EPUSP - Escola Politécnica of University of São Paulo

Telecommunications and Control Engineering Department

Communications and Signals Laboratory

# CDMA - Code Division Multiple Access
# DS/SS - Direct Sequence Spread Spectrum
# and
# Related Topics

by

Paul Jean Etienne Jeszensky

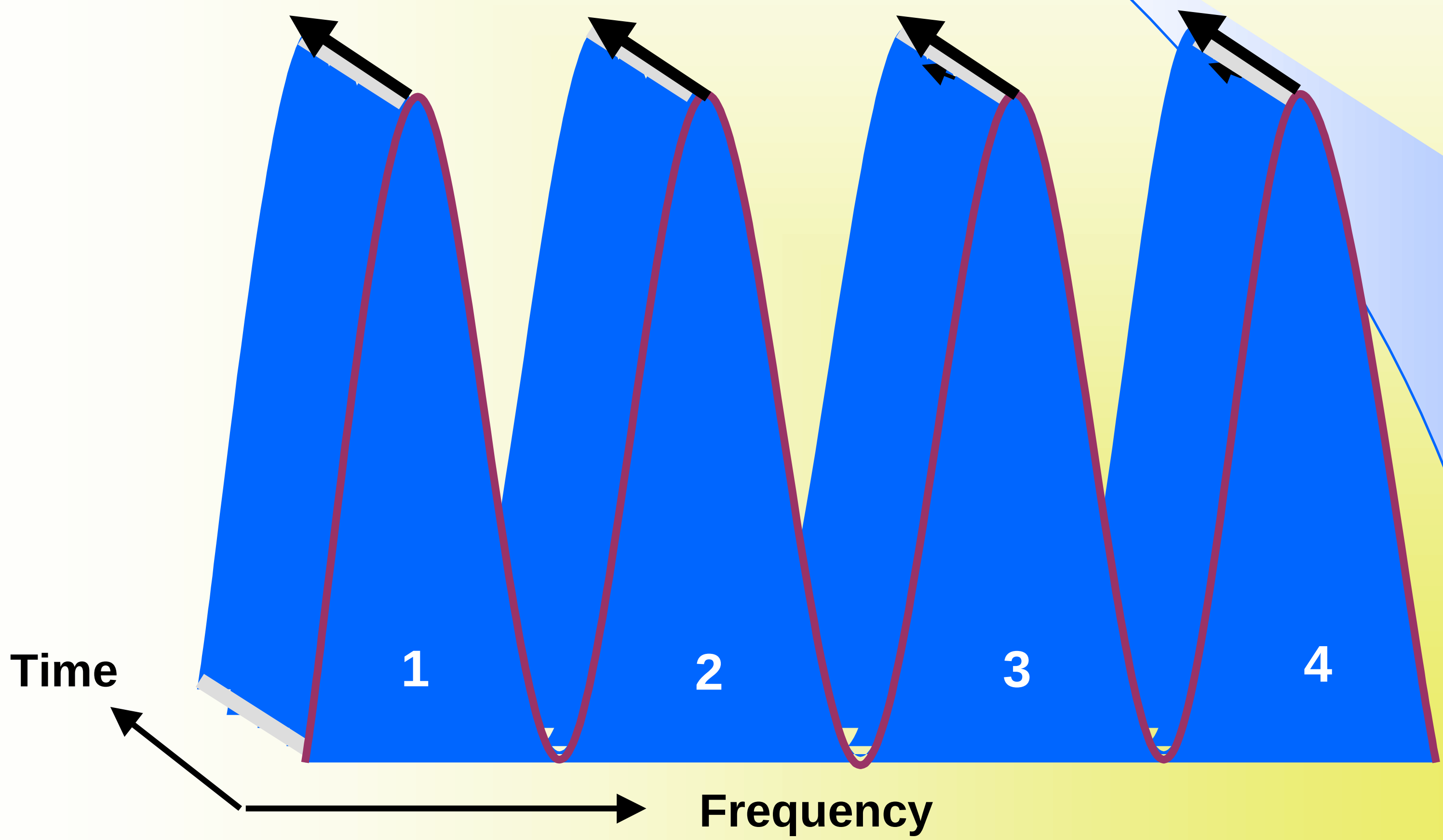
First edition in October 2001; revised in January 2004.

# Contents

# Multiple Access Techniques

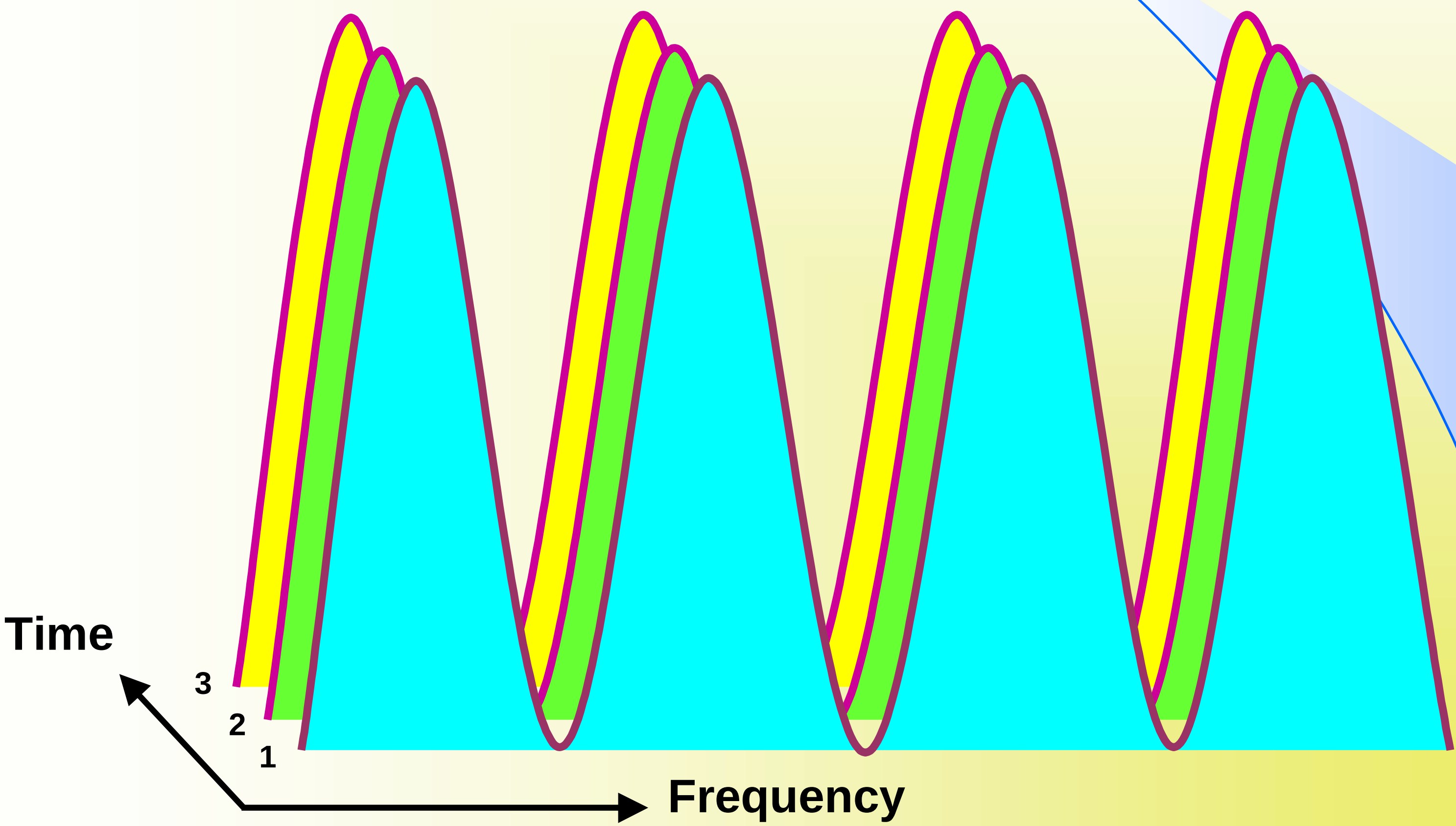
## FDMA: Frequency Division Multiple Access

(one carrier for each user for all connection time)



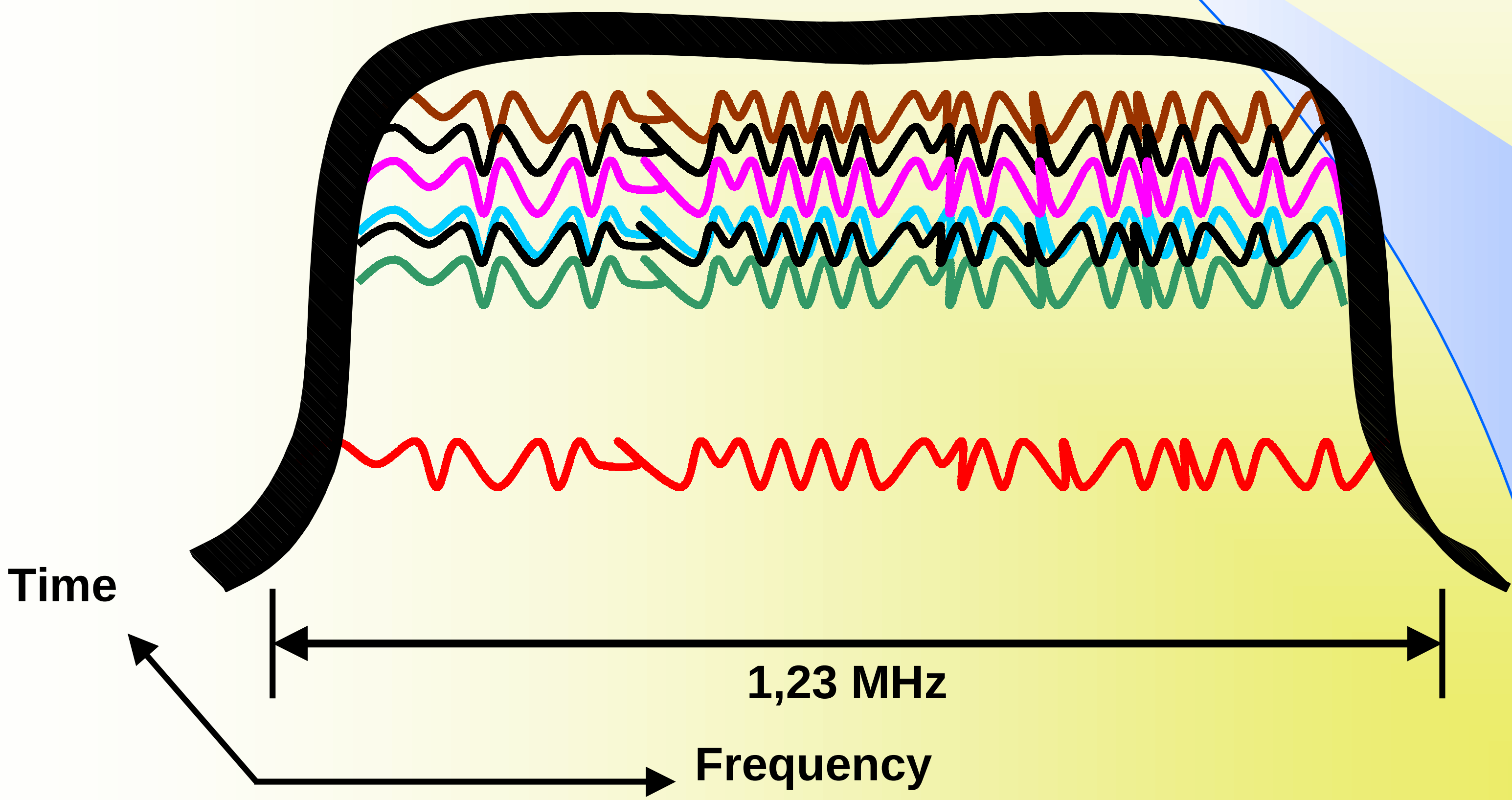
# TDMA: Time Division Multiple Access

(one carrier for a group of users in a time division principle)

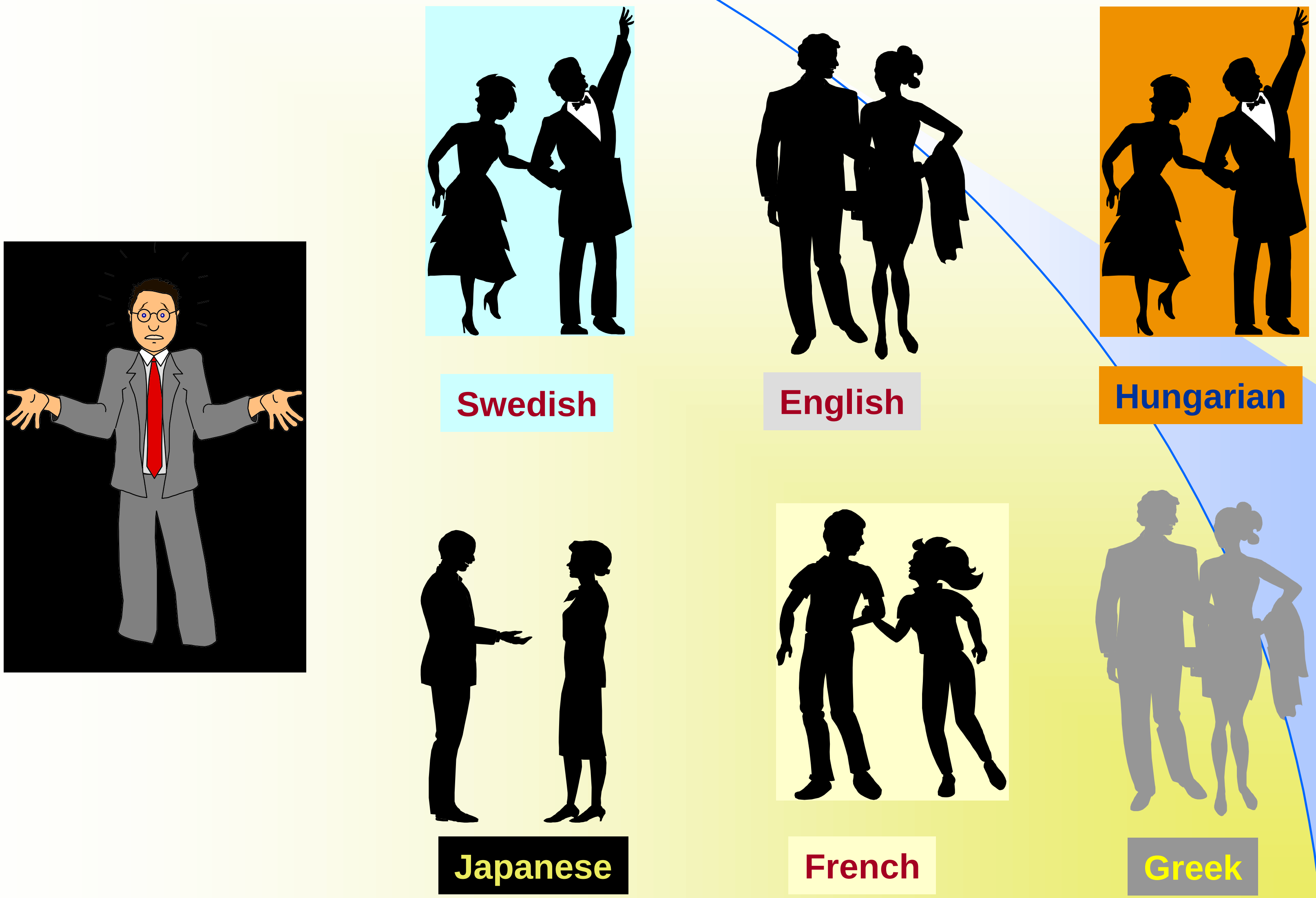


# CDMA: Code Division Multiple Access

(one carrier for all users for all time in a code division principle)

