Contents

1 FIGURE 6 – BACCALA ET AL. (2016) DTF: UNIFIED ASYMPTOTIC THEORY

DESCRIPTION:

Routine **figure6_example2_ipdc_idtf_ns2000.m** publish

Linear trivariate VAR(2) model

LA Baccala, DY Takahashi, K Sameshima (2016) Directed Transfer Function: Unified Asymptotic Theory and Some of its Implications. *IEEE Transactions on Biomedical Engineering* **PP**.

```
http://dx.doi.org/10.1109/TBME.2016.2550199
```

Example 2: Trivariate loop VAR(2) model x1 ==> x2 ==> x3 ˆ———–/

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Generating data set for analysis

```
clear; clc; format compact
flgPlotStyle = 'Print'; % or 'Screen' mode
flgRandomize = 0;  % Generate the specific data set used in Fig. 1.
ns = 2000;          % number of sample points
nDiscard = 20000;  % number of points discarded at beginning of simulation
p = 2;             % model order

if (exist('figure6_example2_ipdc_idtf_ns2000.mat') == 2) & is_octave & ~
   flgRandomize,
   load figure6_example2_ipdc_idtf_ns2000
else
   [u] = fbaccala2016_example2( ns, nDiscard, flgRandomize );
   if ~is_octave & ~flgRandomize,
      save figure6_example2_ipdc_idtf_ns2000 u
   end;
```

```
end;

chLabels = [];     % Using default labeling schema for channel identification
```

```
==============================================================================
            "DTF Unified Asymptotic Theory" Example 2
                        x1 ==> x2 ==> x3
                         ^-----------/
==============================================================================
```

Equation

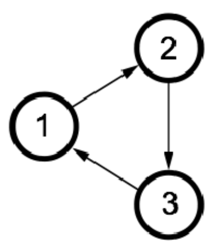
$$
\begin{aligned}
x_1(n) &= 0.95\sqrt{2}x_1(n-1) - 0.9025x_1(n-2) \\
&\quad +0.35x_3(n-1) + w_1(n) \\
x_2(n) &= 0.5x_1(n-1) + 0.5x_2(n-1) + w_2(n) \\
x_3(n) &= x_2(n-1) - 0.5x_3(n-1) + w_3(n)
\end{aligned}
$$

Equation (9) from Baccala et al. *IEEE Trans Biomed Engin.*, 2016.

with with the innovations covariance matrix given by

$$
\Sigma_{\mathbf{w}} = \begin{bmatrix} 1 & 5 & 0.3 \\ 5 & 100 & 2 \\ 0.3 & 2 & 1 \end{bmatrix}
$$

Connectivity diagram



Example 2 loop connectivity structure following (9) and (10). Signals from any structure reach all other structures. from Baccala et al. *IEEE Trans Biomed Engin.*, 2016.

Data pre-processing: detrending and normalization options

```
flgDetrend = 1;      % Detrending the data set
flgStandardize = 0; % No standardization
[nChannels,nSegLength] = size(u);
if nChannels > nSegLength, u = u.';
```

```
   [nChannels,nSegLength] = size(u);
end;
if flgDetrend,
   for i=1:nChannels, u(i,:) = detrend(u(i,:)); end;
   disp('Time series were detrended.');
end;
if flgStandardize,
   for i=1:nChannels, u(i,:) = u(i,:)/std(u(i,:)); end;
   disp('Time series were scale-standardized.');
end;
```

```
        Time series were detrended.
```

MVAR model estimation

```
maxIP = 30;         % maximum model order to consider.
alg = 1;            % 1: Nutall-Strand MVAR estimation algorithm
criterion = 1;      % 1: AIC, Akaike Information Criteria
disp('Running MVAR estimation and GCT analysis routines.')
[IP,pf,A,pb,B,ef,eb,vaic,Vaicv] = mvar(u,maxIP,alg,criterion);
disp(['Number of channels = ' int2str(nChannels) ' with ' ...
   int2str(nSegLength) ' data points; MAR model order = ' int2str(IP) '.']);
```

```
        Running MVAR estimation and GCT analysis routines.
maxOrder limited to 30
IP=1  vaic=60428.213072
IP=2  vaic=54036.613963
IP=3  vaic=54039.508093

Number of channels = 3 with 2000 data points; MAR model order = 2.
```

