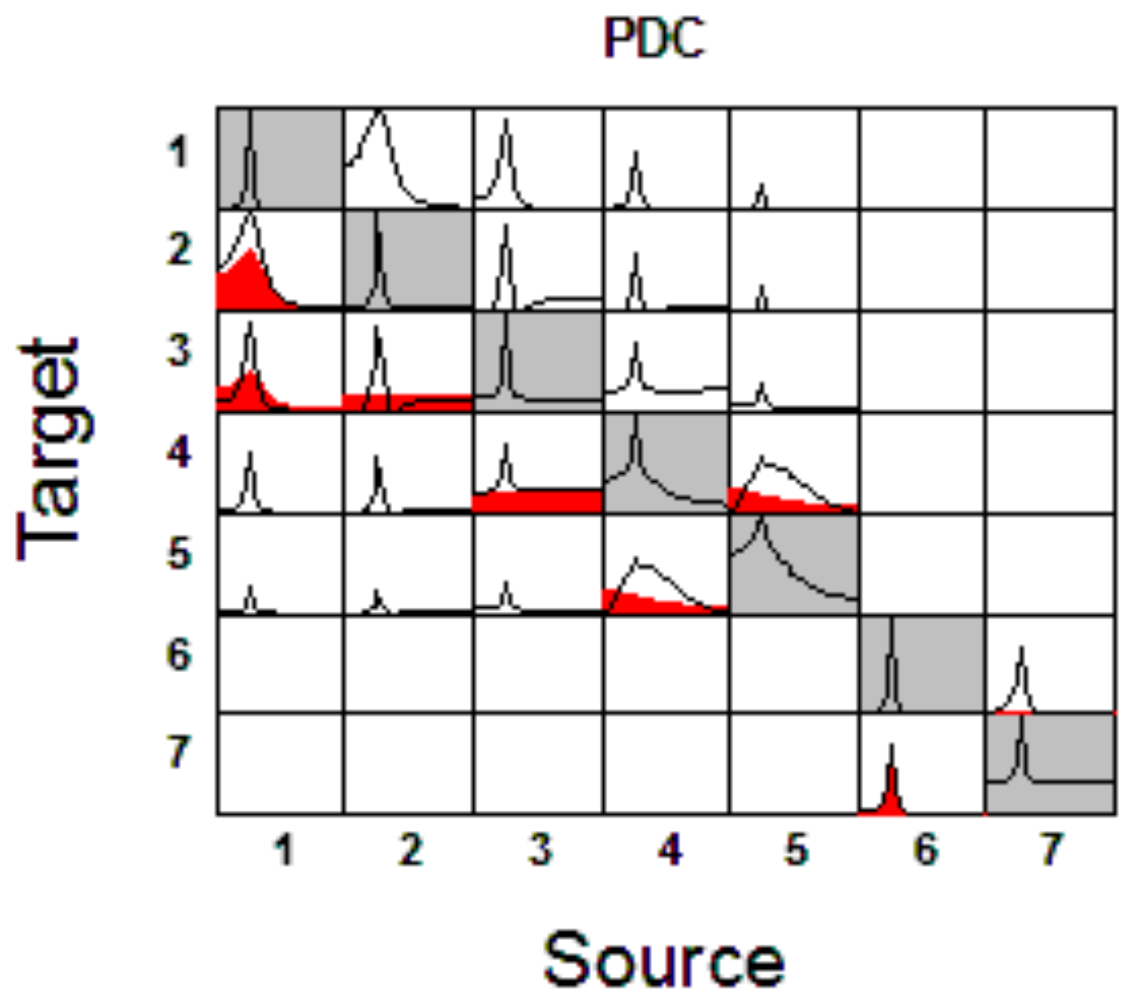
(Partial Directed Coherence - PDC)

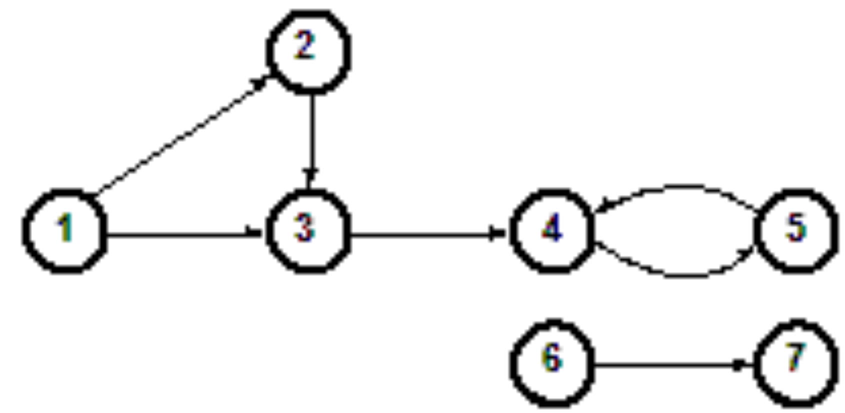
PDC from j to i

$$\boxed{\pi_{ij}(f) \triangleq \frac{\bar{A}_{ij}(f)}{\sqrt{\sum_{k=1}^{N} \bar{A}^{*}_{kj}(f)\bar{A}_{kj}(f)}}}$$

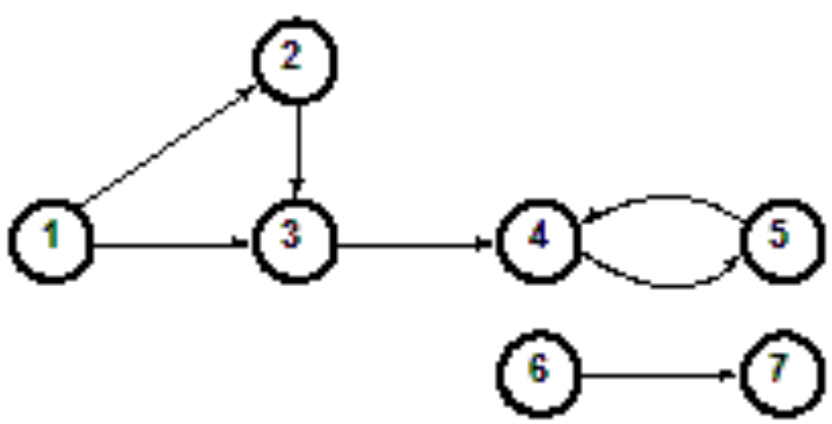
- derived from factoring partial coherence

Partial Directed Coherence (PDC)