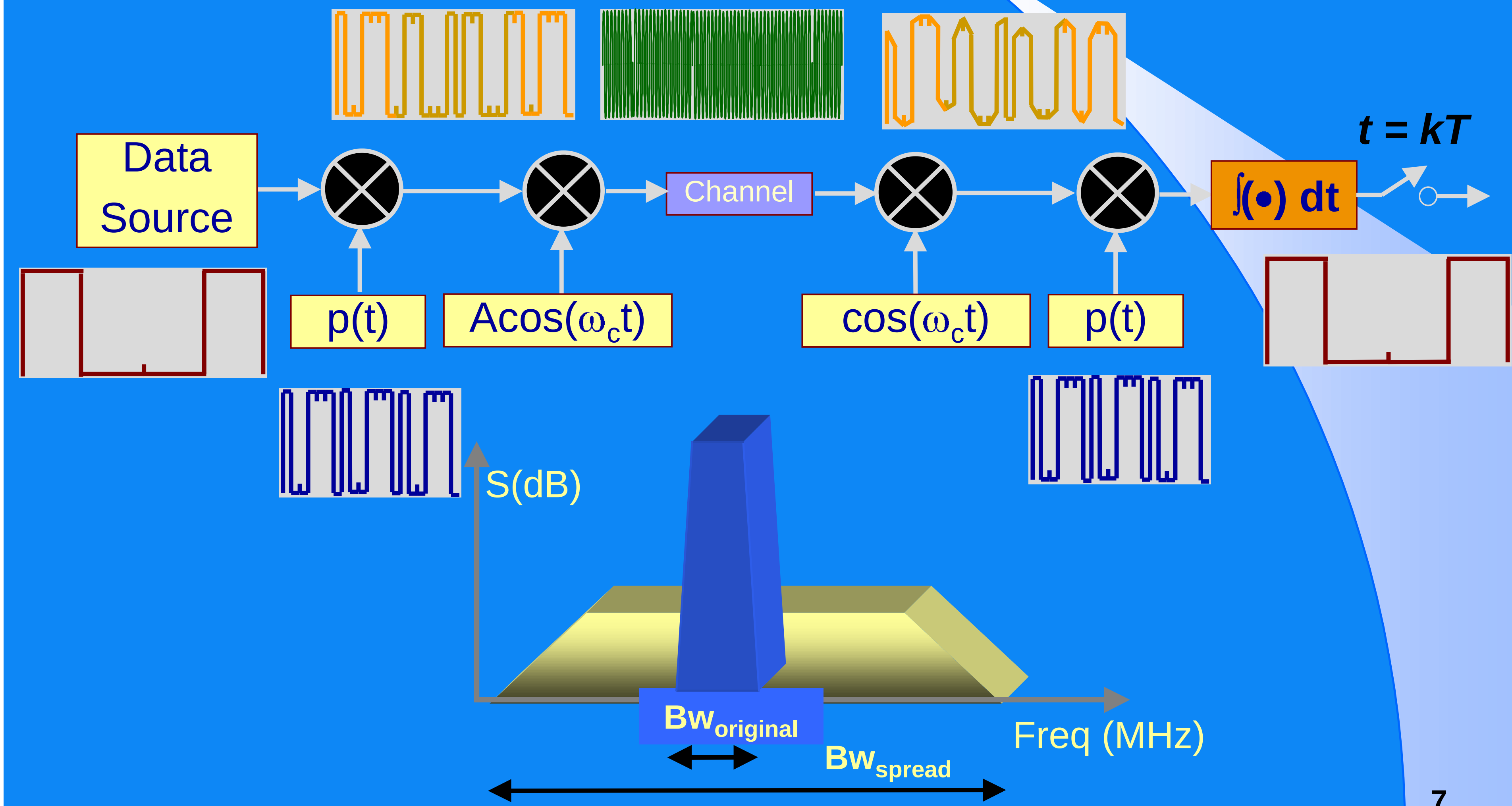


# CDMA Philosophy

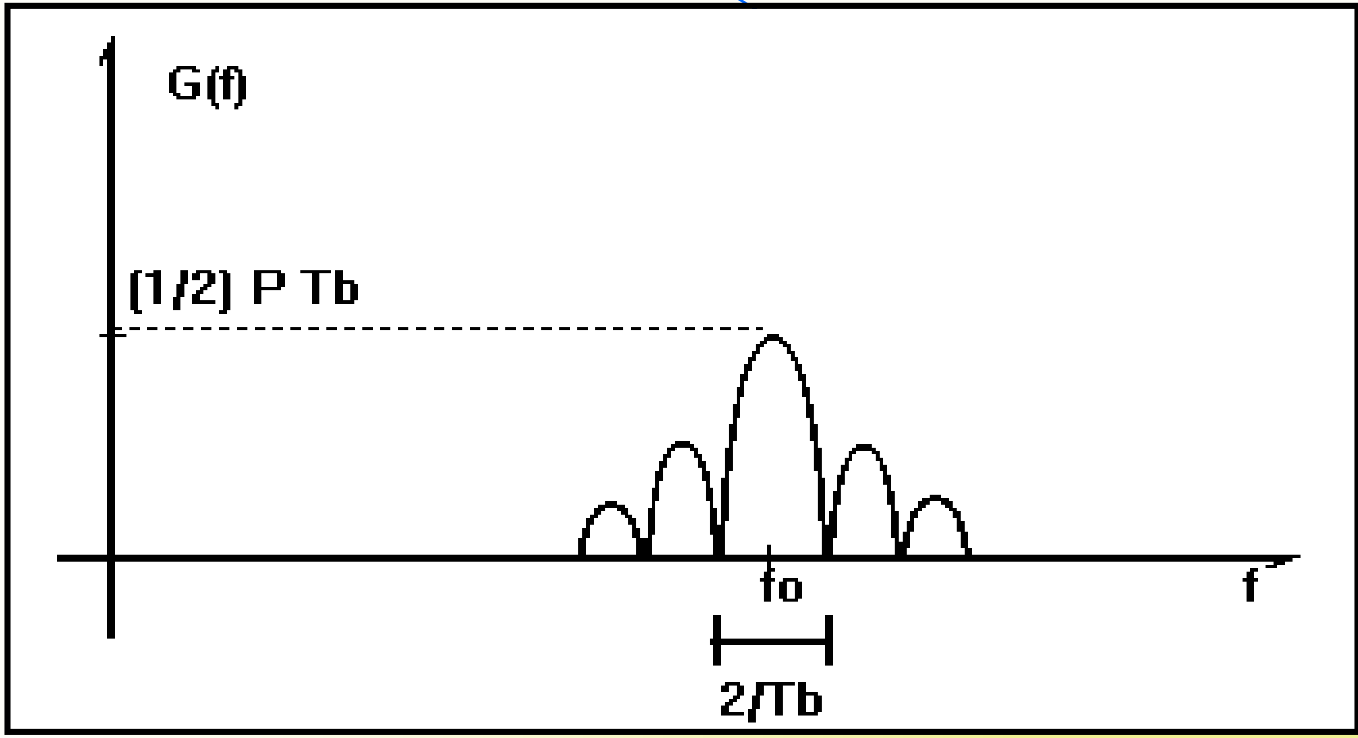


# Some General Characteristics

## DS/SS Block Diagram

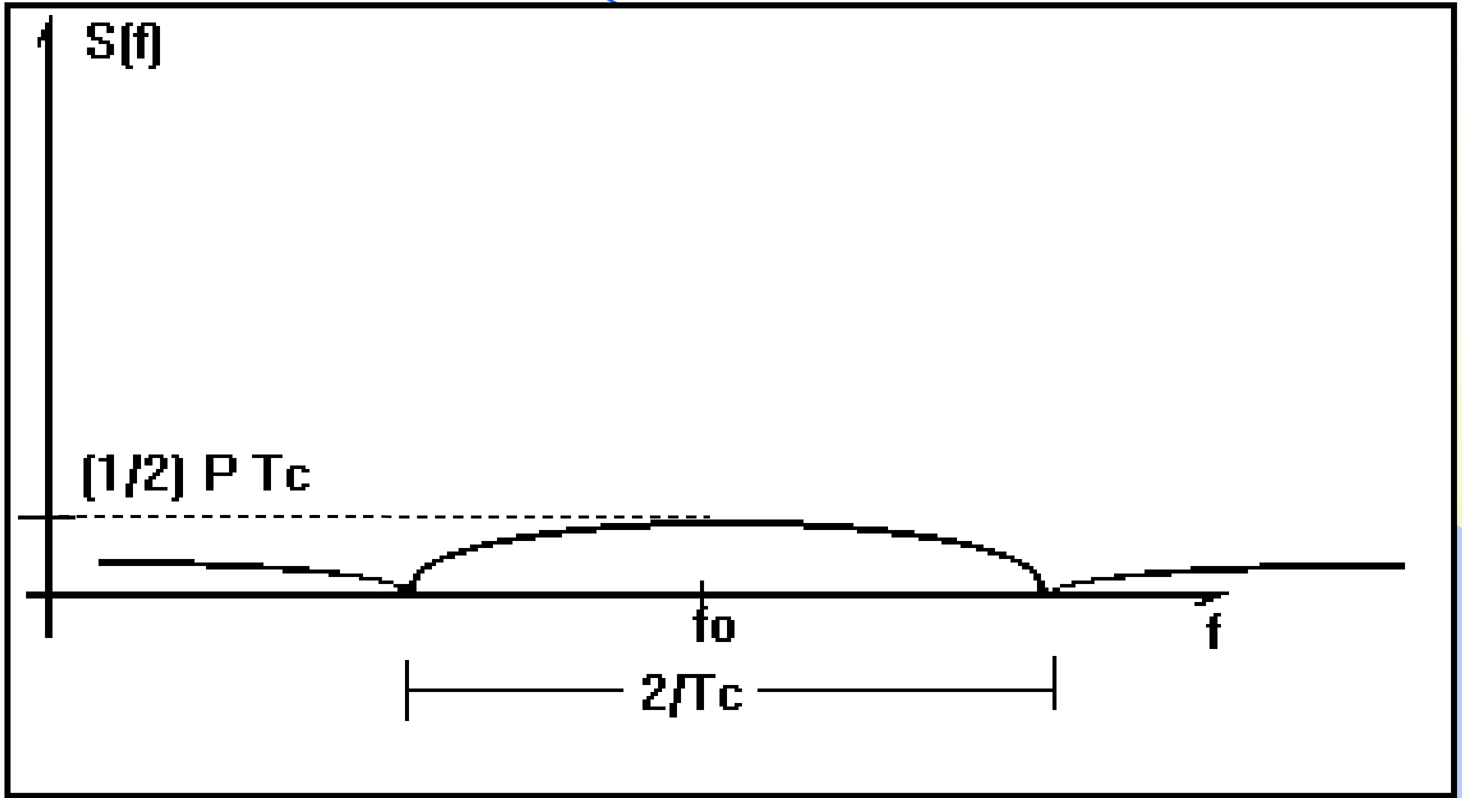


# Power Spectral Densities (PSD) of DS/SS Signals




BPSK signal with power P, carrier frequency $f_o$ and a data rate $R_b=1/T_b$

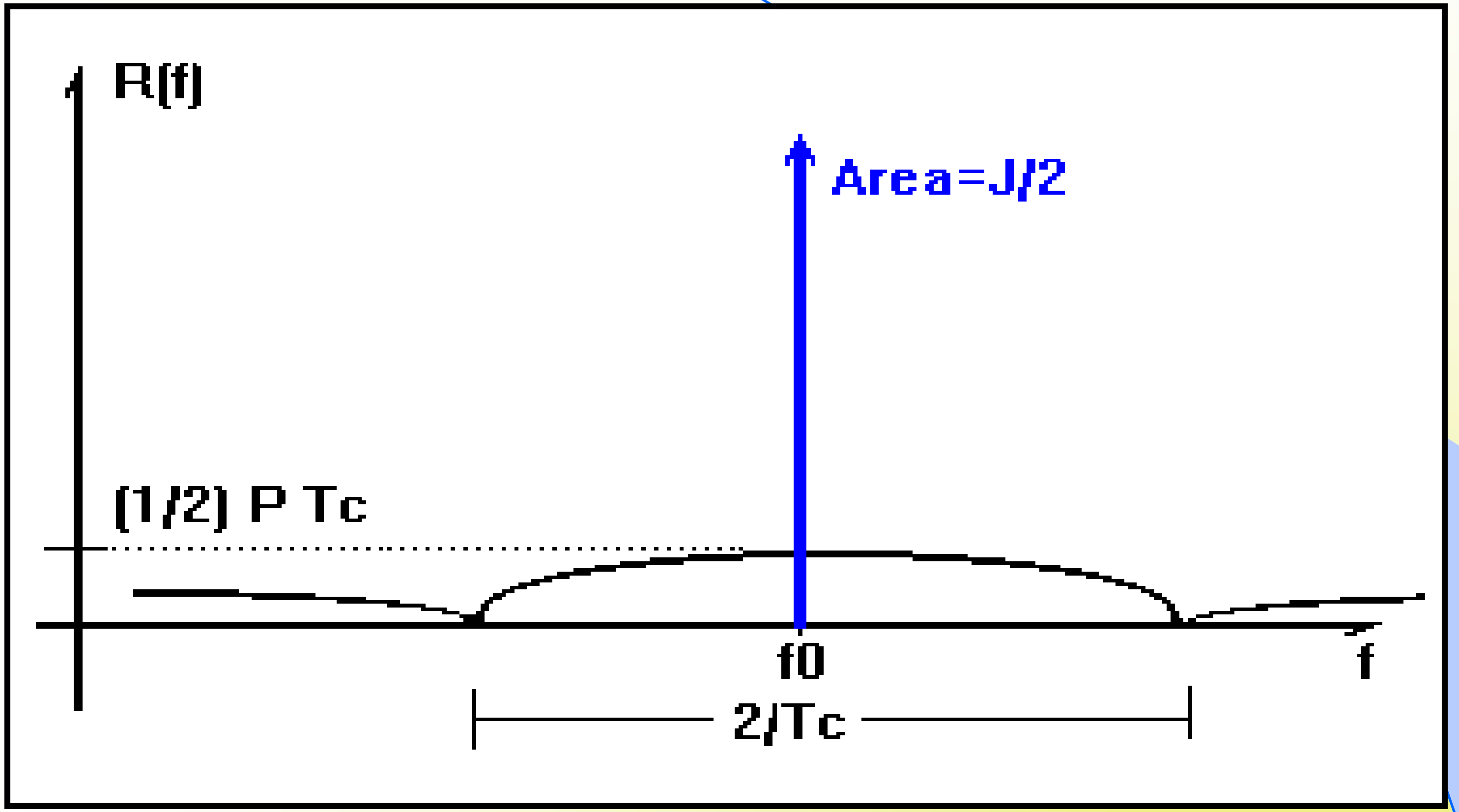
$$G(f) = \frac{PT_b}{2}[\mathrm{sinc}^2(f - f_0)T_b + \mathrm{sinc}^2(f + f_0)T_b]$$




Previous BPSK signal spread by a code with a chip rate $R_c=1/T_c$

- Note that spreading maintains unchanged the total power P;

- The ratio $G = R_c/R_b = T_b/T_c$ is known as processing gain and determines the interference rejection capability.




