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FIGURE 11 – BACCALA ET AL. (2016) DTF: UNIFIED ASYMPTOTIC THEORY

DESCRIPTION:

Routine **figure11_example4_dtf_ns50_500.m** publish

Linear bivariate VAR(2) model

LA Baccala, DY Takahashi, K Sameshima (2016) Directed Transfer Function: Unified Asymptotic Theory and Some of its Implications. *IEEE Transactions on Biomedical Engineering* **PP**.

```
http://dx.doi.org/10.1109/TBME.2016.2550199
```

Example 1: Reachability Example 4, ns = {50 500}

```
                    x1 <==> x2 -->x3
```

```
clear; clc; format compact
flgPlotStyle = 'Print'; % or 'Screen' mode
flgRandomize = 0; % Generate the specific data set used in Fig. 1.
nd = [50 500];          % number of sample points
nDiscard = 2000;  % number of points discarded at beginning of simulation
p = 2;            % model order
```

Equation

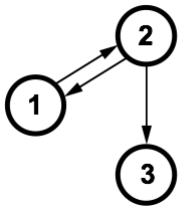
$$
\begin{aligned}
x_1(n) &= 0.25\sqrt{2}x_1(n-1) + 0.25\sqrt{2}x_2(n-1) + w_1(n) \\
x_2(n) &= -0.25\sqrt{2}x_1(n-1) + 0.25\sqrt{2}x_2(n-1) + w_2(n) \\
x_3(n) &= 0.5x_2(n-1) + w_3(n) \qquad (12)
\end{aligned}
$$

Equation (12) from Baccala et al. *IEEE Trans Biomed Engin.*, 2016.

Connectivity diagram

Example 4 reachability example following (11) from Baccala et al. *IEEE Trans Biomed Engin.*, 2016.

```
for ns = nd,
```



Generating data set for analysis

```
file_name = ['figure11_example4_dtf_ns' int2str(ns) '.mat'];

if (exist(file_name) == 2) & is_octave & ~flgRandomize,
   eval(['load ' file_name])
else
   [u] = fbaccala2016_example4( ns, nDiscard, flgRandomize );
   if ~is_octave & ~flgRandomize,
      eval(['save ' file_name ' u'])
   end;
end;

chLabels = [];    % Using default labeling schema for channel identification
```

```
=========================================================================
     "Asymptotic DTF" Reachability Example 4, ns = 50
                  x1 <==> x2 -->x3
=========================================================================
```

```
=========================================================================
     "Asymptotic DTF" Reachability Example 4, ns = 500
                  x1 <==> x2 -->x3
=========================================================================
```

Data pre-processing: detrending and normalization options

```
flgDetrend = 1;      % Detrending the data set
flgStandardize = 0; % No standardization
[nChannels,nSegLength] = size(u);
if nChannels > nSegLength, u = u.';
   [nChannels,nSegLength] = size(u);
end;
if flgDetrend,
   for i=1:nChannels, u(i,:) = detrend(u(i,:)); end;
   disp('Time series were detrended.');
end;
if flgStandardize,
   for i=1:nChannels, u(i,:) = u(i,:)/std(u(i,:)); end;
   disp('Time series were scale-standardized.');
end;
```

```
        Time series were detrended.
```

```
        Time series were detrended.
```

MVAR model estimation

```
   maxIP = 30;          % maximum model order to consider.
   alg = 1;             % 1: Nutall-Strand MVAR estimation algorithm
   criterion = 1;       % 1: AIC, Akaike Information Criteria
   disp('Running MVAR estimation and GCT analysis routines.')
   [IP,pf,A,pb,B,ef,eb,vaic,Vaicv] = mvar(u,maxIP,alg,criterion);
   disp(['Number of channels = ' int2str(nChannels) ' with ' ...
      int2str(nSegLength) ' data points; MAR model order = ' int2str(IP) '.']);
```

```
        Running MVAR estimation and GCT analysis routines.
maxOrder limited to 30
IP=1  vaic=575.897394
IP=2  vaic=582.310858

Number of channels = 3 with 50 data points; MAR model order = 1.
```

```
        Running MVAR estimation and GCT analysis routines.
maxOrder limited to 30
IP=1  vaic=9366.231496
IP=2  vaic=9370.571107

Number of channels = 3 with 500 data points; MAR model order = 1.
```