Testing for adequacy of MAR model fitting through Portmanteau test

```
h = 20; % testing lag
MVARadequacy_signif = 0.05; % VAR model estimation adequacy significance
                            % level
aValueMVAR = 1 - MVARadequacy_signif; % Confidence value for the testing
flgPrintResults = 1;
```

Granger causality test (GCT) and instantaneous GCT

```
gct_signif  = 0.01;  % Granger causality test significance level
igct_signif = 0.01;  % Instantaneous GCT significance level
flgPrintResults = 1;
[Tr_gct, pValue_gct, Tr_igct, pValue_igct] = gct_alg(u,A,pf,gct_signif, ...
                                              igct_signif,flgPrintResults);
```

```

----------------------------------------------------------------------
                         GRANGER CAUSALITY TEST
======================================================================
Connectivity matrix:
          NaN             0          1.00
         1.00           NaN             0
            0          1.00           NaN
Granger causality test p-values:
          NaN          0.56             0
```

```
            0            NaN            0.71
         0.55              0             NaN
----------------------------------------------------------------------
                  INSTANTANEOUS GRANGER CAUSALITY TEST
======================================================================
Instantaneous connectivity matrix:
          NaN           1.00           1.00
         1.00            NaN           1.00
         1.00           1.00            NaN
Instantaneous Granger causality test p-values:
          NaN              0              0
            0            NaN              0
            0              0            NaN
>>>> There are 3 pairs of channels with
      significant Instantaneous Causality.
```

DTF estimation

DTF analysis results are saved in **c** structure. See asymp_dtf.m or issue >> help asymp_dtf command for more detail.

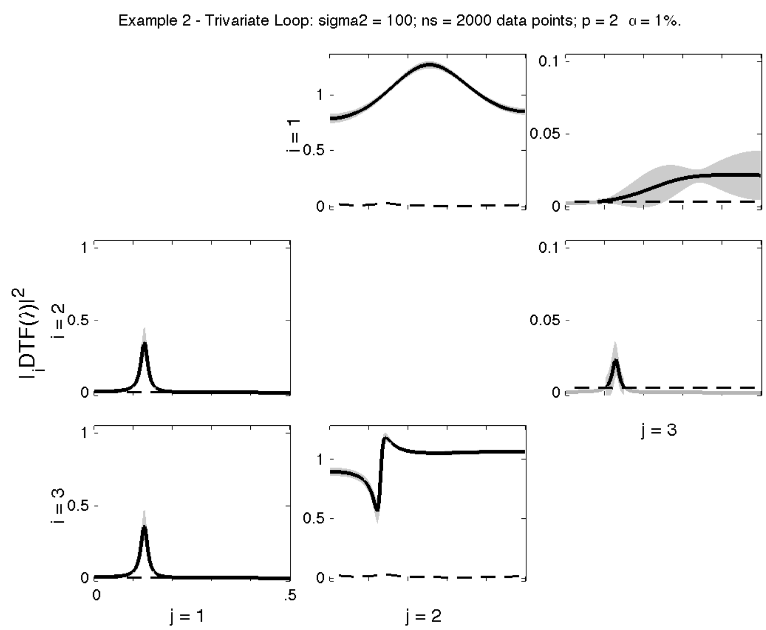
```
metric = 'info';  % euc  = original PDC or DTF;
                 % diag = generalized PDC (gPDC) or directed coherence (DC);
                 % info = information PDC (iPDC) or iDTF.

nFreqs = 128;
alpha = 0.01;

c = asymp_dtf(u,A,pf,nFreqs,metric,alpha);
```

```
        * Information DTF and asymptotic statistics
```