Previous signal and a centred tonal jammer with power J at receiver’s input

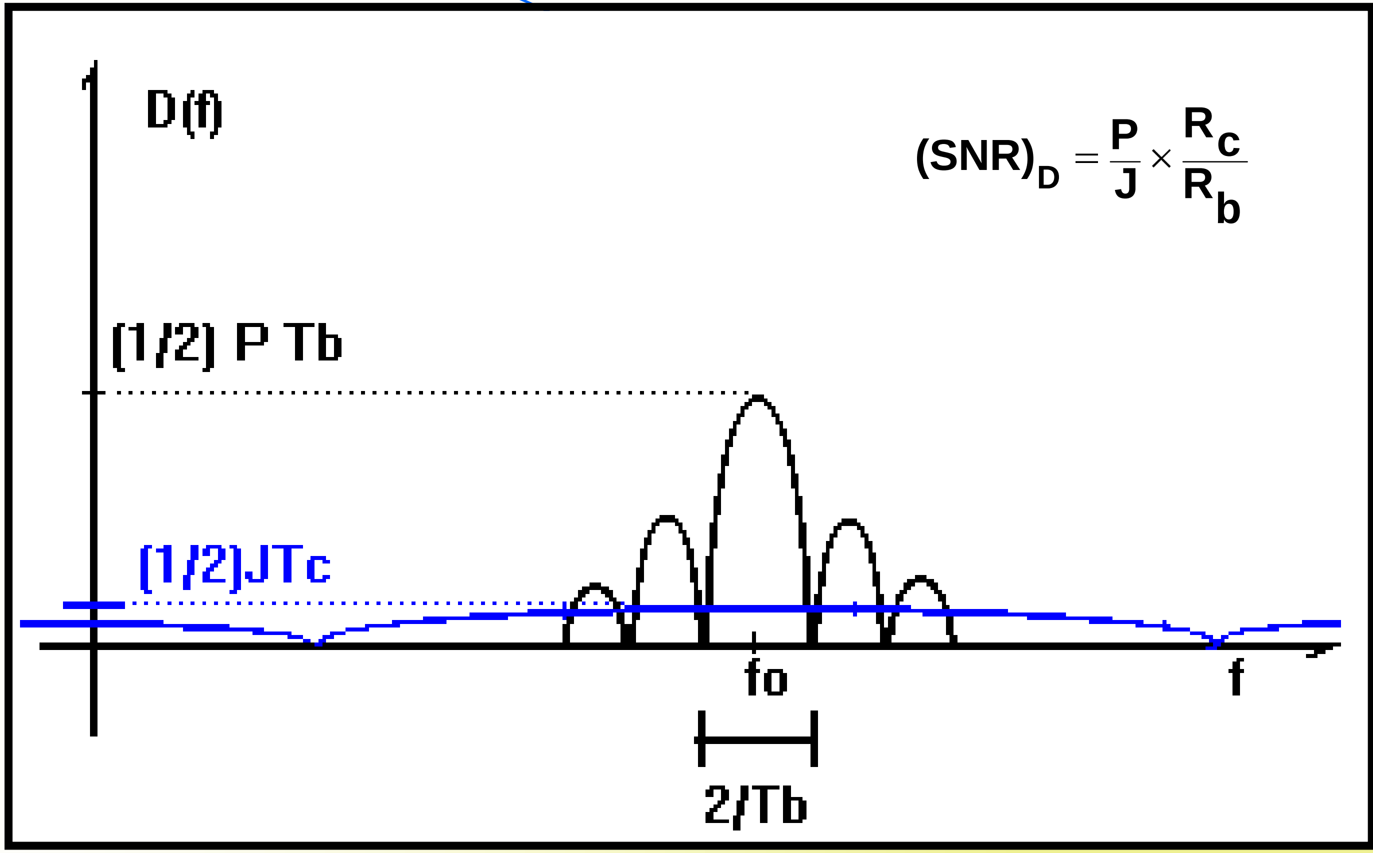
The composed signal at detector's input, r(t), can be written as

$$r(t) = s(t) + j(t)$$
$$s(t) = \sqrt{2P}d(t)p(t)co(\omega_0 t + \varphi)$$
$$j(t) = \sqrt{2J}\cos\omega_0 t$$

Admitting a perfect code synchronism (i. e., p(t) has exactly recovered in the synchronism stage $\Rightarrow p^2(t) = 1$) after de-spreading we have

$$r'(t) = r(t)p(t) = s'(t) + j'(t)$$
$$s'(t) = \sqrt{2P}d(t)co(\omega_0 t + \varphi)$$
$$j'(t) = \sqrt{2J}p(t)\cos\omega_0 t$$

Therefore the de-spread effect is to return the desirable signal to its original form and to spread the interference (next slide).




Previous signals now at detector’s output

This set of PSD figures shows the interference rejection capability and also the low probability of interception (LPI) for DS-SS signals.

# DS/SS Signals Synchronism

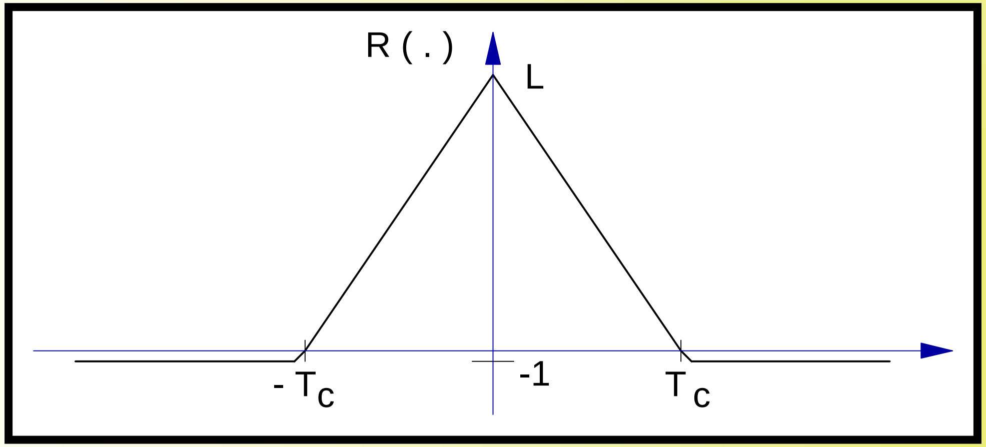
Synchronism is a two step task: acquisition and tracking.

The two more usual acquisition methods are:
Serial Search and Matched Filter.

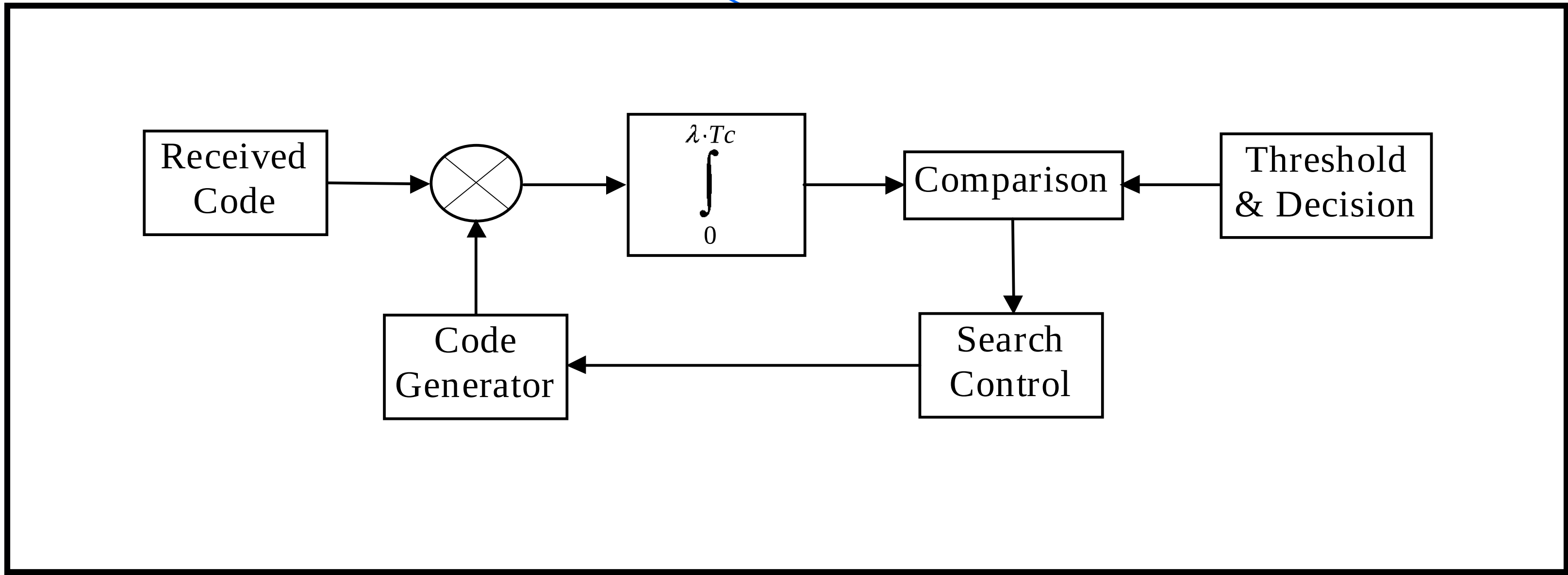
After the acquisition stage (which guarantees a $T_c/2$ uncertainty for the delay between received and local codes) the tracking loop is started.

The two more usual tracking loops are:
DLL-Delay Lock Loop and Tau-Dither.

Acquisition and tracking are based on the well known code sequences auto-correlation function (maximal length in the figure)



## Acquisition by Serial Search (Sliding Window)



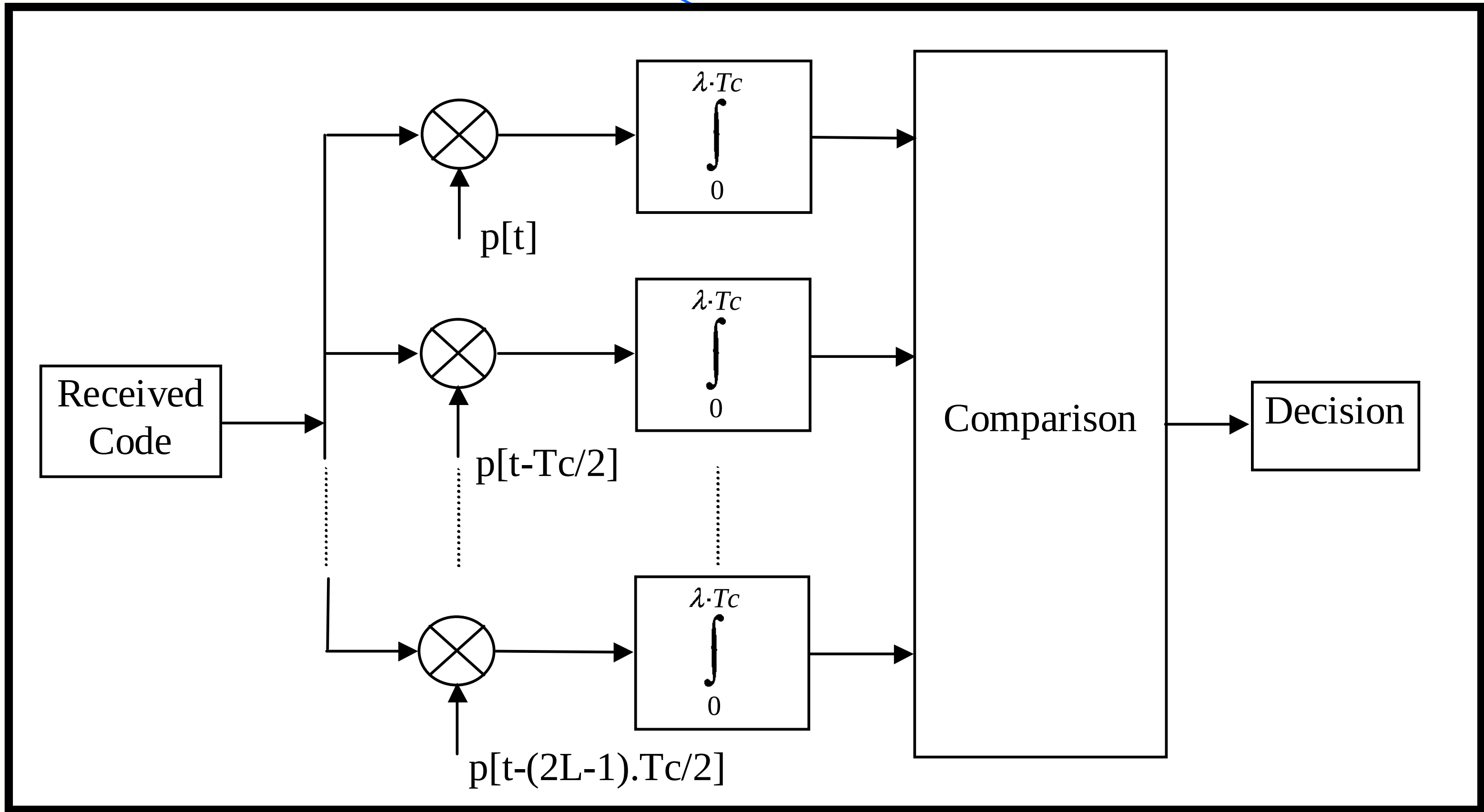

- Each successive search is carried out at $T_c/2$ intervals (i. e., 2L possible intervals where L is the code length);

- The mean acquisition time is given by $\overline{T}_{acq} = L\lambda T_c$

where $\lambda$ is a fraction of full sequence period, i. e., $0 < \lambda \leq L$ and its value is a compromise between $T_{acq}$ and false alarm probability;

- Less complexity × Greater acquisition time.