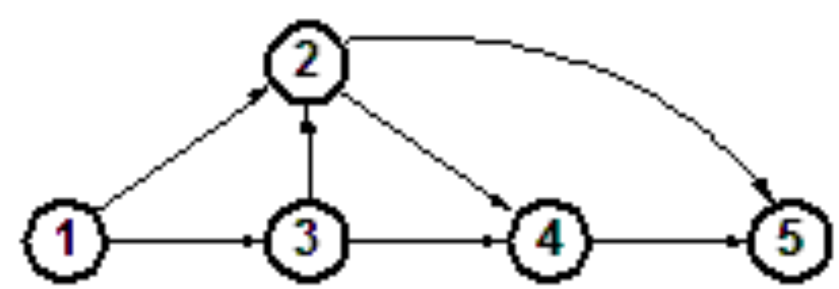




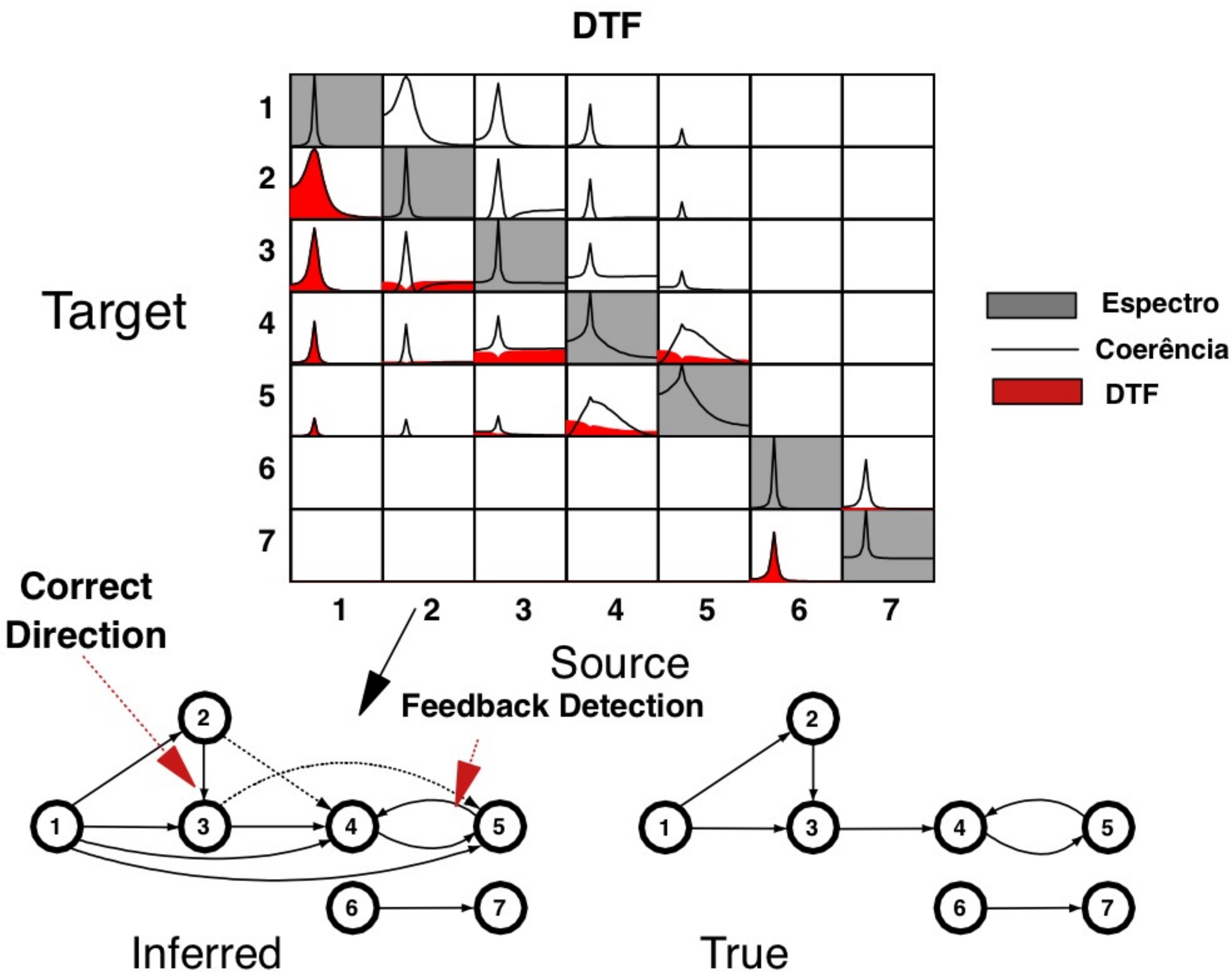
Deduced



True



Correlation Based Picture



DTF
Target
Source
1
2
3
4
5
6
7
Espectro
Coerência
DTF
Correct Direction
Feedback Detection
Inferred
True

$$DTF_{ij}(f) = \frac{H_{ij}(f)}{\sqrt{\sum_{j=1}^{K} \left| H_{ij}(f) \right|^2}}$$

Directed Transfer Function (Kaminski & Blinowska, 1991)

$$\gamma_{ij}(f) = \frac{\sigma_{jj} H_{ij}(f)}{\sqrt{\sum_{j=1}^{K} \sigma_{jj}^2 \left| H_{ij}(f) \right|^2}}$$

Directed Coherence
(Baccala et al, 1998)

Information DTF
(Takahashi et al, 2010)

$$\longrightarrow \quad \imath DTF_{ij}(\omega) = \frac{\sigma_{jj} H(\omega)}{\sqrt{h_j^H(\omega)\Sigma_w h_j(\omega)}}$$

$$\pi_{ij}(f) \triangleq \frac{\bar{A}_{ij}(f)}{\sqrt{\sum_{k=1}^{N} \bar{A}^{*}_{kj}(f)\bar{A}_{kj}(f)}}$$

PDC 2001

$$_{g}\pi_{ij}(f) \triangleq \frac{\frac{1}{\sigma_{ii}}\bar{A}_{ij}(f)}{\sqrt{\sum_{k=1}^{K} \frac{1}{\sigma^{2}_{kk}}\bar{A}^{*}_{kj}(f)\bar{A}_{kj}(f)}}$$

gPDC 2007

Information PDC 2011 $\longrightarrow$ $$\iota\pi_{ij}(\omega) = \frac{\sigma_{ii}^{-1}\bar{A}_{ij}(\omega)}{\sqrt{a_j^H(\omega)\Sigma_w^{-1}a_j(\omega)}}$$

iPDC

Multivariate Spectral Time Series

$$S(f) = \boldsymbol{H}(f)\boldsymbol{\Sigma}\boldsymbol{H}^H(f)$$

$$\boldsymbol{H}(f) = \begin{bmatrix} H_{11}(f) & H_{12}(f) & \dots & H_{1N}(f) \\ H_{21}(f) & H_{22}(f) & \dots & H_{2N}(f) \\ \vdots & \vdots & \ddots & \vdots \\ H_{N1}(f) & H_{N2}(f) & \dots & H_{NN}(f) \end{bmatrix} \qquad \boldsymbol{\Sigma} = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12} & \dots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22}^2 & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \dots & \dots & \sigma_{NN}^2 \end{bmatrix}$$

- Coherence (Counterpart to Correlation in Time)
  - Pairwise case
- Factorization
  - DTF
  - PDC

Apply to More than 2 time series

What do they reveal?

- Identical for time series pairs (*N=2*)
  DTF/MA=PDC/AR=Granger Causality
- Expose **distinct** interaction aspects when *N>2*

DTF=PDC

=

Granger Causality

$N=2$ time series

Illustration of the Differences

DTF/MA

PDC/AR

Information

$$ {}_{\iota}\gamma_{ij}(\omega) = \frac{\sigma_{jj} H_{ij}(\omega)}{\sqrt{h_i^H(\omega)\Sigma_w h_i(\omega)}} \qquad \iota\pi_{ij}(\omega) = \frac{\sigma_{ii}^{-1} \bar{A}_{ij}(\omega)}{\sqrt{a_j^H(\omega)\Sigma_w^{-1} a_j(\omega)}} $$

(Takahashi et al, 2010)