$|DTF(\lambda)|^2$ Matrix Layout Plotting

```
switch lower(flgPlotStyle)
   case 'print'
      flgColor =[0];       % white background
      flgMax = 'TCI';
      flgSignifColor = 1; % black + gray
      flgScale = 2;        % [0 max(flgMax)]
   otherwise % 'screen'
      flgColor =[1];       % Colored background
      flgMax = 'TCI';
      flgSignifColor = 3; % red + green
      flgScale = 2;        % [0 1]/[0 .1]/[0 .01]
end;

% ---------------Plotting options flag setting-----------------------------
%            [1 2 3 4 5 6 7]
flgPrinting=[1 1 1 2 2 0 0];
%             | | | | | | 7 Spectra(0: w/o SS; 1: Linear; 2: log-scale)
%             | | | | | 6 Coherence
%             | | | | 5 Plot lower confidence limit (legacy)
%             | | | 4 Plot upper confidence limit
%             | | 3 Significant DTF(w) in red line  (legacy)
%             | 2 Patnaik threshold level in black dashed-line
```

```
%              1 plot DTF
%----------------------------------------------------------------------------

fs = 1;        % sampling frequency

w = fs*(0:(nFreqs-1))/2/nFreqs;
w_max = fs/2;

h = figure;
set(h,'NumberTitle','off','MenuBar','none', ...
   'Name','[Asymptotic DTF] Fig 6. Example 2 - Trivariate loop iDTF, ns = 2000')
[hxlabel hylabel] = xplot(c,...
   flgPrinting,fs,w_max,chLabels,flgColor,flgScale,flgMax,flgSignifColor);
%xplot_title(alpha,metric, measure(c));

[ax,hT] = suplabel(['Example 2 - Trivariate Loop: sigma2 = 100; ns = ' ...
           int2str(ns) ' data points; p = ' int2str(p) ' \alpha = ' ...
           int2str(100*alpha) '%.'],'t');
set(hT,'FontSize',10); % Subtitle font size
```



Uncomment the command line bellow to generate an eps output file

% print -depsc2 -painters Fig6_example2_dtf_ns2000.eps

PDC estimation

PDC analysis results are saved in **d** structure. See asymp_dtf.m or issue >> help asymp_pdc command for more detail.

```
nFreqs = 128;
metric = 'info'; % euc   = Euclidian = original PDC;
%                 % diag = diagonal   = generalized PDC or gPDC;
%                 % info = information = iPDC
```

```
alpha = 0.01;

d = asymp_pdc(u,A,pf,nFreqs,metric,alpha);

% Power spectra and coherence calculation
d.SS = ss_alg(A, pf, nFreqs);
d.coh = coh_alg(d.SS);

if alpha ~= 0,
   % Statistically significant PDC on frequency scale
   pdc_temp = ((abs(d.pdc)-d.th) > 0).*d.pdc + ((abs(d.pdc)-d.th) <= 0)*(-1);
   pdc_temp(ind2sub(size(pdc_temp),find(pdc_temp == -1))) = NaN;
   d.pdc_th = pdc_temp;
end;
```

```
        * Information PDC and asymptotic statistics
```

$|PDC(\lambda)|^2$ Matrix Layout Plotting

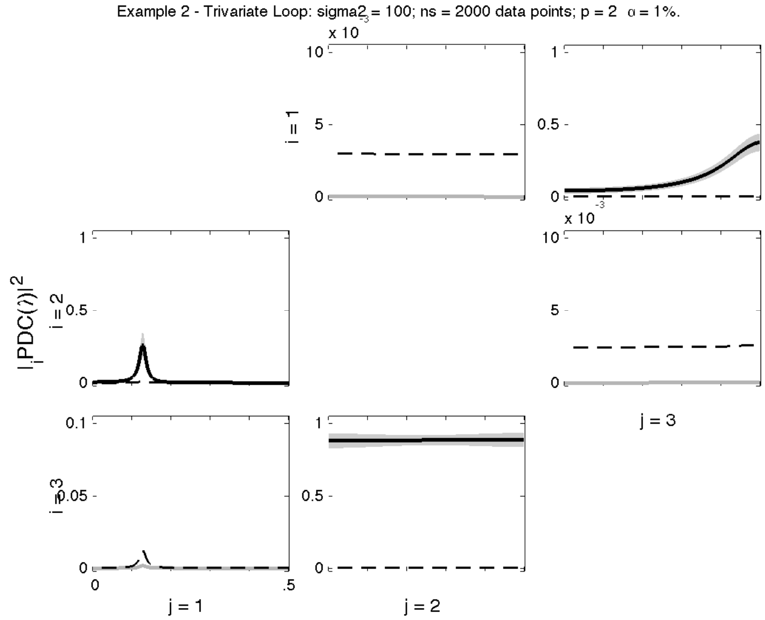
```
% ---------------Plotting options flag setting-----------------------------
%            [1 2 3 4 5 6 7]
flgPrinting=[1 1 1 2 2 0 0];
%             | | | | | | 7 Spectra(0: w/o SS; 1: Linear; 2: log-scale)
%             | | | | | 6 Coherence
%             | | | | 5 Plot lower confidence limit (legacy)
%             | | | 4 Plot upper confidence limit
%             | | 3 Significant DTF(w) in red line  (legacy)
%             | 2 Patnaik threshold level in black dashed-line
%             1 plot DTF
%--------------------------------------------------------------------------
fs = 1;       % sampling frequency
w_max = fs/2; % x-axis maximum value
h = figure;
set(h,'NumberTitle','off','MenuBar','none', ...
   'Name','[Asymptotic DTF] Fig 6. Example 2 - Trivariate loop iPDC, ns = 2000'
      )
[hxlabel hylabel] = xplot(d,...
   flgPrinting,fs,w_max,chLabels,flgColor,flgScale,flgMax,flgSignifColor);

%xplot_title(alpha,metric, measure(c));

[ax,hT]=suplabel(['Example 2 - Trivariate Loop: sigma2 = 100; ns = ' ...
         int2str(ns) ' data points; p = ' int2str(p) ' \alpha = ' ...
         int2str(100*alpha) '%.'],'t');
set(hT,'FontSize',10); % Subtitle font size
```