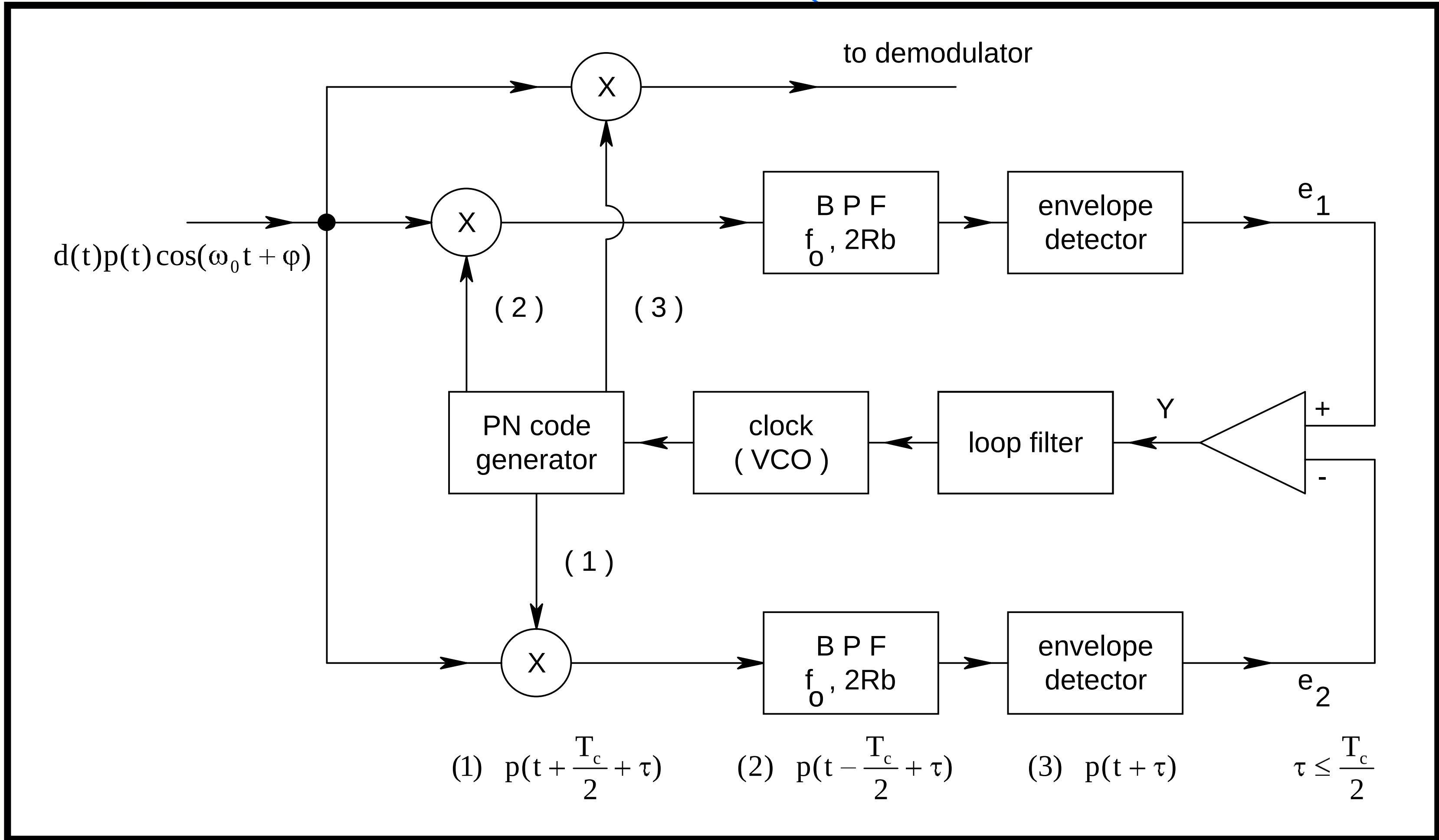
# Acquisition by Matched Filters



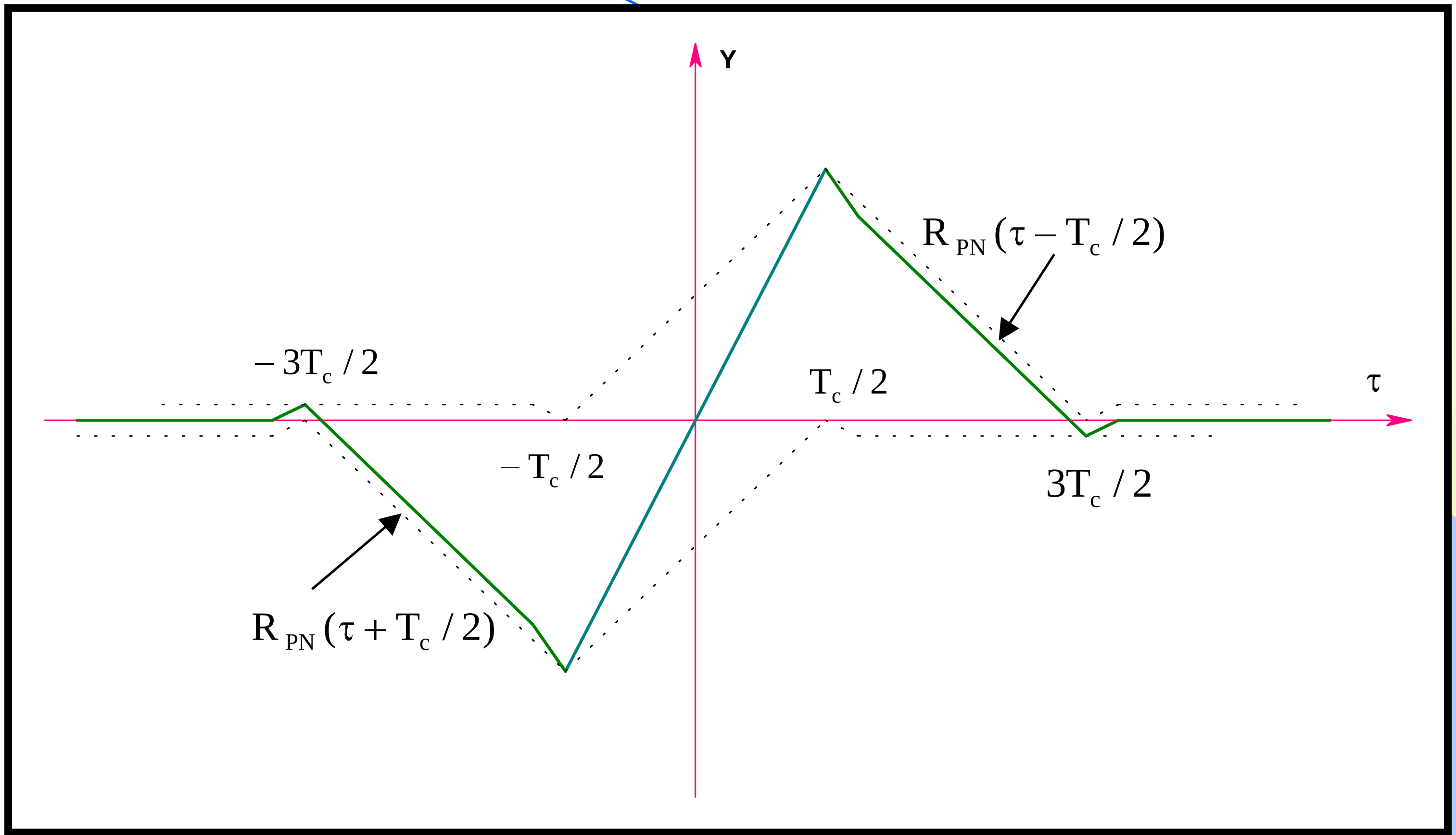

- All 2L possible search positions are checked in a parallel way;
- The acquisition time is given by $\overline{T}_{acq} = \lambda T_c$
- More complexity × Smaller acquisition time.

# DLL-Delay Lock Tracking Loop

The DLL starts after the acquisition stage which means that $|\tau| < T_c / 2$.



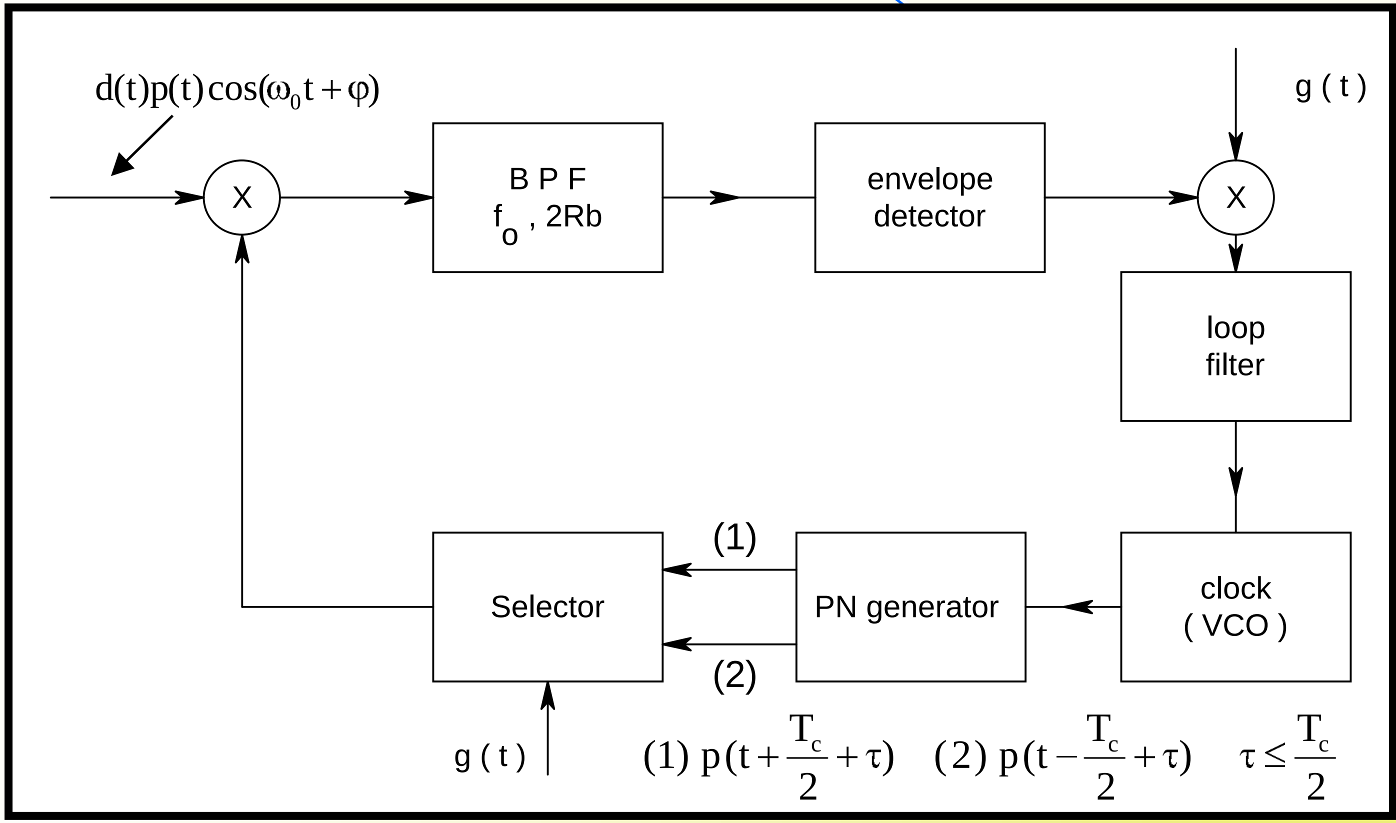

The tracking performance is based on the Y signal which acts as the VCO control signal (next figure).




In the range $\pm T_c/2$ the Y signal is a linear function of the delay and therefore this signal can be used to drive the VCO whose equilibrium point is Y=0.

## Tau-Dither Tracking Loop

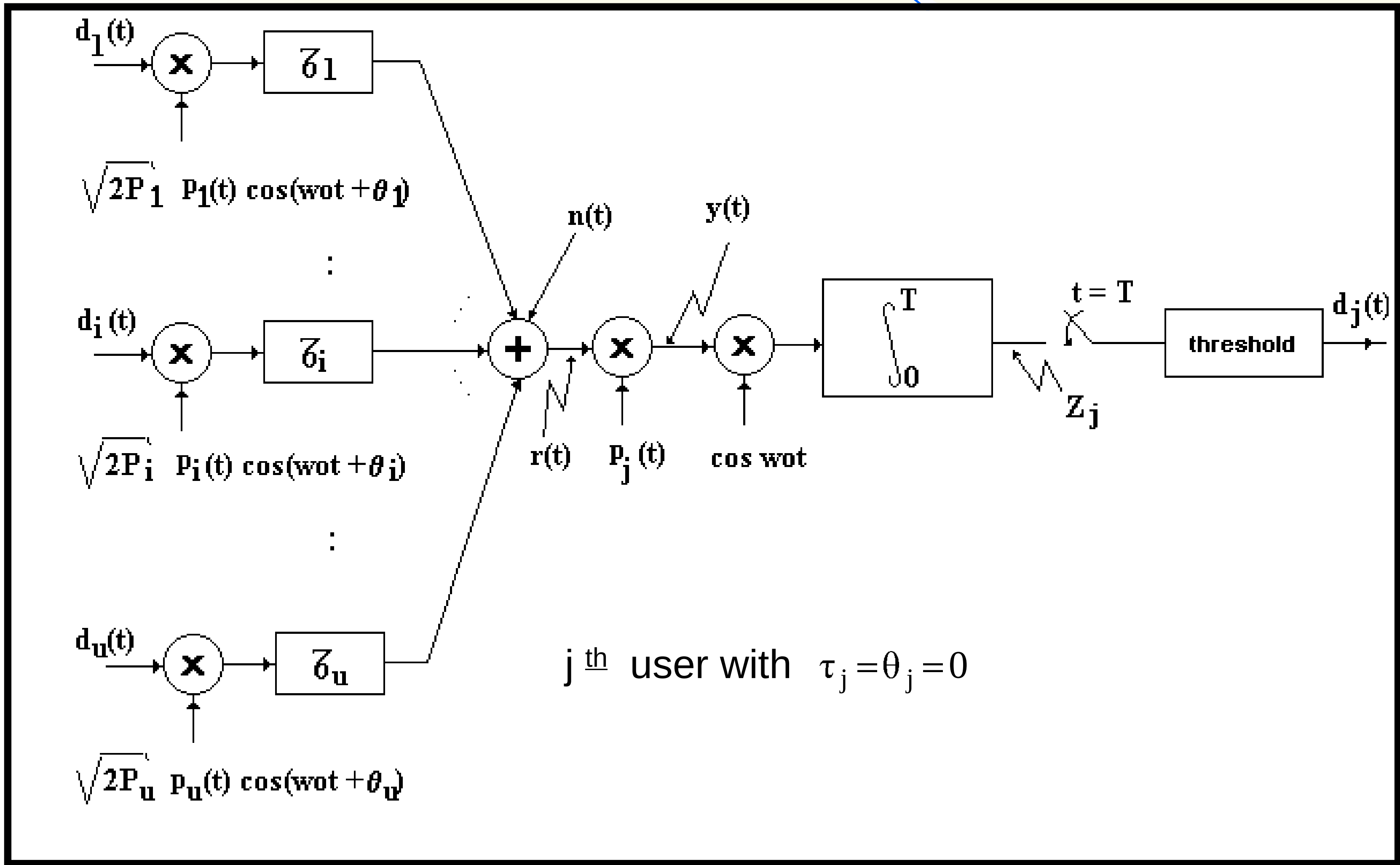
The Tau-Dither tracking loop is a time sharing version of DLL with only one correlator circuit avoiding therefore any mismatch between correlators.




The dither function g(t) (unitary bipolar square wave signal) selects the local code early or late version and also, in correspondence, the signal of the envelope detector output. See reference [22] for additional details.

# Asynchronous CDMA System Model / Single User Receiver (Pursley's Approach)

The single user performance determination will be based on next model



Where $$r(t) = \sum_{i=1}^{U} \sqrt{2P_i}\, d_i(t-\tau_i)\, p_i(t-\tau_i) \cos(\omega_0 t + \varphi_i) + n(t)$$

$p_i(.)$ = ± 1 in the interval $kT_c<t<(k+1)T_c$ with k=0;1;2;....... is the code sequence assumed with length L for all users;

$d_i(.)$ = ± 1 in the interval jT<t< (j+1)T with j=0;1;2; ....... is the information bit;

$T_c$ : is the chip interval;

T : is the information bit interval;

$N=T/T_c$ : is the processing gain and represents the number of chips during one bit interval (not to be confused with L, but here assumed equal to);

n(t) : is an Addictive White Gaussian Noise (AWGN) with bilateral PSD (Power Spectral Density) $S_n(f)=N_0/2$;

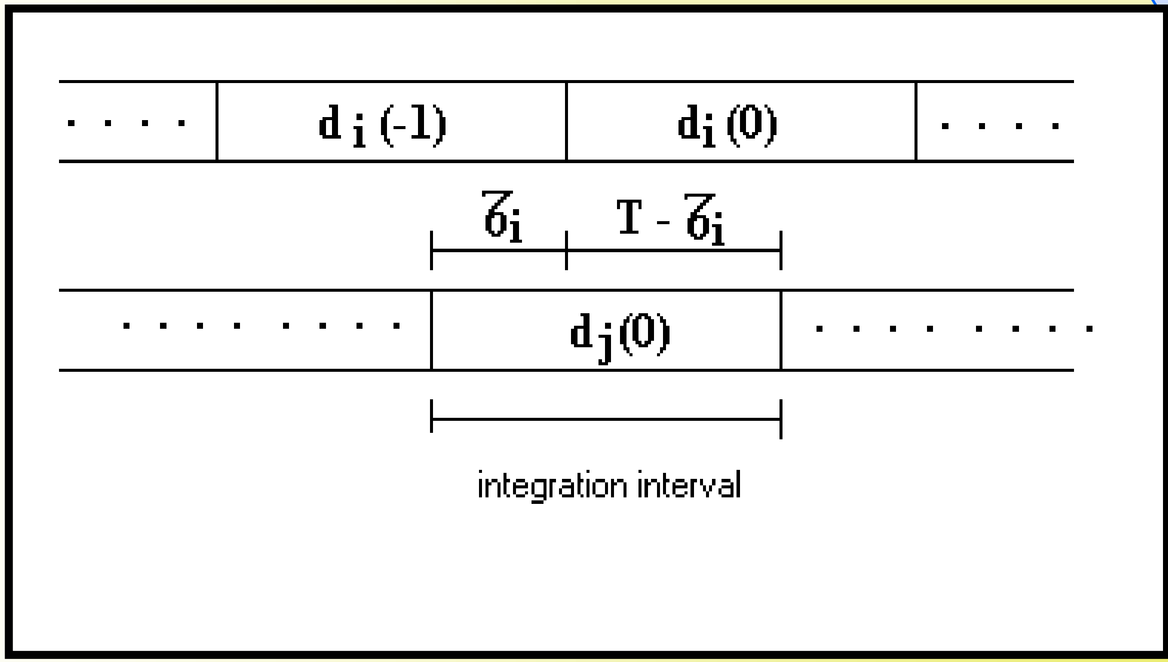
$\varphi_i = \theta_i - \omega_0 \tau_i$ : is the relative phase for the $i^{th}$ user.