The Connectivity Problem

Detection

Characterization

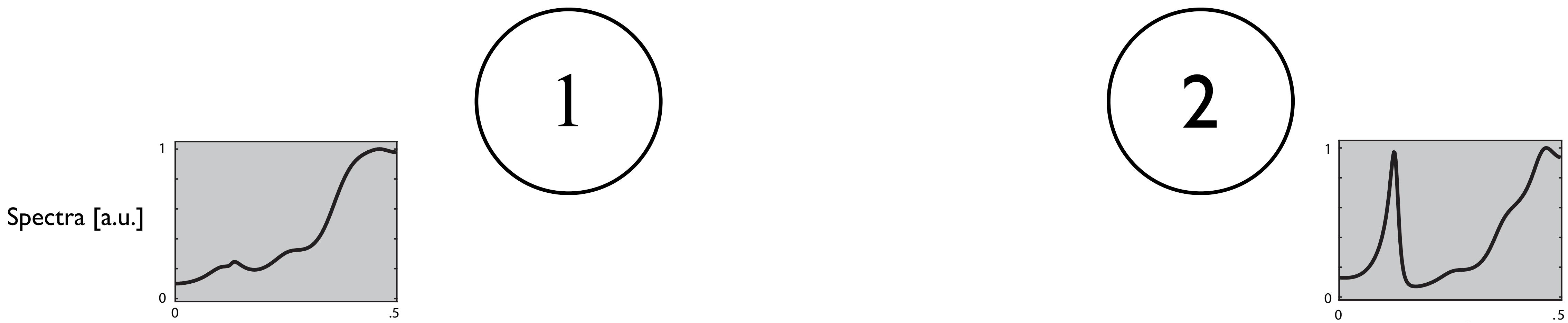
Prevalent Paradigm

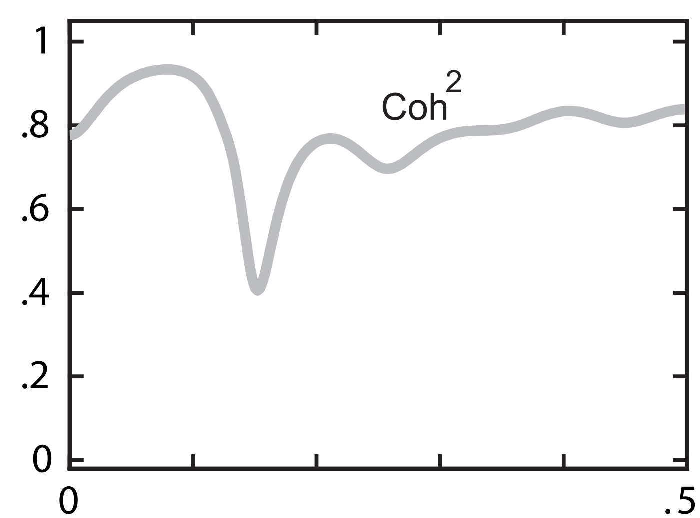
Amount of Correlation/Coherence

Toy Example 1

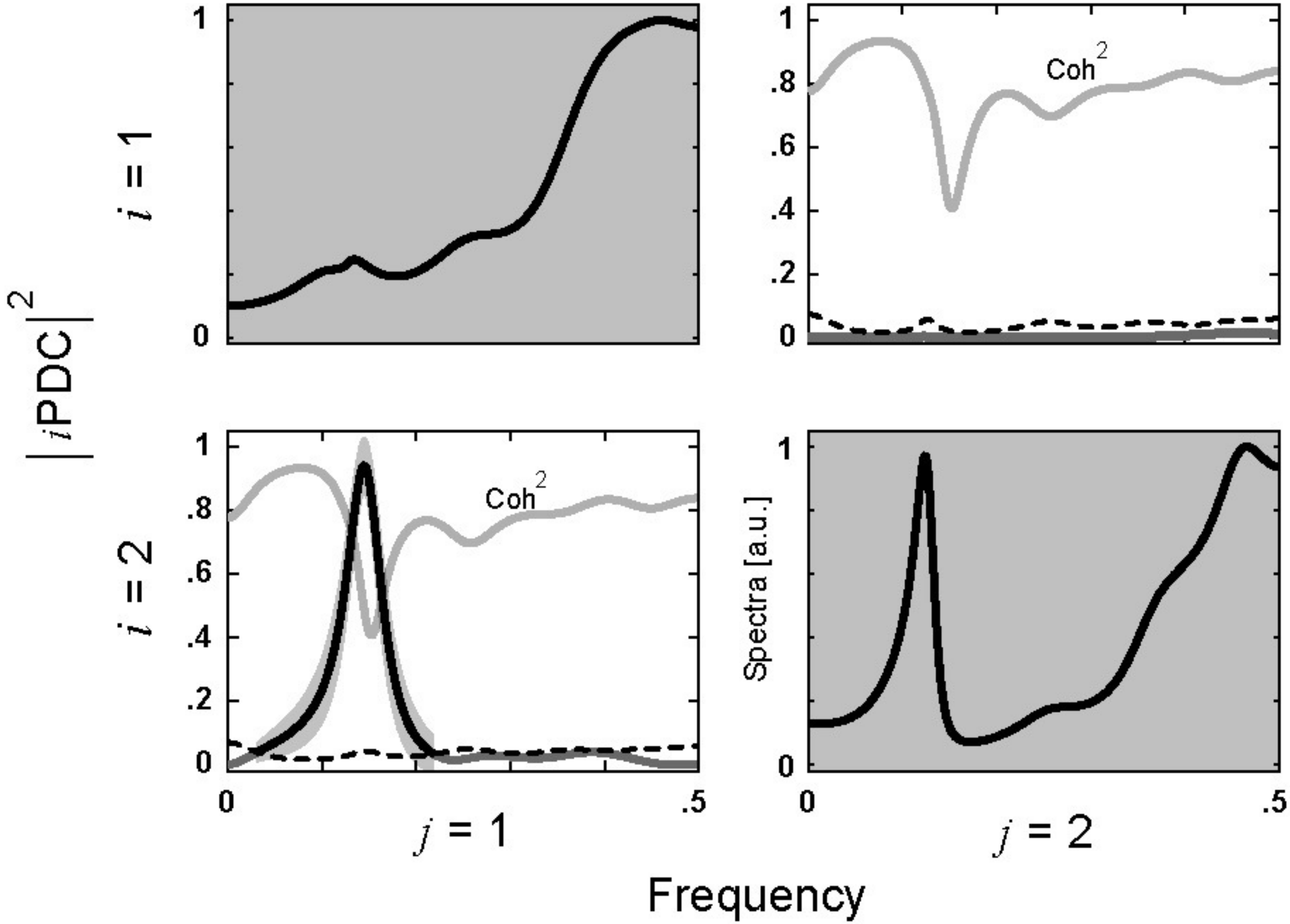
DTF=PDC=Granger Causality

2 time series