Testing for adequacy of MAR model fitting through Portmanteau test

```
   h = 20; % testing lag
   MVARadequacy_signif = 0.05; % VAR model estimation adequacy significance
   % level
   aValueMVAR = 1 - MVARadequacy_signif; % Confidence value for the testing
   flgPrintResults = 1;
```

Granger causality test (GCT) and instantaneous GCT

```
   gct_signif  = 0.01;  % Granger causality test significance level
   igct_signif = 0.01;  % Instantaneous GCT significance level
   flgPrintResults = 1;
   [Tr_gct, pValue_gct, Tr_igct, pValue_igct] = gct_alg(u,A,pf,gct_signif, ...
      igct_signif,flgPrintResults);
```

```
-------------------------------------------------------------------------
                         GRANGER CAUSALITY TEST
=========================================================================
Connectivity matrix:
           NaN          1.00          1.00
             0           NaN             0
             0          1.00           NaN
Granger causality test p-values:
           NaN          0.00          0.01
          0.02           NaN          0.80
          0.77          0.00           NaN
-------------------------------------------------------------------------
                  INSTANTANEOUS GRANGER CAUSALITY TEST
=========================================================================
Instantaneous connectivity matrix:
           NaN             0             0
```

```
             0           NaN             0
             0             0           NaN
Instantaneous Granger causality test p-values:
           NaN          0.23          0.17
          0.23           NaN          0.34
          0.17          0.34           NaN
>>>> Instantaneous Granger causality NOT detected.
```

```
------------------------------------------------------------------------
                         GRANGER CAUSALITY TEST
========================================================================
Connectivity matrix:
           NaN          1.00             0
          1.00           NaN             0
             0          1.00           NaN
Granger causality test p-values:
           NaN             0          0.40
             0           NaN          0.29
          0.47             0           NaN
------------------------------------------------------------------------
                  INSTANTANEOUS GRANGER CAUSALITY TEST
========================================================================
Instantaneous connectivity matrix:
           NaN             0             0
             0           NaN             0
             0             0           NaN
Instantaneous Granger causality test p-values:
           NaN          0.41          0.99
          0.41           NaN          0.23
          0.99          0.23           NaN
>>>> Instantaneous Granger causality NOT detected.
```

DTF estimation

DTF analysis results are saved in **c** structure. See asymp_dtf.m or issue >> help asymp_dtf command for more detail.

```
metric = 'euc'; % euc  = Euclidian   = original DTF;
%               % diag = diagonal    =  DC;
%               % info = information = iDTF.
nFreqs = 128;
alpha = 0.01;

d = asymp_dtf(u,A,pf,nFreqs,metric,alpha);

eval(['c' int2str(ns) '=d;'])
```

```
      * Original DTF and asymptotic statistics
```

```
      * Original DTF and asymptotic statistics
```