With the assumption that we have a integer number of carrier cycles in a chip interval and perfect synchronism for desired j<u>th</u> signal, the integrator's output for this user can be expressed as

$$Z_j = \sum_{\substack{i=1 \\ i \neq j}}^{U} \sqrt{\frac{P_i}{2}} \int_0^T p_i(t-\tau_i)\, p_j(t)\, d_i(t-\tau_i) \cos\varphi_i \, dt + \int_0^T n(t)\, p_j(t) \cos\omega_0 t \, dt + \sqrt{\frac{P_j}{2}} T d_j(0)$$

(return)

<u>Integration interval showing the asynchronism between users</u>



Defining the even and odd continuous partial cross-correlation functions of sequences i and j, respectively, as

$$R_{i,j}(\tau_i) = \int_0^{\tau_i} p_i(t-\tau_i)\, p_j(t)\, dt$$

and

$$\hat{R}_{i,j}(\tau_i) = \int_{\tau_i}^{T} p_i(t-\tau_i)\, p_j(t)\, dt$$

and assuming that $d_j(0)=1$, we can express $Z_j$ as

$$Z_j = \sum_{\substack{i=1 \\ i\neq j}}^{U} \sqrt{\frac{P_i}{2}} [d_i(-1)\, R_{i,j}(\tau_i) + d_i(0)\, \hat{R}_{i,j}(\tau_i)] \cos\varphi_i + \int_0^T n(t)\, p_j(t) \cos\omega_0 t\, dt + \sqrt{\frac{P_j T^2}{2}} = \sum_{\substack{i=1 \\ i\neq j}}^{U} \alpha_i \cos\varphi_i + N_j + S_j$$

- the first term, named MAI - Multiple Access Interference, corresponds to the interference of $i^{\underline{th}}$ user in the $j^{\underline{th}}$ desired user and has zero mean value;

- the second term corresponds to interference dues to the AWGN, with zero mean value also;

- and finally the third term corresponds to the desired signal (information bit) and its value is the mean value of $Z_j$.

Now in order to find the signal to noise ratio ($SNR_j$) we should calculate the variance of $Z_j$. For this purpose we will assume that

- $\tau_i$ is uniformly distributed in the interval [ 0,T [

- $\varphi_i$ is uniformly distributed in the interval [ 0,2$\pi$ [

- $d_i(.)$ has equal probabilities of occurrence $\Rightarrow$ P(-1)=P(+1)=0,5

The desired information is a constant so we can write

$$V_{ar}\left[Z_j\right] = V_{ar}\left[\sum_{\substack{i=1 \\ i \neq j}}^{U} \alpha_i \cos\varphi_i\right] + V_{ar}\left[N_j\right]$$

For the second term we have

$$V_{ar}[N_j] = V_{ar}\left[\int_0^T n(t)\, p_j(t) \cos\omega_0 t dt\right] = V_{ar}\left[\sum_{k=0}^{L-1} p_j(k) \int_{kT_C}^{(k+1)T_C} n(t) \cos\omega_0 t dt\right]$$

It is easy to show that

$$\sum_{k=0}^{L-1} p_j(k) \int_{kT_C}^{(k+1)T_C} n(t) \cos\omega_0 t dt = N(0, N_0 T/4)$$

because, considering n(t) as AWGN, we can write

$$E\left\{\int_{kT_c}^{(k+1)T_c} n(t)\cos\omega_0 t\,dt\right\} = \int_{kT_c}^{(k+1)T_c} E[n(t)]\cos\omega_0 t\,dt = 0$$

and

$$E\left\{\int_{k_1T_c}^{(k_1+1)T_c} n(t)\cos\omega_0 t\,dt \int_{k_2T_c}^{(k_2+1)T_c} n(u)\cos\omega_0 u\,du\right\} = E\left\{\iint n(t)\,n(u)\cos\omega_0 t\cos\omega_0 u\,dt\,du\right\}$$

$$= \iint E\left[n(t)\,n(u)\right]\cos\omega_0 t\cos\omega_u\,dt\,du = \iint \frac{N_0}{2}\delta(t-u)\cos\omega_0 t\cos\omega_u\,dt\,du$$

$$= \frac{N_0}{2}\int_{kT_c}^{(k+1)T_c}\cos^2\omega_0\,t\,dt = N_0 T_c/4$$

and finally

$$\sum\sigma_k^2 = \sum_{k=0}^{L-1}\frac{N_0 T_c}{4} = \frac{LT_c N_0}{4} = \frac{N_0 T}{4}$$

Now we need to calculate the variance of the first term of $Z_j$ considering its independent random variables $\tau_i$ and $\varphi_i$ and $d_i(.)$ and its distribution function as defined earlier. The calculation will be done in the three steps. The first one considering $\varphi_i$ influence.

$$\mathrm{Var}\left[\sum_{\substack{i=1\\i\neq j}}^{U} \alpha_i \cos\varphi_i\right]_1 = \frac{1}{2\pi}\int_0^{2\pi}\left\{\sum_{\substack{i=1\\i\neq j}}^{U} \alpha_i \cos\varphi_i\right\}^2 d\varphi_i - \left\{\frac{1}{2\pi}\int_0^{2\pi}\sum_{\substack{i=1\\i\neq j}}^{U} \alpha_i \cos\varphi_i\right\}^2$$

$$= \frac{1}{2\pi}\sum_{\substack{i=1\\i\neq j}}^{U}\left\{\alpha_i^2\int_0^{2\pi}\cos^2\varphi_i\, d\varphi_i + \sum_{\substack{k=1\\k\neq j}}^{U}\alpha_k\cos\varphi_k\int_0^{2\pi}\alpha_i\cos\varphi_i\, d\varphi_i\right\} - \frac{1}{4\pi^2}\left\{\sum_{\substack{k=1\\k\neq j}}^{U}\alpha_i\int_0^{2\pi}\cos\varphi_i\, d\varphi_i\right\}^2$$

$$= \frac{1}{2\pi}\sum_{\substack{i=1\\i\neq j}}^{U}\alpha_i^2\int_0^{2\pi}\left(\frac{1}{2}+\frac{1}{2}\cos 2\varphi_i\right)d\varphi_i = \sum_{\substack{i=1\\i\neq j}}^{U}\frac{\alpha_i^2}{2}$$

In the second step we will consider the influence of the information bit of all other users.

$$\operatorname{Var}\left[\sum_{\substack{i=1\\i\neq j}}^{U}\alpha_i\cos\varphi_i\right]_2=\frac{1}{4}\cdot\frac{1}{2}\left\{\sum_{\substack{i=1\\i\neq j}}^{U}P_i\left(R_{i,j}(\tau_i)+\hat{R}_{i,j}(\tau_i)\right)^2+\sum_{\substack{i=1\\i\neq j}}^{U}P_i\left(R_{i,j}(\tau_i)-\hat{R}_{i,j}(\tau_i)\right)^2\right\}$$

$$=\frac{P}{4}\left\{\sum_{\substack{i=1\\i\neq j}}^{U}\left(R_{i,j}^2(\tau_i)+\hat{R}_{i,j}^2(\tau_i)\right)\right\}$$

For this final expression we additionally have assumed that all users have the same received power (perfect power control to avoid the near far problem; this restriction will be considered later).

Finally for the third and last step, we have to calculate the delays' influence on previous variance. For this objective we need some additional definitions, references [1] and [7]. Given two sequences $a_n$ and $b_n$, with its elements in [1,-1], both periodic with period p

$$\{a_n\} = \{a_0, a_1, \cdots, a_k, \cdots, a_{p-1}, a_0, \cdots\} \qquad \{b_n\} = \{b_0, b_1, \cdots, b_k, \cdots, b_{p-1}, b_0, \cdots\}$$

The periodical cross-correlation between $a_n$ and $b_n$ is another sequence with the same period and defined by

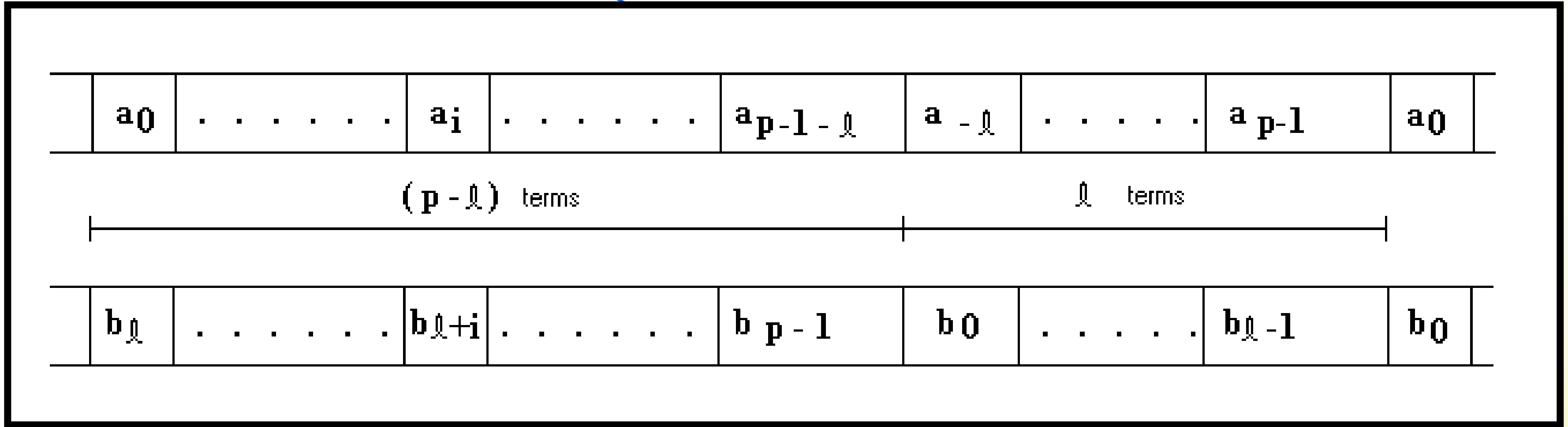
$$\theta_{a,b}(\ell) = \sum_{i=0}^{p-1} a_i b_{i+\ell} \quad \text{with } \ell \in Z$$

With this definition $\theta_{a,b}(.)$ can be interpreted as the number of agreements minus the number of disagreements between the two sequences in a complete period as a function of the relative delay between the two sequences.

The partial cross-correlation between $a_n$ and $b_n$ is a function defined by

$$C_{a,b}(\ell) = \begin{cases} \sum_{i=0}^{p-1-\ell} a_i b_{i+\ell} & 0 \le \ell \le p-1 \\ \sum_{i=0}^{p-1+\ell} a_{i-\ell} b_i & 1-p \le \ell < 0 \\ 0 & |\ell| \ge p \end{cases}$$

The next figure give us an interpretation for this function.




for $0 \le \ell \le p-1$ we have $(p-\ell)$ terms given by

$$C_{a,b}(\ell) = a_0 b_\ell + a_1 b_{\ell+1} + \ldots\ldots\ldots + a_{p-1-\ell} b_{p-1}$$

for $1-p \le \ell < 0$ we have $(p-|\ell|)$ terms given by

$$C_{a,b}(\ell) = a_{-\ell} b_0 + a_{-\ell+1} b_1 + \ldots\ldots\ldots + a_{p-1} b_{p-1+\ell}$$

With these definitions we can easily show that

$$\theta_{a,b}(0) = C_{a,b}(0)$$

$$\theta_{a,b}(\ell) = \theta_{a,b}(\ell + p)$$

$$\theta_{a,b}(-\ell) = \theta_{b,a}(\ell)$$ and $$C_{a,b}(-\ell) = C_{b,a}(\ell)$$

$$\theta_{a,b}(\ell) = C_{a,b}(\ell) + C_{a,b}(\ell - p) \quad \text{for } |\ell| \leq p$$

For two identical sequences we introduce the notation

$$\{a_n\} = \{b_n\} \Rightarrow \theta_{a,a}(.) = \theta_a(.) \text{ and } C_{a,a}(.) = C_a(.)$$

and rename the functions as periodic auto-correlation and partial auto correlation, respectively.