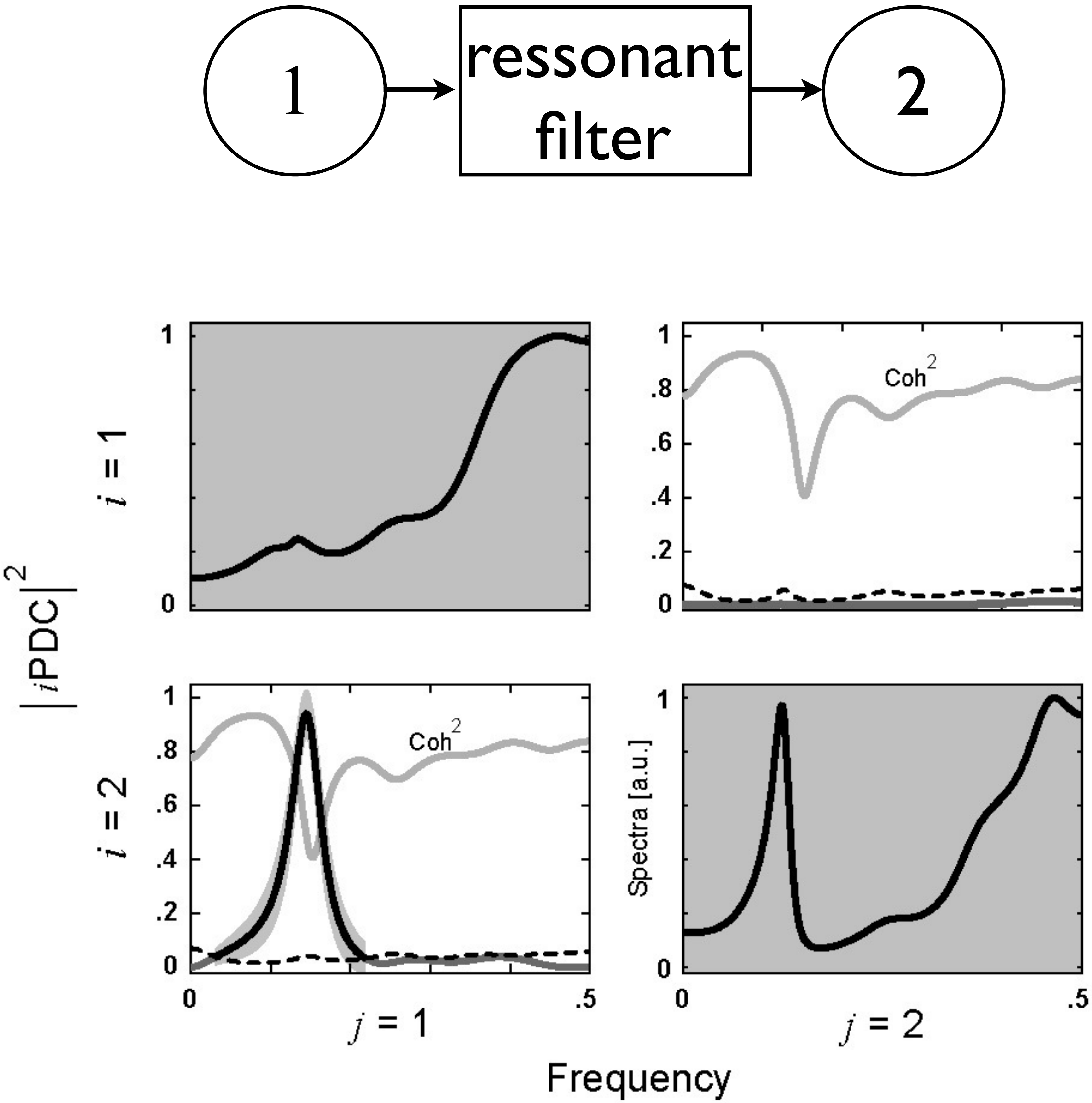




Normalized Freq. (x-axis)



DTF=PDC
(Granger Causality)

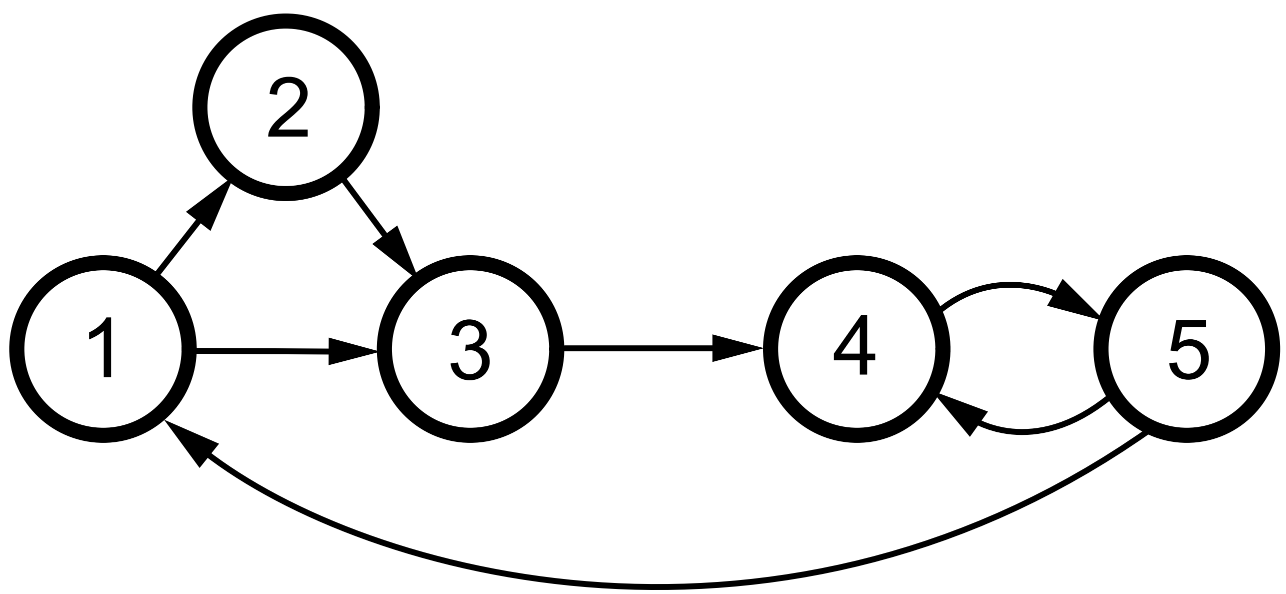


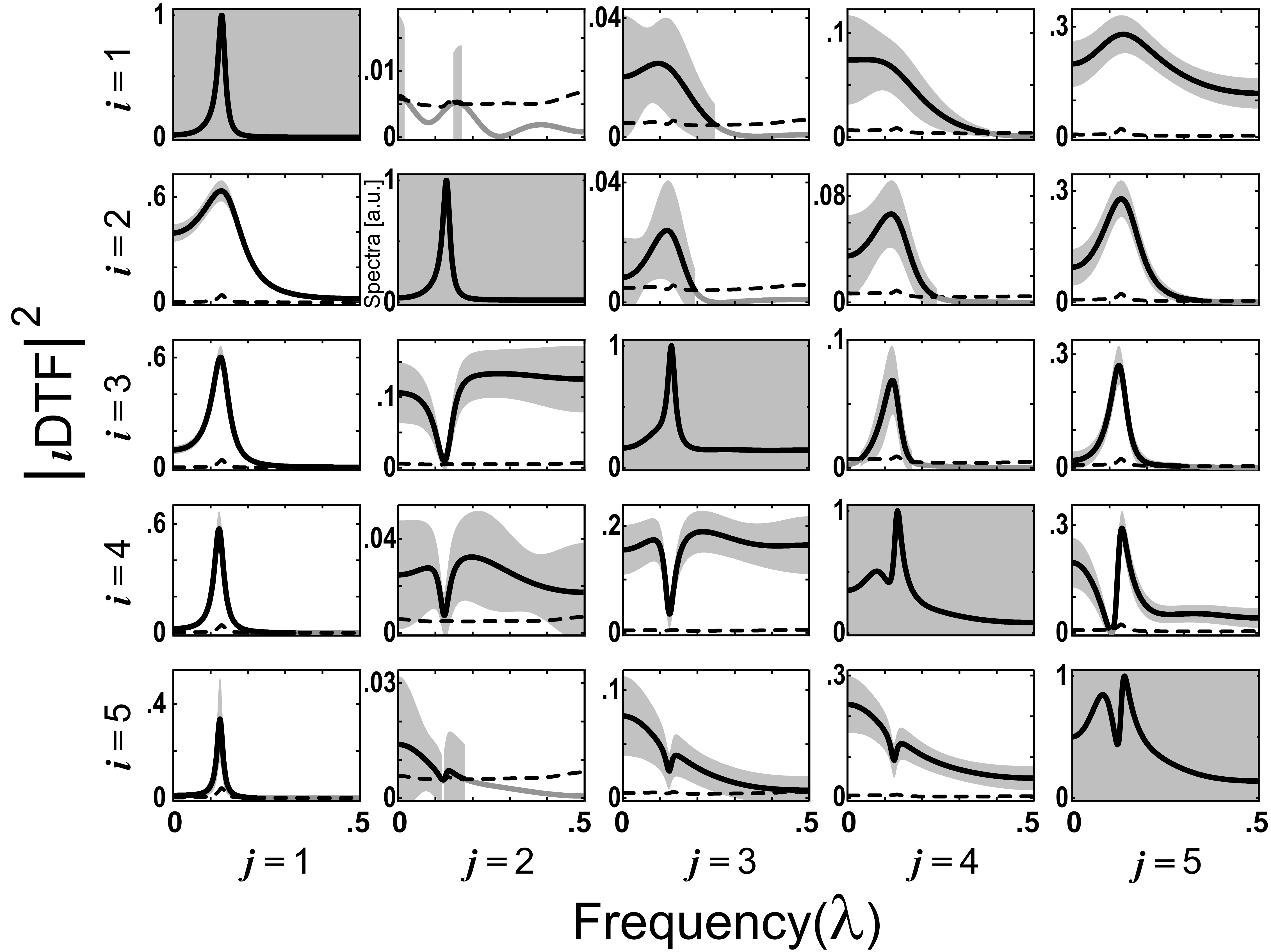
1
ressonant filter
2
Coh²
Coh²
1
.8
.6
.4
.2
0
i = 1
i = 2
|iPDC|²
Spectra [a.u.]
0
.5
j = 1
j = 2
Frequency