Uncomment the command line bellow to generate an eps output file

print -depsc2 -painters Fig6_example2_ipdc_ns2000.eps

Figure depicted in the article Baccala et al (2016)

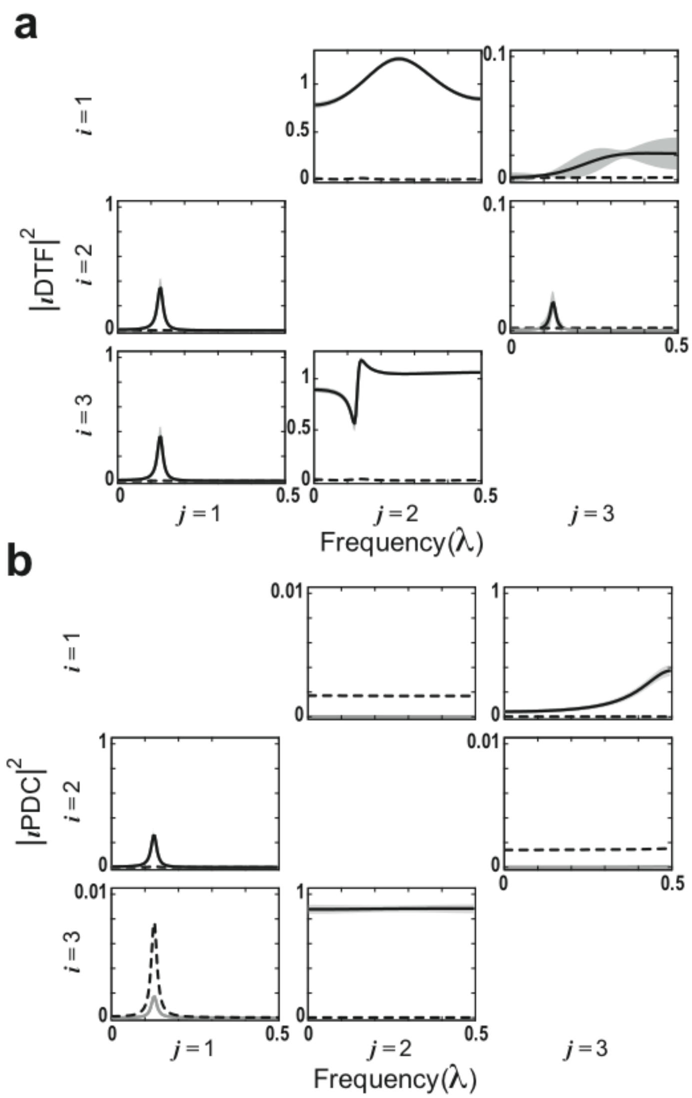
Figure 6, reproduced from article.



Some remarks:

1. As usual, figure 6 underwent some cosmetic edit

This completes the **Figure 6** generation (Baccala et al, 2016)'


a
$|\iota\mathrm{DTF}|^2$
$i=1$
$i=2$
$i=3$
$j=1$
$j=2$
$j=3$
Frequency(λ)
b
$|\iota\mathrm{PDC}|^2$
$i=1$
$i=2$
$i=3$
$j=1$
$j=2$
$j=3$
Frequency(λ)