With these definitions and properties we are able to return to the original problem and for the last step we can write

$$\mathrm{Var}\left[\sum_{\substack{i=1\\ i\neq j}}^{U} \alpha_i \cos\varphi_i\right]_3 = \frac{P}{4T}\sum_{\substack{i=1\\ i\neq j}}^{U}\int_0^T \left(R_{i,j}^2(\tau_i) + \hat{R}_{i,j}^2(\tau_i)\right) d\tau_i$$

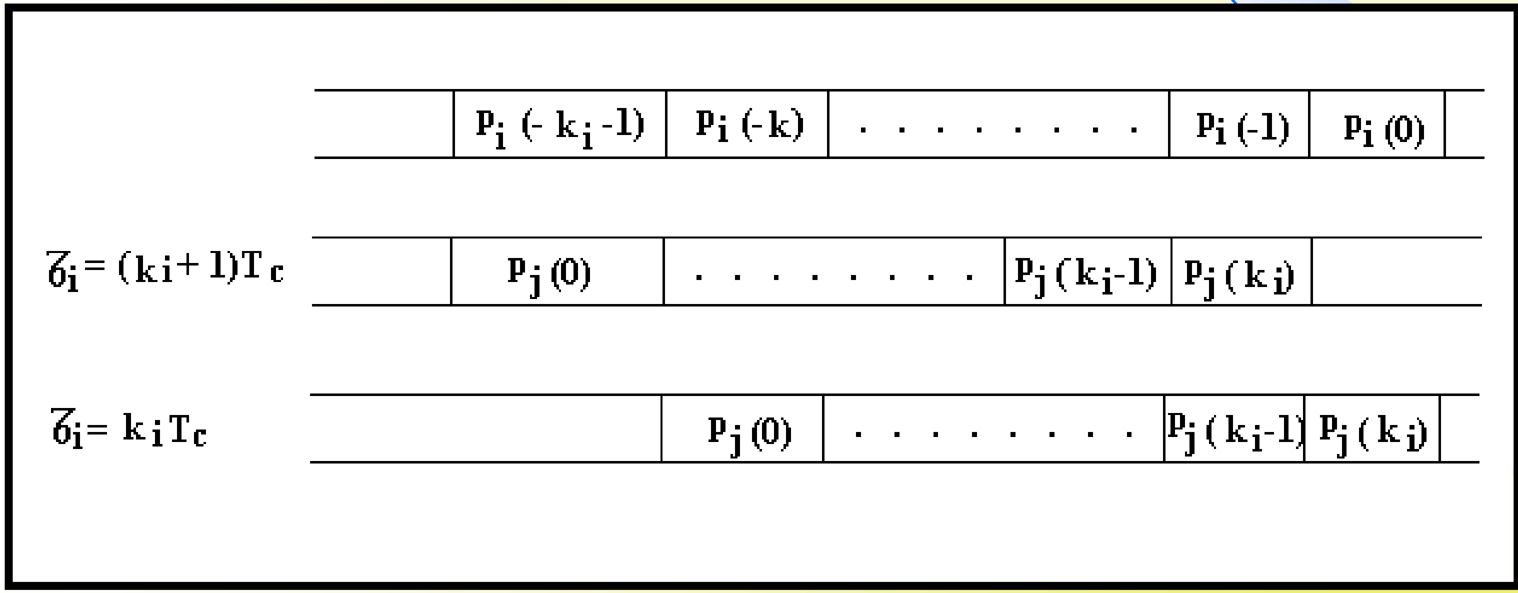
$$= \frac{P}{4T}\sum_{\substack{i=1\\ i\neq j}}^{U}\sum_{m=0}^{L-1}\int_{mT_C}^{(m+1)T_C} \left(R_{i,j}^2(\tau_i) + \hat{R}_{i,j}^2(\tau_i)\right) d\tau_i$$

Now it is necessary to establish the relation between $R_{i,j}(.)$ and $C_{i,j}(.)$

The relative delay between users i and j can be written as

$$0 \le k_i T_c \le \tau_i < (k_i + 1)T_c < LT_c$$

Considering two consecutive chip intervals for this two users we have




Evaluating $R_{i,j}(.)$ in these two extremes values for $\tau_i$ and considering rectangular pulses for the chips we have

$$R_{i,j}(\tau_i = k_i T_c) = \int_0^{k_i T_c} p_i(t - k_i T_c)\, p_j(t)\,dt = T_c \sum_{m=0}^{k_i - 1} p_i(m - k_i)\, p_j(m)$$

and with the substitution $k_i - 1 = L - 1 + \ell$ we have

$$R_{i,j}(\tau_i = k_i T_c) = T_c \sum_{m=0}^{L-1+\ell} p_i(m - L - \ell) p_j(m) = T_c \sum_{m=0}^{L-1+\ell} p_i(m - \ell) p_j(m)$$

and then

$$R_{i,j}(\tau_i = k_i T_c) = T_c C_{i,j}(\ell) = T_c C_{i,j}(k_i - L)$$

For the other interval's extreme $\tau_i = (k_i + 1)T_c$ and we can write

$$R_{i,j}\left[\tau_i = (k_i+1)T_c\right] = \int_0^{(k_i+1)T_c} p_i\left[t-(k_i+1)T_c\right]p_j(t)dt = T_c \sum_{m=0}^{k_i} p_i\left[m-(k_i+1)\right]p_j(m)$$

and now putting $k_i = L - 1 + \ell$ we have

$$R_{i,j}\left[\tau_i = (k_i+1)T_c\right] = T_c \sum_{m=0}^{L-1-\ell} p_i(m-L-\ell)\,p_j(m) = T_c \sum_{m=0}^{L-1+\ell} p_i(m-\ell)\,p_j(m)$$

So similar to the other extreme we obtain

$$R_{i,j}\left[\tau_i = (k_i+1)T_c\right] = T_c C_{i,j}(\ell) = T_c C_{i,j}(k_i - L + 1)$$

Now observing that the function is linear between these two points we can establish the relation

$$R_{i,j}[\tau_i] = T_c C_{i,j}(k_i - L) + \left[C_{i,j}(k_i - L + 1) - C_{i,j}(k_i - L)\right](\tau_i - k_i T_c)$$

and with a completely analogous procedure we obtain

$$\hat{R}_{i,j}[\tau_i] = T_c C_{i,j}(k_i) + \left[C_{i,j}(k_i + 1) - C_{i,j}(k_i)\right](\tau_i - k_i T_c)$$

For the variance expression calculation we need to evaluate integrals of $R^2_{i,j}[\tau_i]$ and $\hat{R}^2_{i,j}[\tau_i]$, which are linear in their variables $\tau_i$. So we have an general expression in the form

$$R_{i,j}(\tau) = A + B\,\tau \Rightarrow \int_{\ell T_c}^{(\ell+1)T_c} R_{i,j}^2(\tau)\,d\tau = \left. A^2\,\tau + A\,B\,\tau^2 + B^2\,\frac{\tau^3}{3} \right|_{\ell T_c}^{(\ell+1)T_c}$$

$$= A^2 T_c + A\,B\,(2\ell+1)\,T_c^2 + \frac{B^2}{3}\,(3\ell^2 + 3\ell + 1)\,T_c^3$$

Now with $\ell = k_i$ we obtain the following values for A and B

$$A = (\ell+1)T_c\,C_{i,j}(\ell-L) - \ell\,T_c\,C_{i,j}(\ell-L+1)$$

$$B = C_{i,j}(\ell-L+1) - C_{i,j}(\ell-L)$$

We need to do the same procedure for $\hat{\mathbf{R}}^2_{i,j}[\tau_i]$ and finally we are able to evaluate the expression

$$\sum_{m=0}^{L-1} \int_{mT_C}^{(m+1)T_C} \left( R_{i,j}^2(\tau) + \hat{R}_{i,j}^2(\tau) \right) d\tau$$

This final step is very simple but tedious; the result is

$$V_{ar}\left[Z_j\right] = \frac{P\,T^2}{12\,L^3} \sum_{\substack{i=1 \\ i \neq j}}^{U} \beta_{i,j} + \frac{N_0 T}{4}$$

where

$$\beta_{i,j} = \sum_{\ell=0}^{L-1} \left\{ C_{i,j}^2(\ell-L) + C_{i,j}(\ell-L) C_{i,j}(\ell-L+1) + C_{i,j}^2(\ell-L+1) + C_{i,j}^2(\ell) + C_{i,j}(\ell) C_{i,j}(\ell+1) + C_{i,j}^2(\ell+1) \right\}$$

With this expression we are able to determine Var[$Z_j$] if we know the code sequences used by each user in the system. The result depends on the partial cross-correlation between sequences and can be simplified. This simplification, originally done by Pursley, references [3] and [4], is strongly based in correlation properties and its details are beyond the scope of this presentation. Complete details for this derivation can be found in [1]. The result is

$$\beta_{i,j} = 2L^2 + 4\sum_{\ell=1}^{L-1} C_i(\ell)\, C_j(\ell) + \sum_{\ell=1-L}^{L-1} C_i(\ell)\, C_j(\ell+1)$$

Where now the calculation is based only on partial auto-correlation functions instead of cross-correlation functions. This is a substantial simplification for $\beta_{i,j}$ determination. Finally with this variance determination we can express the signal to noise ratio as

$$(SNR)_j = \frac{E[Z_j]}{\sqrt{Var[Z_j]}} = \left\{ \frac{1}{6L^3} \sum_{\substack{i=1 \\ i \neq j}}^{U} \beta_{i,j} + \frac{N_0}{2PT} \right\}^{-1/2} = \left\{ \frac{1}{6L^3} \sum_{\substack{i=1 \\ i \neq j}}^{U} \beta_{i,j} + \frac{N_0}{2E_b} \right\}^{-1/2}$$

and if we have a great number of users we can assume a gaussian approximation for the BER (Bit Error Rate) determination as

$$P_{e,j} = Q[(SNR)_j]$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp(-y^2/2)dy$$

This gaussian assumption causes some problems concerning Central Limit Theorem applicability, see references [5] and [6]. For practical purpose we can use it and later we will introduce a more general calculation that will overcome some eventual theoretical limitations.

With an additional simplification in mind we will consider two random sequences selected from the set of all $2^L$ binary sequences of length L, where each sequence has the same probability for its selection from the set. It is a simple matter to show that for two randomly selected sequences we have

$$E[\beta_{i,j}] = 2L^2$$

and

$$(SNR)_j = \left[ \left\{ \frac{U-1}{3L} + \frac{N_0}{2E_b} \right\}^{-1/2} \right]$$

Which allows us to express the BER in an approximate, but very useful form

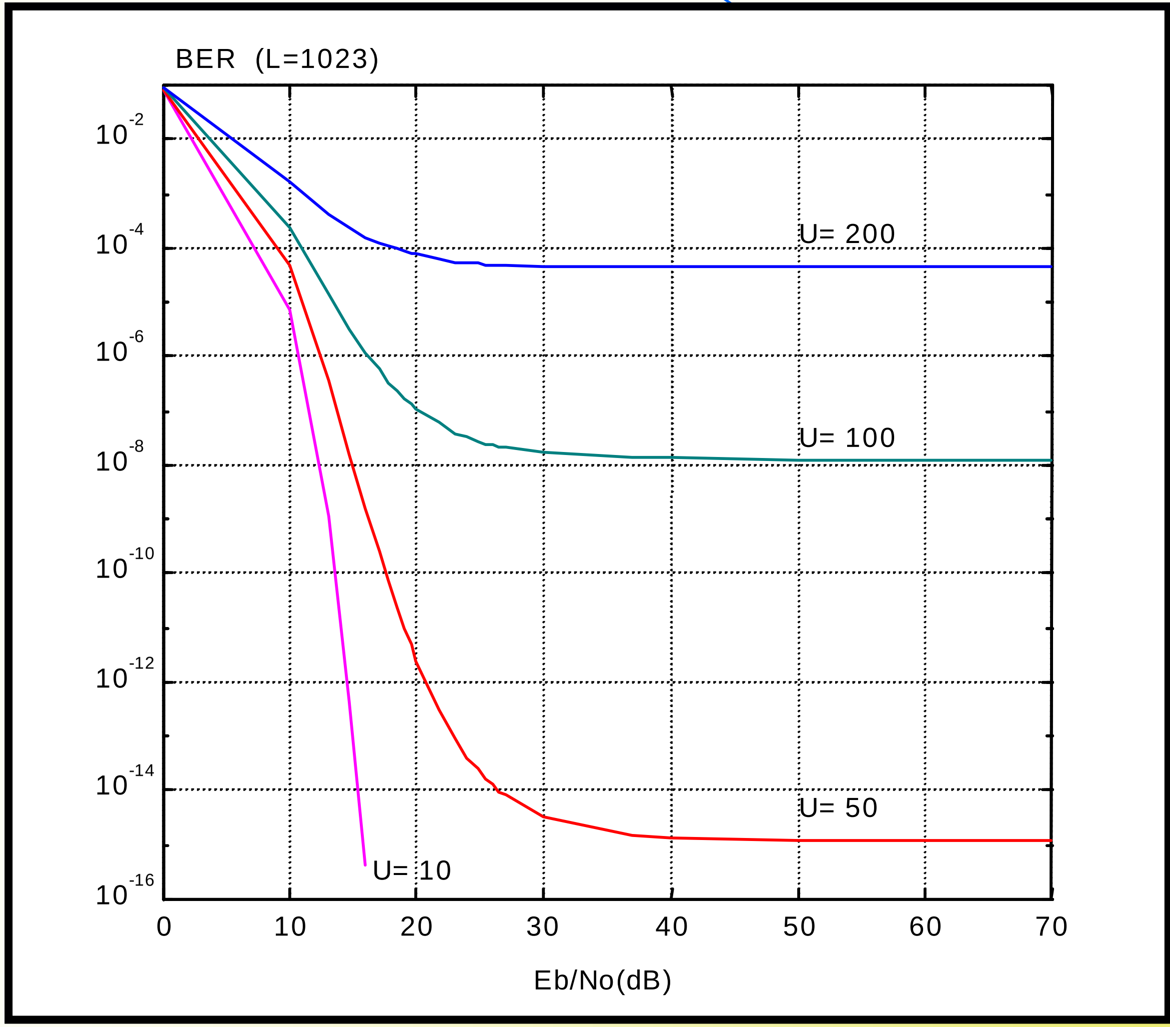
$$P_{e,j} = Q[(SNR)_j] = Q\left[ \left( \frac{U-1}{3L} + \frac{N_0}{2E_b} \right)^{-1/2} \right]$$

(return)

obs.: with U=1 we obtain the well known result

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

In the next figure we show this final result for L=1023 and particular set of values for U.



For a given U value, the asymptotic result (when the relation $E_b/N_o$ grows) shows us basically the MAI limitation. For this region the BER is given by

$$P_e = Q[(SNR)] = Q\left(\sqrt{\frac{3L}{U-1}}\right)$$

<u>Numerical Example</u>

For instance if we impose a typical desirable performance of $P_e=10^{-3}$ we should have $(E_b/N_0)_{eq}=4{,}8$ and in this case (SNR)=3,10.

If we use the value $(E_b/N_0)_{eq}=10{,}0 \Rightarrow U \approx 0{,}162L+1$, and for a system which use L=1023 we could have ≈ 166 users. So, with one user and $(E_b/N_0)_{.}=10{,}0$ we have a BER of $P_e= 4\times10^{-6}$ and with 166 users this performance degrades to $P_e=10^{-3}$.

To get some additional insight on this numerical result it is interesting to compare it with the fundamental limit given by Shannon's channel capacity theorem. As we know the channel capacity of a system is given by

$$C = B \times \log_2\left(1+\frac{S}{N_0\,B}\right) \quad \text{bits/s}$$

For B growing C approaches to $C \Rightarrow \frac{S}{N_0 \ln 2}$ bits/s showing us that we can reach C bits/s increasing the user's bandwidth. In our notation

$$f_0 \ln 2 \Rightarrow \frac{E_b f_0}{(U-1)N_i + N_0} = \frac{E_b f_0}{(U-1)E_b f_0 T_c + N_0}$$

with $f_0$=1/T and $N_i$ representing one user's equivalent power spectral density.

So in this case we have $U \Rightarrow \left[\frac{1}{\ln 2} - \frac{N_0}{E_b}\right] L + 1 \cong 1{,}44\,L$ where we have neglected the second term.

With the previous value of L now we have U≈1470 users. These numbers show us a poor performance (only 11%) if we compare our former solution with this theoretical limit.

# Asynchronous CDMA System Model / Single User Receiver (Worst Case Approach)

Returning to the previous complete expression for $Z_j$ (compare) and considering again the equal powers case we can rewrite it as

$$Z_j = \sqrt{\frac{PT^2}{2}}[d_j(0) + \gamma_j(\underline{d},\underline{\tau},\underline{\varphi})] + N_j$$

with

$$\gamma_j(\underline{d},\underline{\tau},\underline{\varphi}) = \sum_{\substack{i=1 \\ i \neq j}}^{U} I_{i,j}(\underline{d_i},\tau_i,\varphi_i)$$

Where the normalised interference from user i on user j can be expressed as

$$I_{i,j}(\underline{d_i},\tau_i,\varphi_i) = T^{-1}[d_i(-1)R_{i,j}(\tau_i) + d_i(0)\hat{R}_{i,j}(\tau_i)]\cos\varphi_i$$

Considering equal probabilities for the information bits

$$P_e = \frac{1}{2}\text{Prob}[d_j(0) = 1 \text{ and } Z_j < 0] + \frac{1}{2}\text{Prob}[d_j(0) = -1 \text{ and } Z_j > 0]$$

Considering the symmetry

$$P_e = \text{Prob}[d_j(0)=1 \text{ and } Z_j < 0] = \text{Prob}[\sqrt{\frac{PT^2}{2}}[1+\gamma_j(\underline{b},\underline{\tau},\underline{\varphi})] + N_j < 0]$$

$$P_e = \text{Prob}\{N_j < -\sqrt{\frac{PT^2}{2}}[1+\gamma_j(\underline{b},\underline{\tau},\underline{\varphi})]\} = \text{Prob}\{N_j > \sqrt{\frac{PT^2}{2}}[1+\gamma_j(\underline{b},\underline{\tau},\underline{\varphi})]\}$$

We have

$$P_e = Q\{\sqrt{\frac{2PT}{N_0}}[1+\gamma_j(\underline{b},\underline{\tau},\underline{\varphi})]\}$$

The function Q(x) varies monotonically with x. So, for its maximisation (worst case) we need to minimise the argument.