Toy Example II

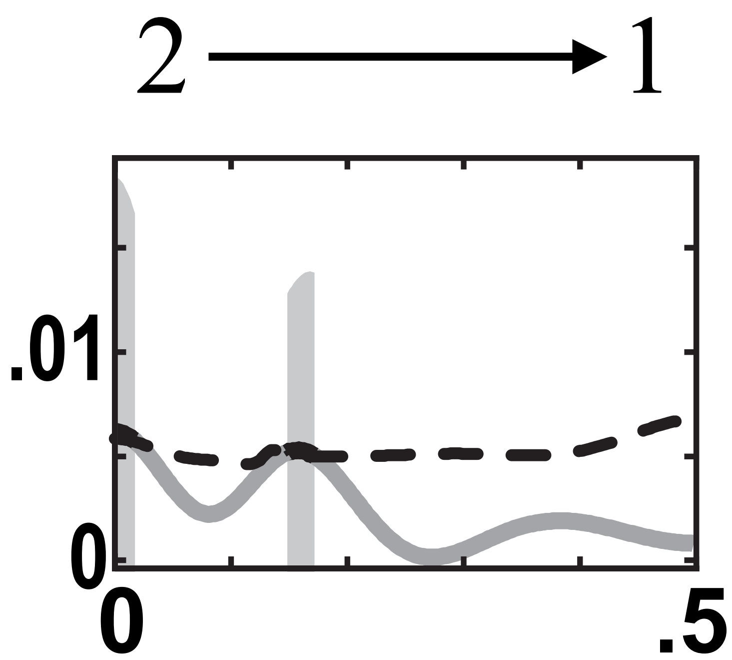
DTF$\neq$PDC

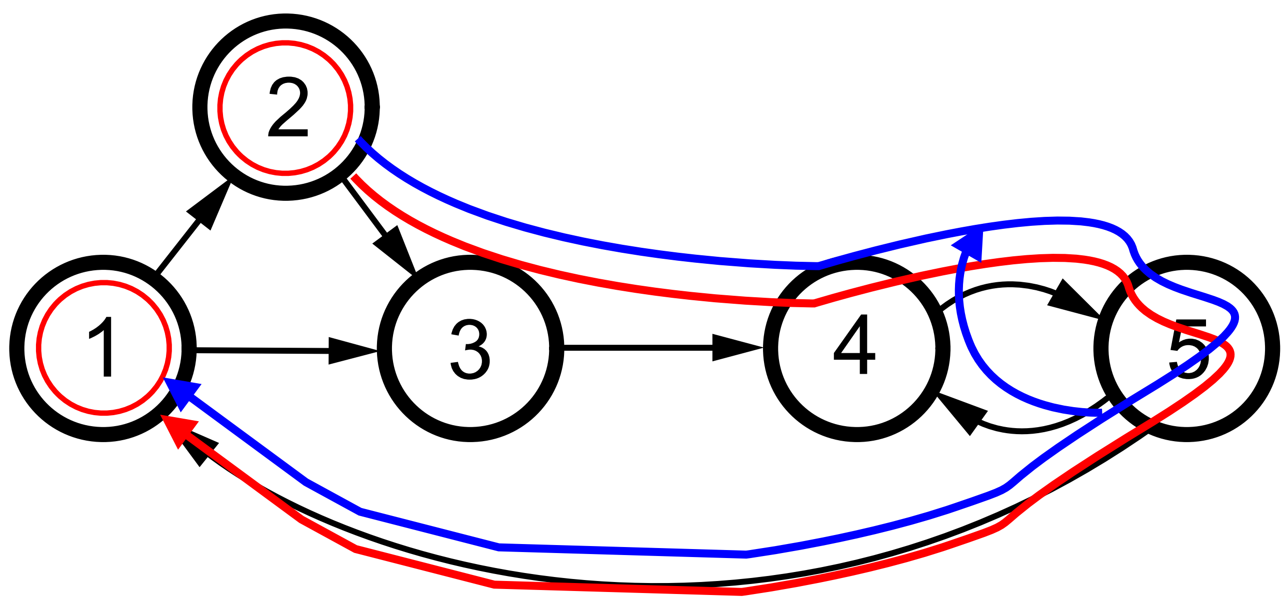
(More than 2 time series)