$|DTF(\lambda)|^2$ Matrix Layout Plotting

```
switch lower(flgPlotStyle)
   case 'print'
      flgColor =[0];      % white background
      flgMax = 'TCI';
```

```
        flgSignifColor = 1; % black + gray
        flgScale = 1;       % [0 max(flgMax)]
    otherwise % 'screen'
        flgColor =[1];      % Colored background
        flgMax = 'TCI';
        flgSignifColor = 3; % red + green
        flgScale = 2;       % [0 1]/[0 .1]/[0 .01]
end;

% ---------------Plotting options flag setting----------------------------
%           [1 2 3 4 5 6 7]
flgPrinting=[1 1 1 2 2 0 1];
%             | | | | | | 7 Spectra(0: w/o SS; 1: Linear; 2: log-scale)
%             | | | | | 6 Coherence
%             | | | | 5 Plot lower confidence limit (legacy)
%             | | | 4 Plot upper confidence limit
%             | | 3 Significant DTF(w) in red line  (legacy)
%             | 2 Patnaik threshold level in black dashed-line
%             1 plot DTF
%------------------------------------------------------------------------

fs = 1;       % sampling frequency

w = fs*(0:(nFreqs-1))/2/nFreqs;
w_max = fs/2;

h=figure;
set(h,'NumberTitle','off','MenuBar','none', ...
    'Name',['[Asymptotic DTF] Fig 11. Example 4 - DTF, ns = ' ...
    int2str(ns)])
[hxlabel hylabel] = xplot(d,...
    flgPrinting,fs,w_max,chLabels,flgColor,flgScale,flgMax,flgSignifColor);
%  xplot_title(alpha,metric, measure(d));

[ax,hT] = suplabel(['Fig11 Example 4 - Tri-VAR Open: ' ...
    int2str(ns) ' data points; p = ' int2str(d.p) '; \alpha = ' int2str(100*
        alpha) '%.'],'t');
set(hT,'FontSize',10); % Subtitle font size
if ns == 50,
    c = d;
end
```

Uncomment the command line bellow to generate an eps output file

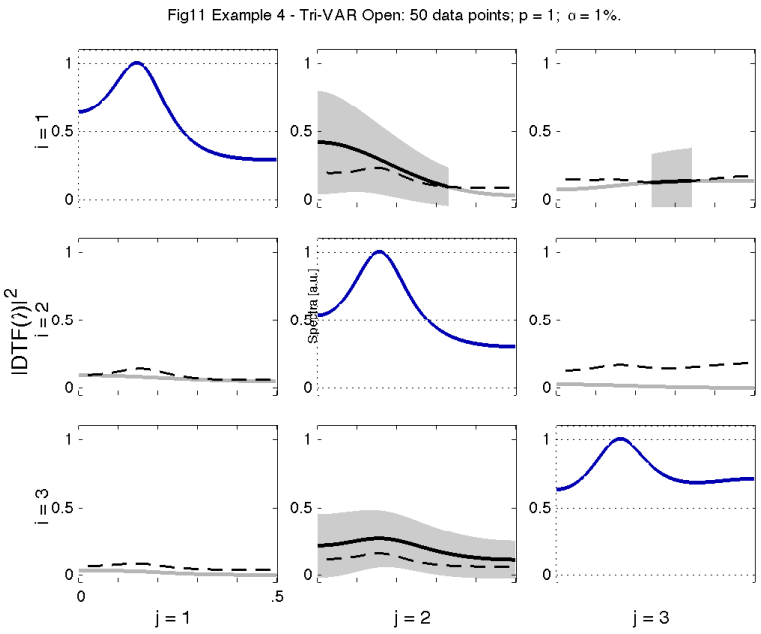
```
eval(['print -depsc2 -painters Fig11_example4_dtf_ns' int2str(ns) '.eps'])

end;
```

Figure depicted in the article Baccala et al (2016)

Figure 11, reproduced from article.

Some remarks:

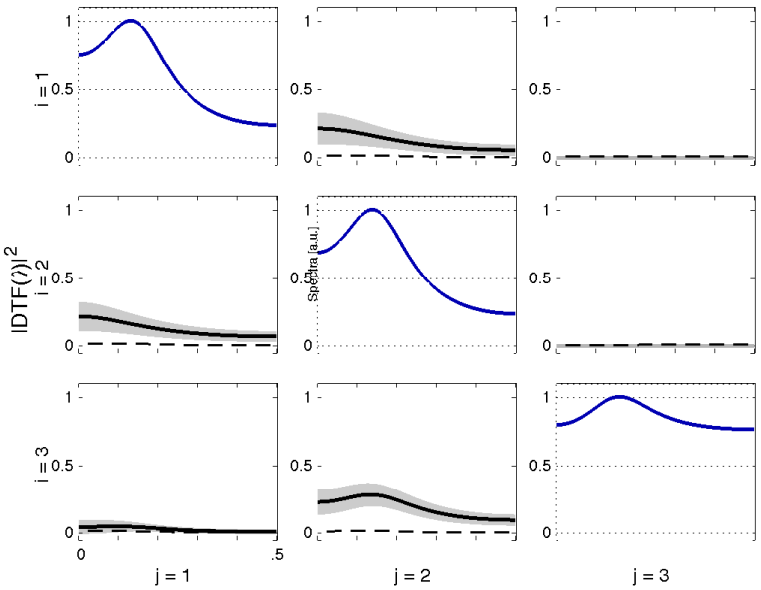
1. For the final figure 1, we did some cosmetic edit with AI