The symmetry of $\gamma_i(\underline{d},\underline{\tau},\underline{\varphi})$ allow us to write

$$\min[\gamma_i(\underline{d},\underline{\tau},\underline{\varphi})] = -\max[\gamma_i(\underline{d},\underline{\tau},\underline{\varphi})] = -\max[|\gamma_i(\underline{d},\underline{\tau},\underline{\varphi})|] = -\sum_{\substack{i=1\\ i\neq j}}^{U} \max[|I_{i,j}(\underline{d_i},\tau_i,\varphi_i)|]$$

Therefore we should maximise

$$I_{i,j}(\underline{d_i},\tau_i,\varphi_i) = T^{-1}.[d_i(-1)R_{i,j}(\tau_i) + d_i(0)\hat{R}_{i,j}(\tau_i)].\cos\varphi_i$$

considering $\varphi_i$'s influence and observing that

$$\max[\cos\varphi_i] = 1 \quad \text{and} \quad \min[\cos\varphi_i] = -1$$

we can write

$$-T^{-1}\max[|d_i(-1)R_{i,j}(\tau_i) + d_i(0)\hat{R}_{i,j}(\tau_i)|] < I_{i,j}(\underline{d_i},\tau,\varphi) < T^{-1}\max[|d_i(-1)R_{i,j}(\tau_i) + d_i(0)\hat{R}_{i,j}(\tau_i)|]$$

Next considering all possible combinations for the interfering bit

$$T^{-1}\max\left[\left|d_i(-1)R_{i,j}(\tau_i)+d_i(0)\hat{R}_{i,j}(\tau_i)\right|\right]=T^{-1}\max\left[\left|R_{i,j}(\tau_i)\right|+\left|\hat{R}_{i,j}(\tau_i)\right|\right]$$

Therefore we have the intermediate result

$$-T^{-1}.\max\left[\left|R_{i,j}(\tau_i)\right|+\left|\hat{R}_{i,j}(\tau_i)\right|\right]<I_{i,j}(\underline{d_i},\tau,\varphi)<T^{-1}.\max\left[\left|R_{i,j}(\tau_i)\right|+\left|\hat{R}_{i,j}(\tau_i)\right|\right]$$

Finally the function $\left|R_{i,j}(\tau_i)\right|+\left|\hat{R}_{i,j}(\tau_i)\right|$ has local extremes in multiples of $T_c$

(remember that this function is linear between points in which the arguments are successive multiples of $T_c$). If we denote by $\varepsilon_{i,j}$ the largest value of the above function calculated in multiples of $T_c$, i. e. ,

$$\tau_i=0,\ T_c,\ 2T_c,\ldots,\ (N-1)T_c$$

Therefore we can bound each normalised interference to the interval

$$-T^{-1}\varepsilon_{i,j} < I_{i,j}(\underline{d}_i, \tau, \varphi) < T^{-1}\varepsilon_{i,j}$$

And finally considering all users

$$P_{e,j} = Q\{\sqrt{\frac{2PT}{N_0}}[1 - T^{-1}\sum_{\substack{i=1\\i\neq j}}^{U}\varepsilon_{i,j}]\}$$

Which is the worst case performance, see reference [2]. Obviously the best case performance is given by

$$P_{e,j} = Q\{\sqrt{\frac{2PT}{N_0}}[1 + T^{-1}\sum_{\substack{i=1\\i\neq j}}^{U}\varepsilon_{i,j}]\}$$

# Asynchronous CDMA System Model / Single User Receiver (Weber's Approach)

In this approach, reference [6], all users are considered independent and degrade other users performance through an equivalent PSD. This approach is based on the following equation

$$(SNR)_U = \frac{2E_b}{N_o + \sum_{i=1}^{U-1} N_i}$$

Where

$(SNR)_U$ : is desired user's SNR in presence of other (U-1) users

$N_o$ : is the unilateral PSD of AWGN

$N_i$ : is the equivalent PSD for one user's interference which can be obtained by

$$N_i = \frac{P_i}{R_c} = P_i\, T_c = \alpha_i\, P_o\, T_c$$

where

$P_i$ : absolute power for i-th user

$R_c$ : chip rate ($R_c = 1/T_c$) common to all users

$\alpha_i$ : power factor with $P_0$ as reference

and from the relation $(SNR)_U$ we can determine the bit error probability with a gaussian assumption

$$P_e = Q[\,(\sqrt{(SNR)_U}\,]$$

From previous equations and a perfect power control assumption ($\alpha_i$=1 for all users) we can derive

$$(SNR)_U = \frac{2E_b}{N_o + \sum_{i=1}^{U-1} N_i} = \frac{1}{\frac{N_0}{2E_b} + \frac{(U-1)P_i T_c}{2E_b}} = \frac{1}{\frac{N_0}{2E_b} + \frac{(U-1)P_i T}{2E_b L}}$$

Therefore

$$P_e = Q\left[\left(\frac{U-1}{2L} + \frac{N_0}{2E_b}\right)^{-1/2}\right]$$

Comparing this result with the Pursley's approach we can see that Weber's approach underestimate the CDMA performance when compared with Pursley's approach.

# (A)synchronous CDMA System Model / Single User Receiver (Morrow and Lehnert 's Results)

Considering random sequences for user's spreading codes Morrow and Lehnert establish four additional results as presented below (see reference [15] for developments' details).

**1** - synchronous carrier's phase and chip's delay.

$$P_e = Q[(\frac{U-1}{N} + \frac{N_o}{2E_b})^{-1/2}]$$

**2** - synchronous carrier's phase and chip's delay uniformly distributed in the interval [0,T].

$$P_e = Q[(\frac{2(U-1)}{3N} + \frac{N_o}{2E_b})^{-1/2}]$$

**3** - synchronous chip's delay and carrier's phase uniformly distributed in the interval [0,2$\pi$]. This result coincides with Weber's approach.

$$P_e = Q[(\frac{U-1}{2N} + \frac{N_o}{2E_b})^{-1/2}]$$

**4** - chip's delay and carrier's phase uniformly distributed in the interval [0,T] and [0,2$\pi$], respectively. Obviously, this result is identical to Pursley's approach with random codes.

$$P_e = Q[(\frac{U-1}{3N} + \frac{N_o}{2E_b})^{-1/2}]$$

# Asynchronous CDMA System Model / Single User Receiver (Nazari and Ziemer's Approach)

In this approach, reference [8], we return to the general normalised interference expression

$$I_{i,j}(\underline{d_i},\tau_i,\varphi_i) = T^{-1}[d_i(-1)R_{i,j}(\tau_i) + d_i(0)\hat{R}_{i,j}(\tau_i)]\cos\varphi_i$$

(return)

and try to find the exact distribution for each interference. After this individual determination we will obtain the final interference distribution by convolution of various contributions. So

1) if $d_i(-1)=d_i(0)$

$$I_{i,j}(\underline{d_i},\tau_i,\varphi_i) = T^{-1} d_i(0)[R_{i,j}(\tau_i) + \hat{R}_{i,j}(\tau_i)]\cos\varphi_i$$

Exactly in a similar form as we have obtained in the Pursley's approach it can be shown that

$$\Re_{i,j}(\tau_i) = R_{i,j}(\tau_i) + \hat{R}_{i,j}(\tau_i) = (T_c - \tau + \ell_i T_c)\theta_{i,j}(\ell_i) + (\tau - \ell_i T_c)\theta_{i,j}(\ell_i + 1)$$

Where $\ell_i$ is the integer part of the relation $\tau_i/T_c$ and

$$\theta_{i,j}(\ell) = C_{i,j}(\ell) + C_{i,j}(\ell - N)$$

2) if $d_i(-1) \neq d_i(0)$

$$I_{i,j}(\underline{d}_i, \tau_i, \varphi_i) = T^{-1} d_i(0)[R_{i,j}(\tau_i) - \hat{R}_{i,j}(\tau_i)]\cos\varphi_i$$

$$\hat{\Re}_{i,j}(\tau) = R_{i,j}(\tau_i) - \hat{R}_{i,j}(\tau_i) = (T_c - \tau + \ell_i . T_c)\hat{\theta}_{i,j}(\ell_i) + (\tau - \ell_i . T_c)\hat{\theta}_{i,j}(\ell_i + 1)$$

where now

$$\hat{\theta}_{i,j}(\ell) = C_{i,j}(\ell) - C_{i,j}(\ell - N)$$

Next we will find the distribution probability for $d_i(-1)=d_i(0)$. First of all we define

$$m_{i,j}=\{\theta_{i,j}(\ell_i)+[\theta_{i,j}(\ell_i+1)-\theta_{i,j}(\ell_i)]u)\}\cos\varphi_i$$

where

$$u=\frac{\tau_i-\ell_i T_c}{T_c}$$

is uniformly distributed in the interval [0,1[ with the assumption that all $\tau_i$ is uniformly distributed in [0,T]. And $\varphi_i$ is also uniformly distributed in the interval $[0,2\pi[$. Now we should consider two cases

a) $\theta_{i,j}(\ell_i)=\theta_{i,j}(\ell_i+1)$

b) $\theta_{i,j}(\ell_i)\neq\theta_{i,j}(\ell_i+1)$

For a) case

$$m_{i,j}=\theta_{i,j}(\ell_i)\cos\varphi_i$$

We have a constant multiplied by a cos(.) function with an argument that is uniformly distributed. We know, reference [9], that in this case the probability distribution for the random variable $m_{i,j}$ is given by

$$f_{m_{i,j}}(d) = \frac{1}{\pi \left|\theta_{i,j}(\ell_i)\right| \left[1 - \left(\frac{d}{\theta_{i,j}(\ell_i)}\right)^2\right]^{1/2}} \quad \text{for} \quad |d| < \left|\theta_{i,j}(\ell_i)\right|$$

and 0 otherwise. Note that this function has a singular point for $d=\theta_{i,j}(l_i)$

For b) case

$$m_{i,j} = Y\cos\varphi_i \quad \text{where} \quad Y = \theta_{i,j}(\ell_i) + [\theta_{i,j}(\ell_i + 1) - \theta_{i,j}(\ell_i)]u)$$

and now Y is uniformly distributed in the interval [A,B] where A and B are given by

$A = \min[\theta_{i,j}(\ell_i), \theta_{i,j}(\ell_i + 1)]$ and $B = \max[\theta_{i,j}(\ell_i), \theta_{i,j}(\ell_i + 1)]$

For this case cos $\varphi_i$ is multiplying Y so we can change the polarity of A and B without any change in the probability density function of $m_{ij}$. With this observation we can consider B>0 and $|B| > |A|$ and obtain, reference [9]

$$f_{m_{i,j}}(d) = \frac{1}{\pi(B - A)} \ln\left|\frac{B + (B^2 - d^2)^{1/2}}{A + (A^2 - d^2)^{1/2}}\right| \quad \text{for} \quad |d| < |A|$$

$$f_{m_{i,j}}(d) = \frac{1}{\pi(B - A)} \ln\left|\frac{B + (B^2 - d^2)^{1/2}}{d}\right| \quad \text{for} \quad |A| < |d| < |B|$$

and zero otherwise.

So we can obtain the probability density function of normalised interference of $i^{\underline{th}}$ user on $j^{\underline{th}}$ desired user for $d_i(-1)=d_i(0)$ case as

$$p1_{m_{i,j}}(d) = \frac{1}{N}\sum_{j=1}^{N} f_{m_{i,j}/\ell_j}(d/\ell_j)$$

Defining now $p2_{m_{i,j}}(d)$ as the probability density function of normalised

interference of $i^{\underline{th}}$ user on $j^{\underline{th}}$ desired user for $d_i(-1)\neq d_i(0)$ we can obtain it

with a simple change: $\theta_{i,j}(\ell_i)$ by $\hat{\theta}_{i,j}(\ell)$. Finally considering equal

probabilities for the interference data bit we can write

$$p_{m_{i,j}}(d) = \frac{1}{2}p1_{m_{i,j}}(d) + \frac{1}{2}p2_{m_{i,j}}(d)$$

Remembering that with our definition we have

$$I_{i,j}(\underline{d_i}, \tau_i, \varphi_i) = T^{-1} T_c d_i(0)\, m_{i,j} = \frac{1}{N} d_i(0)\, m_{i,j}$$

and that the desired $j^{\underline{th}}$ user's total interference is given by

$$\gamma_j(\underline{d}, \underline{\tau}, \underline{\varphi}) = \sum_{\substack{i=1 \\ i \neq j}}^{U} I_{i,j}(\underline{d_i}, \tau_i, \varphi_i)$$

$\Rightarrow$ its distribution can be determined by

$$f_{\gamma_j(\underline{d}, \tau, \underline{\varphi})} = \bigotimes_{\substack{i=1 \\ i \neq j}}^{U} f_{I_{i,j}}$$

where $\otimes$ denotes the convolution operation (valid with the assumed independence among various interference). Observe also that we have constants to consider in this evaluation (normalisation). Finally knowing the MAI distribution we can find the BER easily by

$$P_e = \text{Prob}\left( \left[ \sqrt{\frac{PT^2}{2}}\, \gamma_j(\underline{b}, \underline{\tau}, \underline{\varphi}) + \eta_j \right] > \sqrt{\frac{PT^2}{2}} \right)$$

This last step involves a new convolution and an integral that should be numerically evaluated.

# Sequences Selection for QS - Quasi-Synchronous CDMA Systems / Single User Receiver

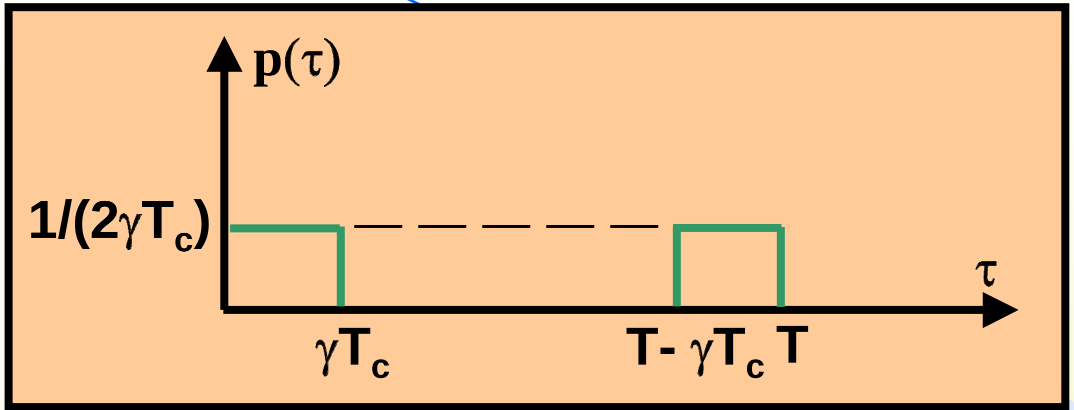
For QS systems the deviations of various random variables are assumed to be constrained within small intervals.

Considering one chip interval ($T_c$) of the code sequence it is difficult to maintain synchronism at carrier level. However with distribution of one pilot signal it is possible to maintain the chip dispersion within predefined boundaries.