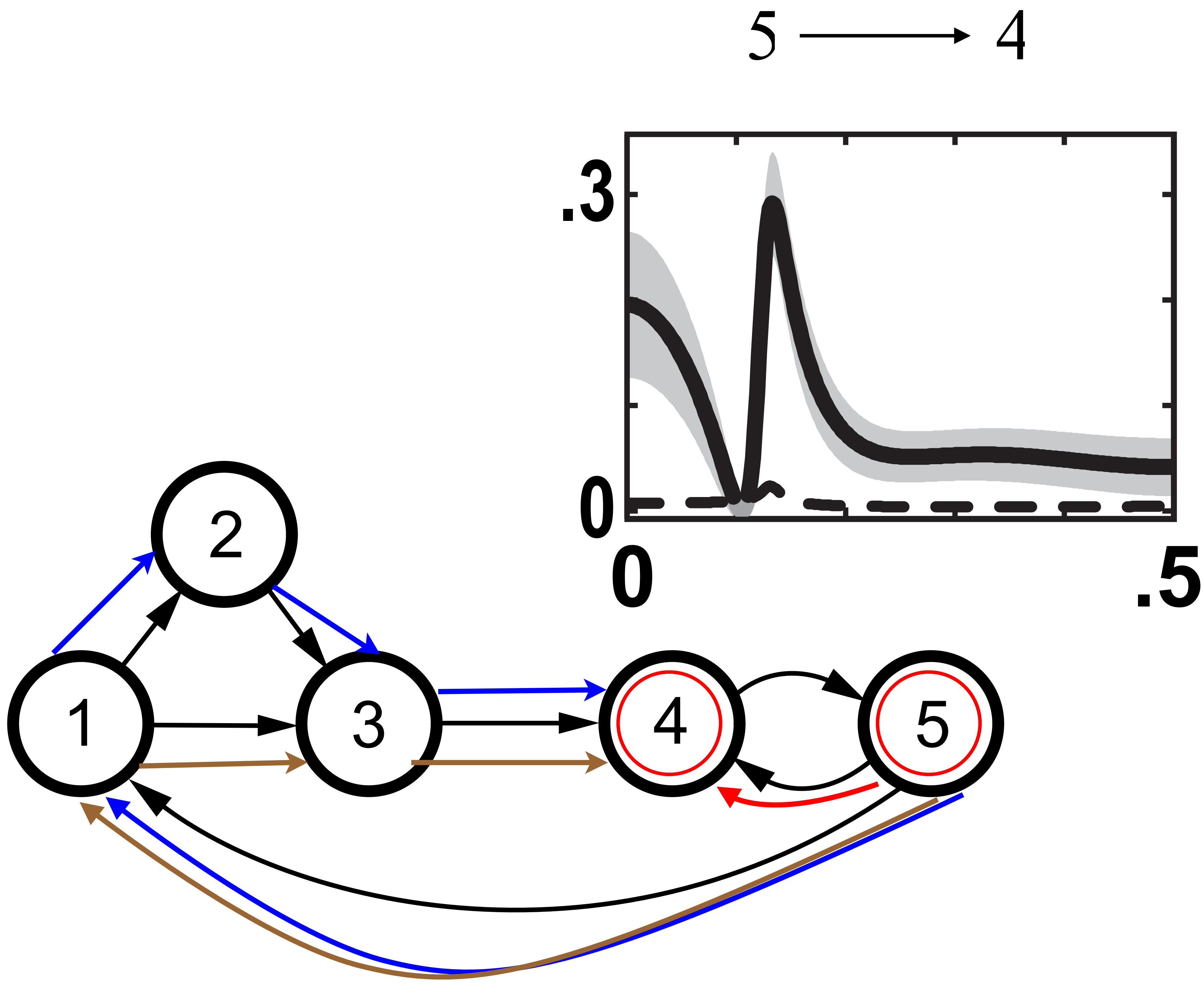




$|{}_\iota\mathrm{DTF}|^2$
$i=1$
$i=2$
$i=3$
$i=4$
$i=5$
$j=1$
$j=2$
$j=3$
$j=4$
$j=5$
Spectra [a.u.]
Frequency(λ)