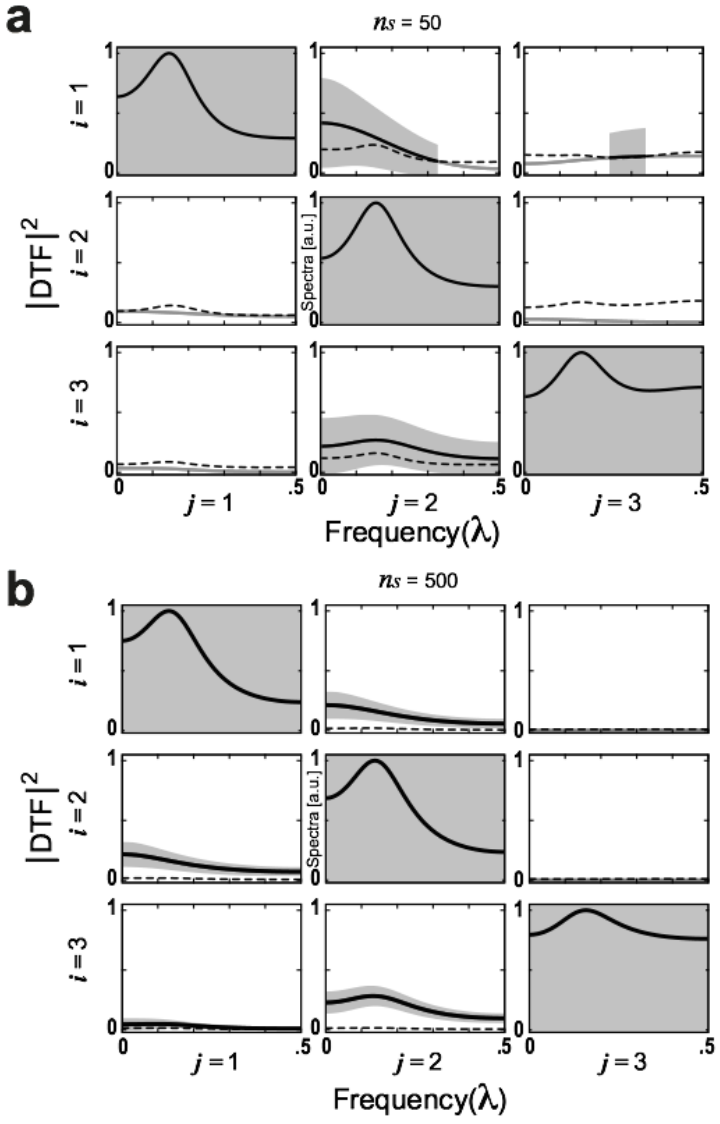


2. Erratum: **If we were not able to correct this typo.** In the caption of Figure 1, it states that "Gray shades describe 95% confidence intervals when above threshold.", however as we have used $\alpha = 1\%$, conversely the confidence interval is actually 99%, instead of 95%.

This completes the Figure 11 routine (Baccala et al, 2016)'

Fig11 Example 4 - Tri-VAR Open: 500 data points; p = 1; α = 1%.




a
ns = 50
i = 1
i = 2
i = 3
|DTF|2
Spectra [a.u.]
j = 1
j = 2
j = 3
Frequency(λ)
b
ns = 500
i = 1
i = 2
i = 3
|DTF|2
Spectra [a.u.]
j = 1
j = 2
j = 3
Frequency(λ)