2
1
.01
0
0
.5


2
1
3
4
5

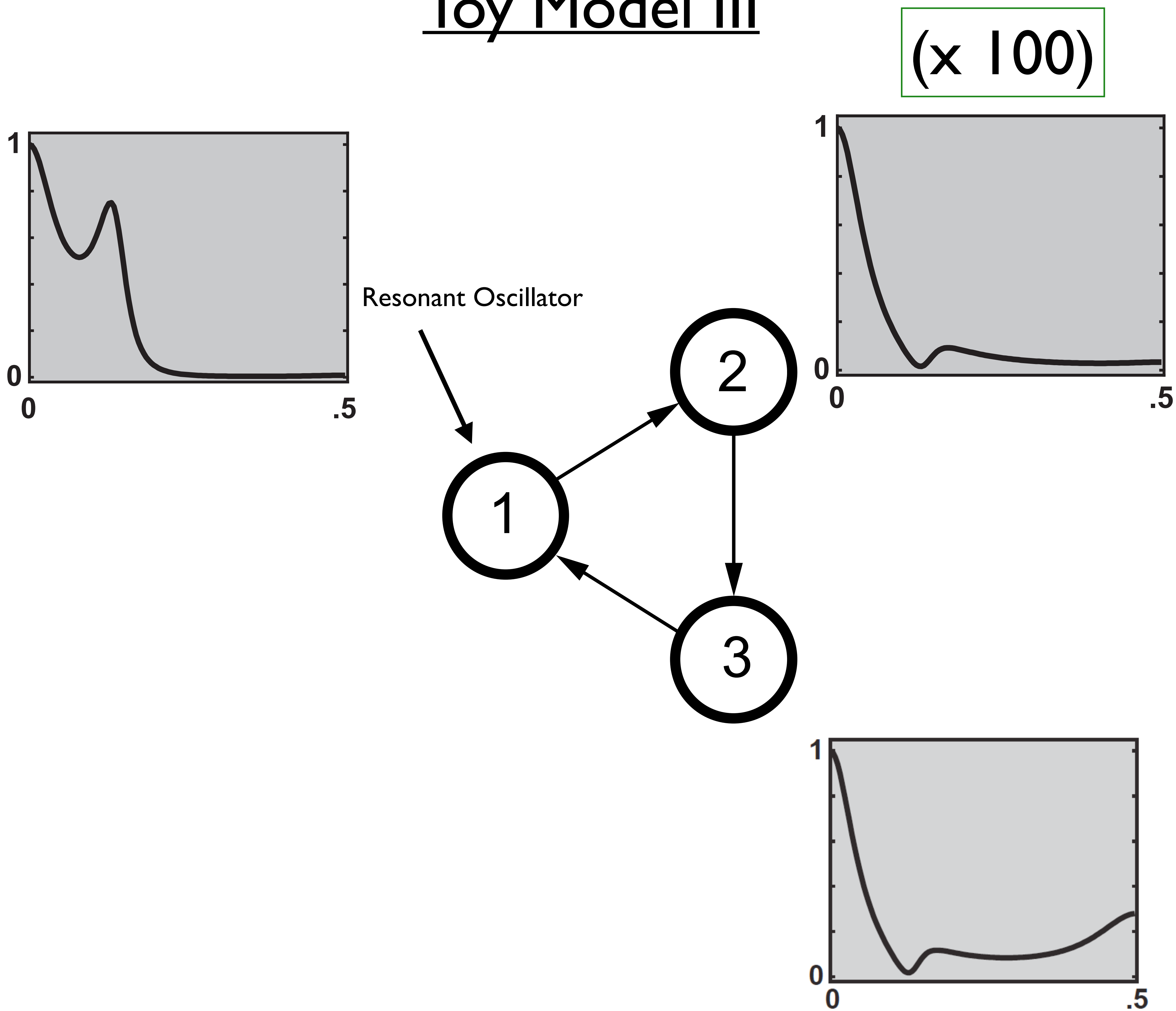

5 → 4
.3
0
0
.5
1
2
3
4
5

DTF

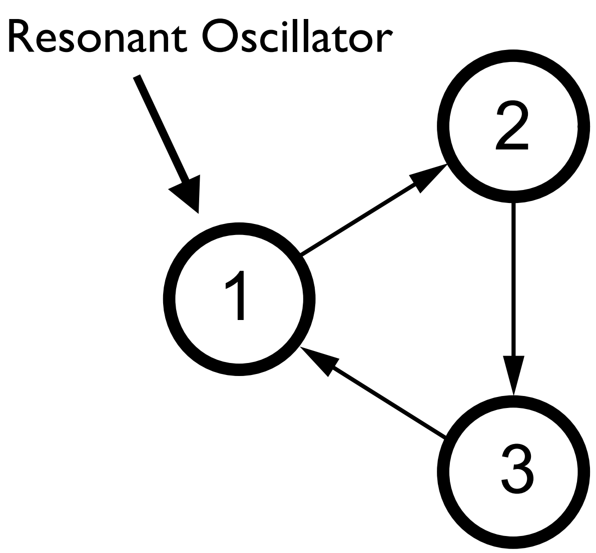
=

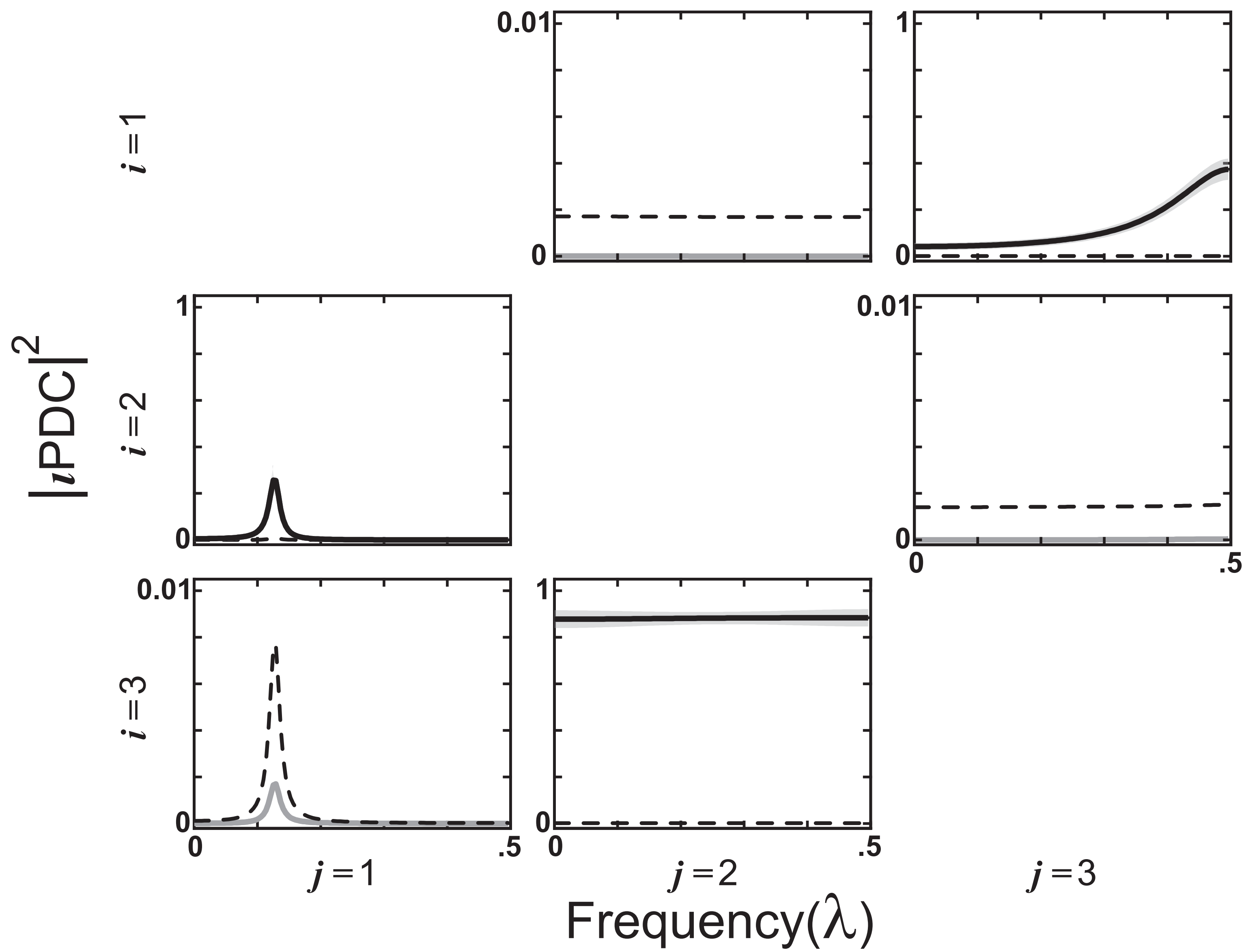
Sum-up of Influences Direct and Indirect

Toy Model III

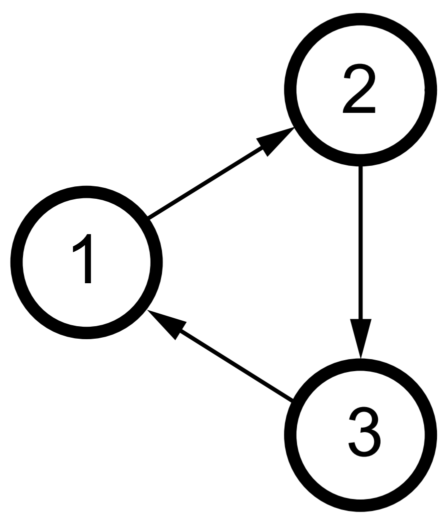


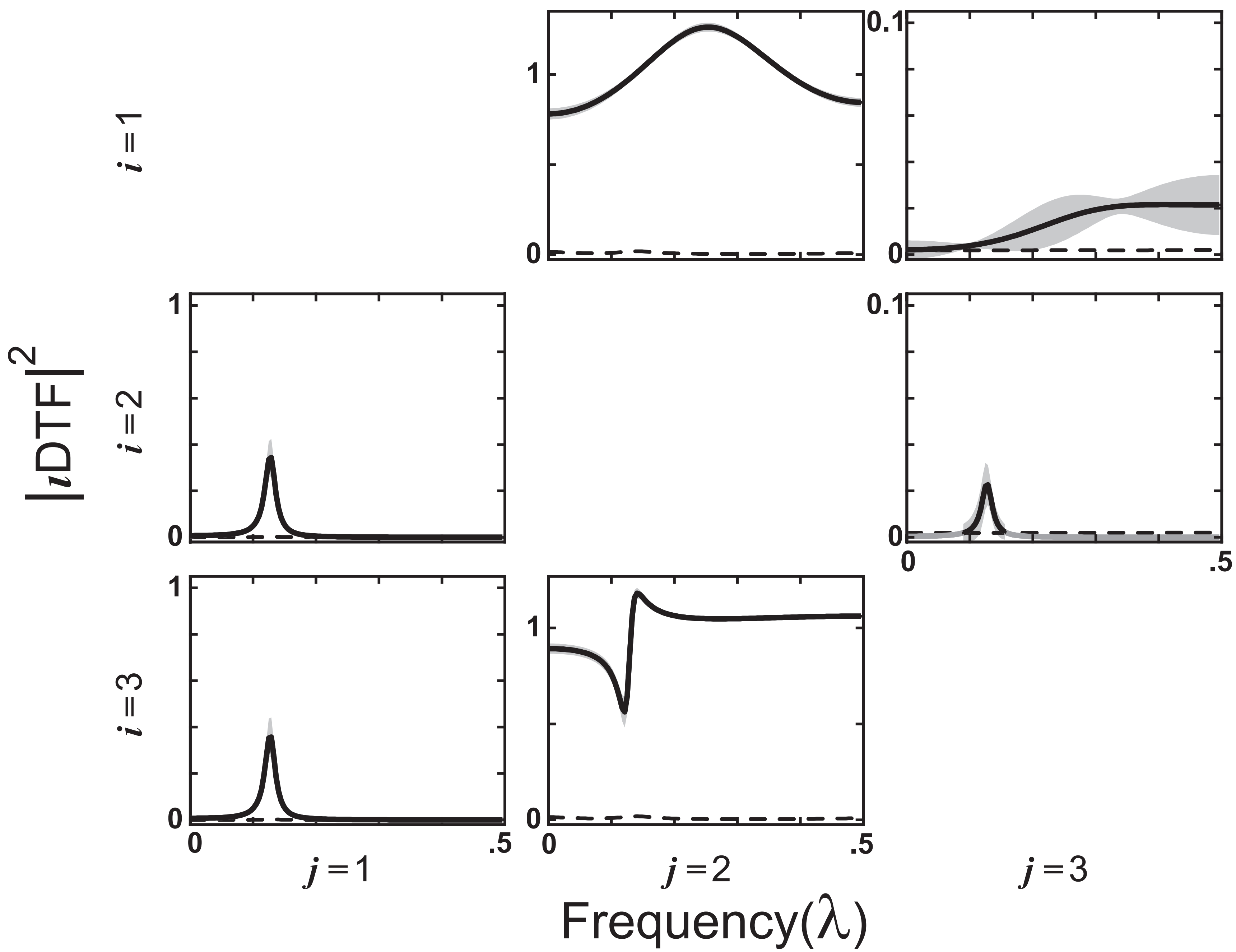
Toy Model III





Toy Model III





DTF and PDC

- Allow focusing on dynamic details of connections
- Work when coherence does not
- Give directional signal-flow information
- Are complementary for more than 2 time series
  - PDC represents immediate adjacent structure descriptions
  - DTF collects direct + indirect signal pathway flows

Effective Connectivity

the influence one neural system exerts over another

from Latin **effectivus,** from efficere 'accomplish'

Functional Connectivity

temporal correlations between remote neurophysiological events

(Friston, 1994)

from Latin *functionem* (nominative *functio*) "performance,

- relating to the way in which something works or operates: *there are important functional differences between left and right brain*.

Effective/Functional Classification

needs

UPGRADING!

Proposal

Link Centered Description

TABLE 13.1
Key to the relationship between connectivity quantifiers and its classification

	Direct	Indirect
Active	$PDC \neq 0$	$PDC = 0$ and $DTF \neq 0$
Inactive	$PDC = 0$	$DTF = 0$

Baccalá LA, Sameshima K (2014b) Multivariate time series brain connectivity: a sum up. In: Sameshima K, Baccalá LA (eds) *Methods in brain connectivity inference through multivariates time series analysis.* Boca Raton: CRC Press, *in press*

DTF=PDC=Granger Causality

2 time series

Granger Causality

> 2 time series

Granger Connectivity → PDC

Granger Influenciability → DTF

Which one to use?

PDC if link description is desired

DTF to sum-up the net effect

The Connectivity Problem

Detection

Characterization

Host of Options

PDC/DTF

rPDC

GRANGER TIME DOMAIN MEASURES

Merely different units?

<u>Connection Detection</u>

H_0: No PDC (DTF) exists at f can be rejected at α

<u>Connection Characterization</u> (if H_0 is rejected)

PDC (DTF) ~ Normally distributed

Information Flow Interpretation

DTF/MA

PDC/AR

Information

$$ {}_{\iota}\gamma_{ij}(\omega) = \frac{\sigma_{jj} H_{ij}(\omega)}{\sqrt{h_i^H(\omega)\Sigma_w h_i(\omega)}} \qquad \iota\pi_{ij}(\omega) = \frac{\sigma_{ii}^{-1} \bar{A}_{ij}(\omega)}{\sqrt{a_j^H(\omega)\Sigma_w^{-1} a_j(\omega)}} $$

(Takahashi et al, 2010)

Diagnostics and Interpretation

- Reliance on model fitting

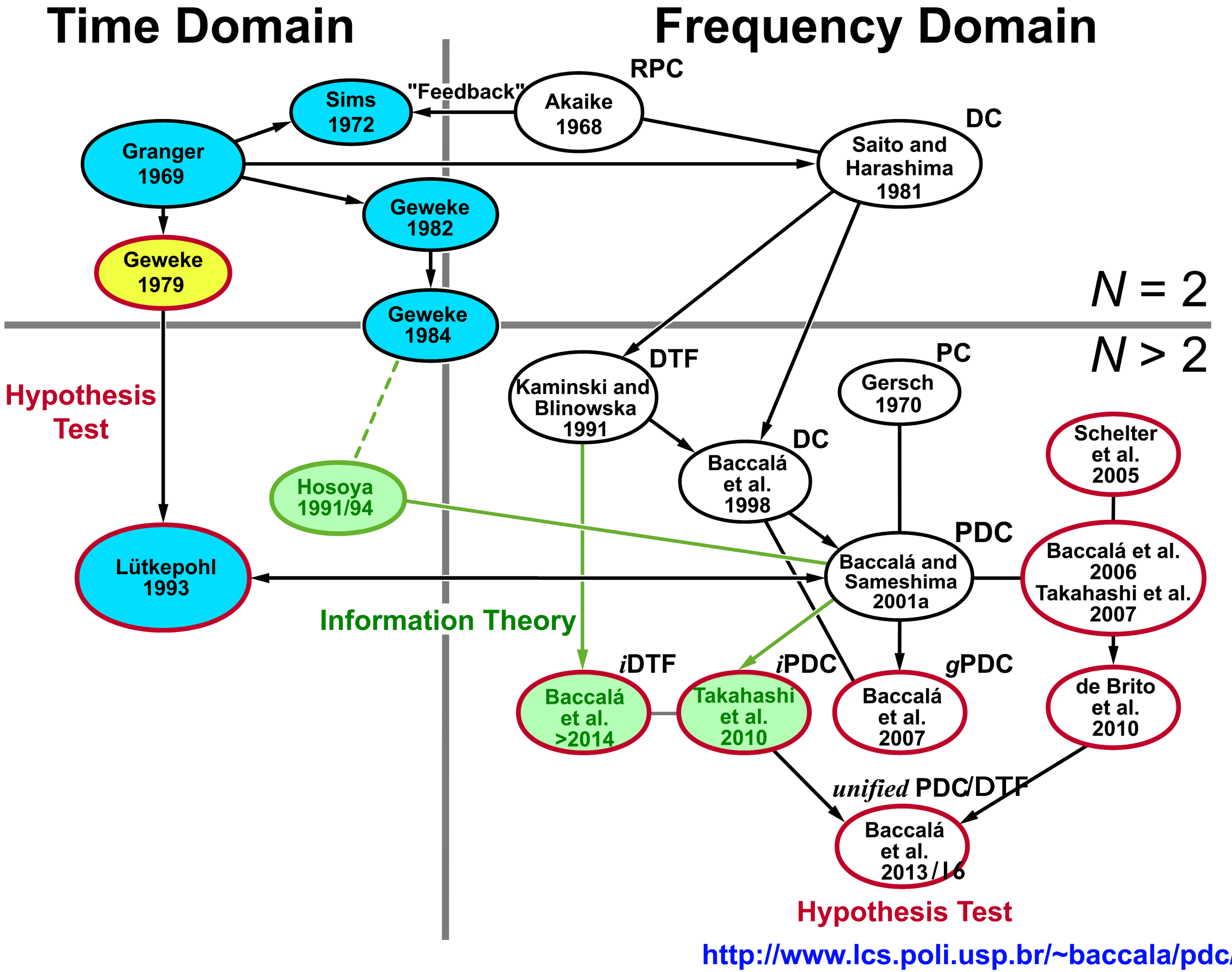
  Model quality appraisal

- “truthfulness” needs including “all” variables of interest

  Implications on model size and signal length

- Limited to ‘linear models’

  (as is correlation)



Time Domain
Frequency Domain
RPC
"Feedback"
Sims 1972
Akaike 1968
DC
Granger 1969
Saito and Harashima 1981
Geweke 1982
Geweke 1979
Geweke 1984
N = 2
N > 2
DTF
PC
Kaminski and Blinowska 1991
Gersch 1970
Hypothesis Test
DC
Schelter et al. 2005
Baccalá et al. 1998
Hosoya 1991/94
PDC
Baccalá et al. 2006 Takahashi et al. 2007
Lütkepohl 1993
Baccalá and Sameshima 2001a
Information Theory
iDTF
iPDC
gPDC
Baccalá et al. >2014
Takahashi et al. 2010
Baccalá et al. 2007
de Brito et al. 2010
unified PDC/DTF
Baccalá et al. 2013/16
Hypothesis Test
http://www.lcs.poli.usp.br/~baccala/pdc/

Some Developments at EMBC 2016

- Time Variant Linear Models for Short Segment ERPs
- Nonlinear Connectivity (Symposium)

Program Overview

Morning

- Introduction and Overview - L.A. Baccalá
- Applications of Granger Causality to Neuroscience - Mingzhou Ding
- Statistical and Software Applications - K. Sameshima
- Power User Applications - L. Astolfi

Afternoon

Data Analysis Challenges

Challenge Resolution and Discussion

Acknowledgments

My Co-authors

Daniel Yasumasa Takahashi

Koichi Sameshima

Web Page:

http://www.lcs.poli.usp.br/~baccala/pdc

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Thank You!

http://www.lcs.poli.usp.br/~baccala/pdc