The BER calculation for QS systems will be done with the chip delay of various subscribers maintained in the range

$|\tau| \leq \gamma \, T_c$ with $0 \leq \gamma \leq 1$

and with uniform distribution given by




$$p(\tau) = \begin{cases} (2\gamma T_c)^{-1} & \text{for} \quad 0 \le \tau < \gamma T_c \quad \text{and} \quad T - \gamma T_c \le \tau < T \\ 0 & \text{for} \quad \gamma T_c \le \tau < T - \gamma T_c \end{cases}$$

Now following exactly the same procedure as we have developed for the Pursley's approach we obtain for the $j^{\underline{th}}$ user in a QS system

$$P_e = Q[(SNR)_j] = Q[(\frac{1}{4L^2}\sum_{\substack{i=1 \\ i\neq j}}^{U}\rho_{i,j} + \frac{N_0}{2E_b})^{-1/2}]$$

where

$$\rho_{i,j} = 2\left(1-\gamma+\frac{\gamma^2}{3}\right)C^2_{i,j}(0)+\gamma\left(1-\frac{2\gamma}{3}\right)C_{i,j}(0)\left[C_{i,j}(1)+C_{i,j}(-1)\right] + \frac{\gamma^2}{3}\left[C^2_{i,j}(1)+C^2_{i,j}(-1)+C^2_{i,j}(N-1)+C^2_{i,j}(1-N)\right]$$

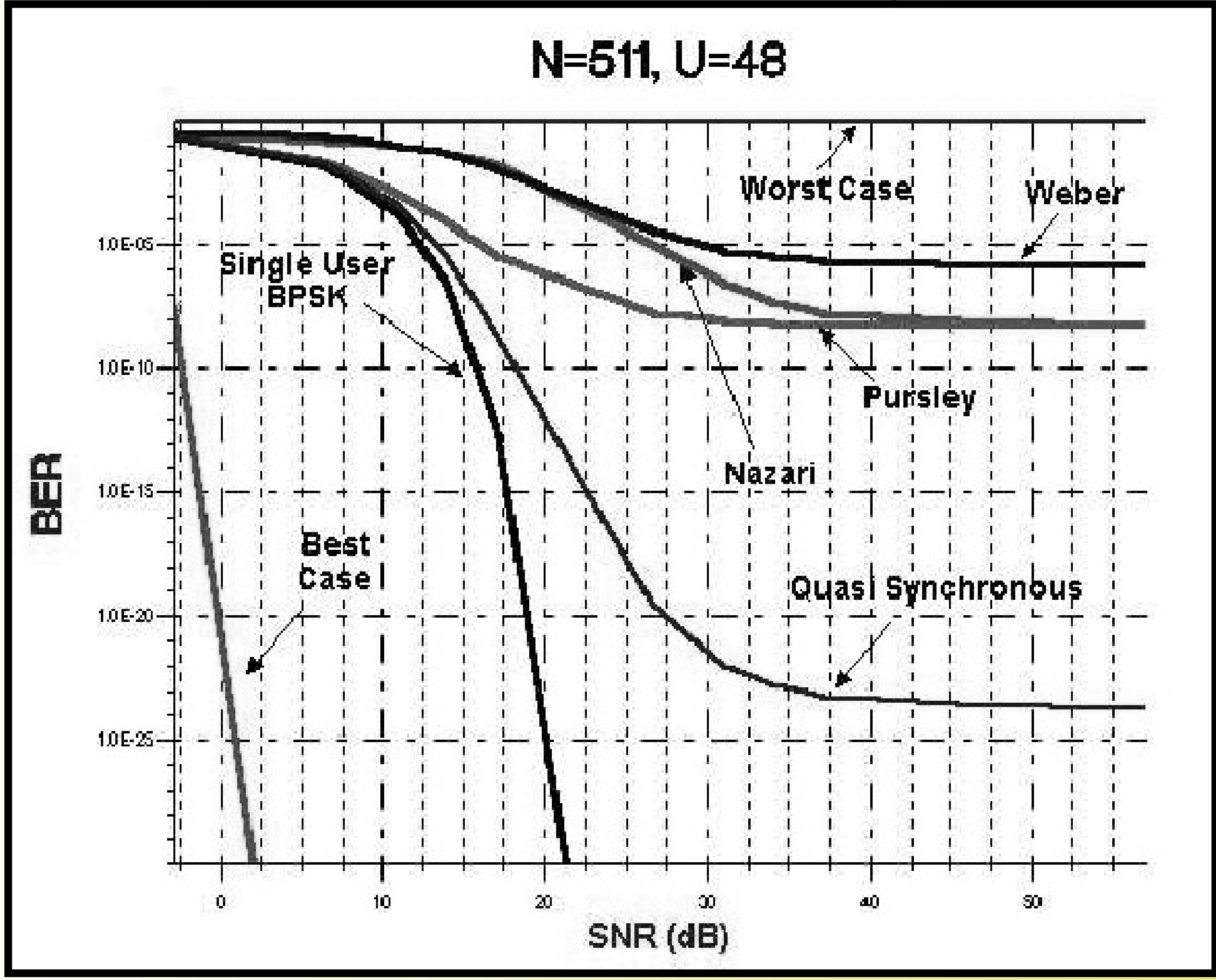
Using this equation it's possible to establish objective rules for sequences selection to be used in a given QS system, based on a few and simple parameters (more details in reference [2]).

<u>Given U sequences a sub-optimum strategy for selection could be</u>

- The first sequence is selected as the first in the list with arbitrary phase;
- The second sequence is selected as the second of the list and its relative phase is calculated considering the minimisation of its influence in the first;
- The $i^{th}$ sequence is selected as the $i^{th}$ of the list and its relative phase is calculated considering the minimisation of its influence in the previous (i-1).

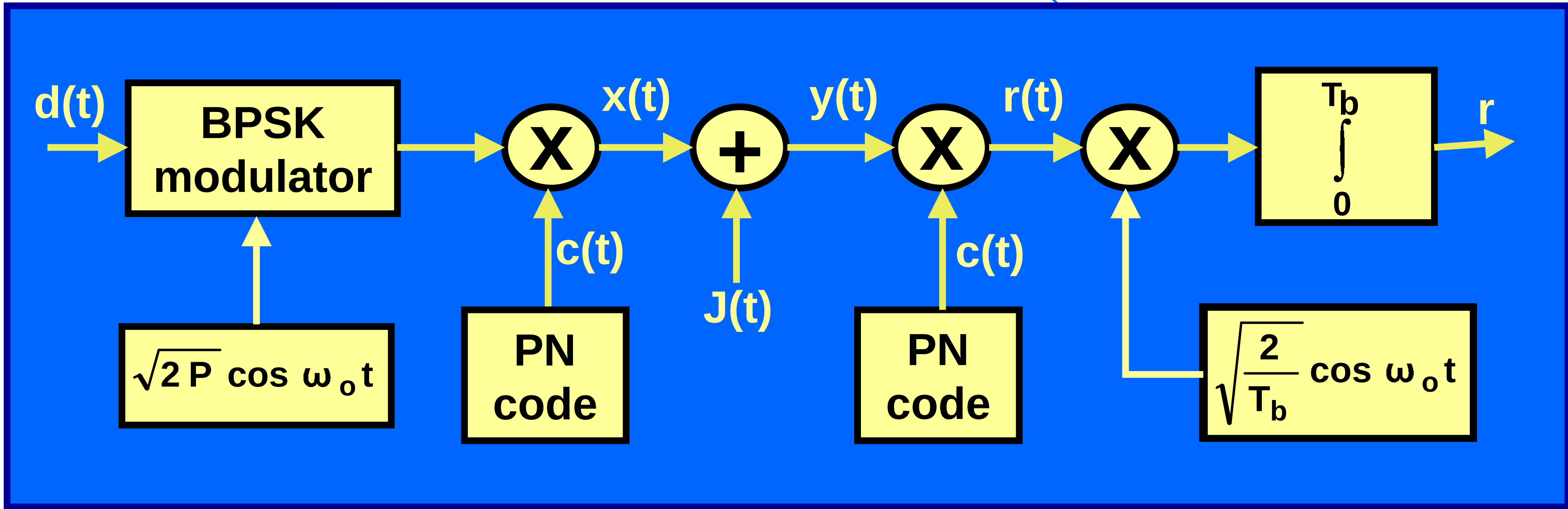
This strategy corresponds to a real scenario where the subscribers are introduced one by one (more details in reference [21]).

The next figure illustrates the various capacity determination processes until this point (reference [2]) for a specific set of values for N and U (Gold codes was used for deterministic sequences). These results confirm the superior performance of QS as expected and this advantage is 10dB for BER=$10^{-7}$ in comparison with an asynchronous system with the same parameters.



# Gaussian Approximation

The main objective of this section is to establish a general expression for BER determination in a SS system with an arbitrary jammer J(t). For this analysis we will consider the following model




Where d(t), P, $\omega_0$, c(t), x(t), y(t), r(t), $T_b$ and r are the data information, the transmitted power, the carrier frequency, the code sequence, the transmitted signal, the received signal, the received signal after de-spreading, the information bit's duration and the sampled output signal, respectively.

In our notation we have

$$d(t) = d_n \quad \text{for} \quad nT_b \le t < (n+1)T_b \quad \text{and with } d_n \in \{-1,1\}$$

$$c(t) = c_K \quad \text{for} \quad KT_c \le t < (K+1)T_c \quad \text{and with } c_K \in \{-1,1\}$$

with $N = T_b/T_c$ denoting the processing gain or information bit duration to chip duration ratio. We may write for x(t) and y(t)

$$x(t) = \sqrt{2P}\, d(t)\, c(t) \cos\omega_o t = c(t)\, s(t)$$

and

$$y(t) = x(t) + J(t)$$

where s(t) represents a conventional BPSK signal. For r(t) we can write

$$r(t) = c(t)[x(t) + J(t)] = s(t) + c(t)\, J(t)$$

The conventional BPSK detector signal's output has a form $r = d\sqrt{E_b} + n$

where $d \in \{-1, 1\}$ and $E_b = PT_b$ is the bit's energy and n is given by

$$n = \sqrt{2/T_b} \int_0^{T_b} c(t)\, J(t) \cos\omega_0 t\, dt$$

Without codification the circuit that follows the integrator has the rule decision

$$\hat{d} = \begin{cases} 1 & \text{for } r \geq 0 \\ -1 & \text{for } r < 0 \end{cases}$$

and admitting equal probabilities for the information bits transmission the BER can be evaluated by

$$P_e = P\{r \geq 0 \mid d = -1\} = P\{n \geq \sqrt{E_b}\}$$

This result is dependent on n, which can be rewritten as

$$n = \sqrt{2/T_b} \sum_{K=0}^{N-1} \int_{KT_C}^{(K+1)T_C} c(t)J(t)\cos\omega_0 t dt = \sqrt{2/T_b} \sum_{K=0}^{N-1} c_K \int_{KT_C}^{(K+1)T_C} J(t)\cos\omega_0 t dt$$

where $c_0$, $c_1$..., $c_{N-1}$ are the N chips from the code sequence that occur during one information bit interval $T_b$. Defining $J_K$ as

$$J_K = \sqrt{2/T_c} \int_{KT_C}^{(K+1)T_C} J(t)\cos\omega_0 t dt$$

we have

$$n = \sqrt{1/N} \sum_{K=0}^{N-1} c_K J_K$$

Considering each code sequence's chip as independent and equally distributed $P\{c_K=1\}=P\{c_K=-1\}=1/2$ for any fixed jammer $\vec{J} = \{J_0, J_1, \dots J_{N-1}\}$ the n sum represents a sum of independent random variables. So we need to evaluate

$$P_e(\vec{J}) = P\{n \geq \sqrt{E_b} \;|\vec{J}\}$$

and next

$$P_e = E\{P_e(\vec{J})\}$$

(the expectation is based on the jammer's statistics). Invoking the **Chernoff bound** we may write

$$P_e(\vec{J}) = P\{n - \sqrt{E_b} \geq 0 \,|\, \vec{J}\} \leq \frac{1}{2} E\{e^{\lambda(n-\sqrt{E_b})} \;|\vec{J}\} = \dots$$

$$\dots \frac{1}{2} e^{-\lambda\sqrt{E_b}} E\{\exp[\lambda \sqrt{\frac{1}{N}} \sum_{K=0}^{N-1} c_K J_K ] \,|\, \vec{J}\} = \frac{1}{2} e^{-\lambda\sqrt{E_b}} \prod_{K=0}^{N-1} E\{\exp[\lambda \sqrt{\frac{1}{N}} c_K J_K \,|\, J_K\}$$

Considering random sequences we have $p(c_K=1)=p(c_K=-1)=1/2$ so the last expression can be written as

$$\frac{1}{2} e^{-\lambda\sqrt{E_b}} \prod_{K=0}^{N-1} E\{\frac{1}{2}\exp[\lambda \sqrt{\frac{1}{N}} J_K ] + \frac{1}{2}\exp[-\lambda \sqrt{\frac{1}{N}} J_K ]\} = \frac{1}{2} e^{-\lambda\sqrt{E_b}} \prod_{K=0}^{N-1} \cosh(\frac{\lambda J_K}{\sqrt{N}})$$

Now it is known that $\cosh x \le \exp(x^2/2)$ ... (if you don't know prove it!)

$$P_e(\vec{J}) \le \frac{1}{2} e^{-\lambda\sqrt{E_b}} \exp\left(\frac{\lambda^2}{2N}\sum_{K=0}^{N-1} J_K^2\right)$$

And this expression should be minimised using $\lambda$ as a parameter. It is easy to show that the $\lambda$ value that minimises this expression is given by

$$\lambda^* = \sqrt{E_b} \Big/ \left(\frac{1}{N}\sum_{K=0}^{N-1} J_K^2\right)$$

and with this $\lambda^*$ value we have

$$P_e(\vec{J}) \le \frac{1}{2}\exp\left(\frac{-E_b}{\frac{2}{N}\sum_{K=0}^{N-1} J_K^2}\right)$$

For this general expression derivation we have only admitted that we are using random code sequences so it is valid for any N or $\{J_i\}$ . Now for our final conclusion we will develop an expression based on a gaussian approximation.

Invoking the Central Limit Theorem we can say that for a fixed $\{J_i\}$ the gaussian approximation means that we can consider n as a gaussian random variable with mean and variance given by

$$E[c_K J_K \mid J_K] = \frac{1}{2}J_K - \frac{1}{2}J_K = 0$$

and

$$Var\left[\frac{1}{\sqrt{N}}\sum_{K=0}^{N-1} c_K J_K \mid J_K\right] = \sum_{K=0}^{N-1} Var\left[\frac{1}{\sqrt{N}} c_K J_K \mid J_K\right] = \frac{1}{N}\sum_{K=0}^{N-1} J_K^2$$

so

$$P_e(\vec{J}) = Q\left(\sqrt{\frac{E_b}{\frac{1}{N}\sum_{K=0}^{N-1} J_K^2}}\right)$$

This expression is valid for N obeying Central Limit Theorem's limitations. Now considering the usual expansion for Q(x) function

$$Q(x) \le \frac{1}{2} e^{-x^2/2}$$

we can finally write

$$P_e(\vec{J}) \le \frac{1}{2}\exp\left(\frac{-E_b}{\frac{2}{N}\sum_{K=0}^{N-1} J_K^2}\right)$$

This final result is the the same as we have obtained with Chernoff bound (without gaussian considerations or any restrictions for N). The former result was independent of N so we can conclude that the gaussian approximation is valid independently of N.

# Chernoff Bound

Given the two functions

$$u(t) = \begin{cases} 1 & \text{for} \quad t \geq 0 \\ 0 & \text{for} \quad t < 0 \end{cases}$$

and

$$v_\lambda(t) = e^{\lambda t} \quad \forall\, t$$

we know that

$$u(t) \leq v_\lambda(t) \quad \forall \lambda \geq 0$$

for every t.

Now considering a random variable X with a pdf-probability density function given by $p_X(x)$ we may write

$$P\{X \geq 0\} = \int_0^{\infty} p_x(t)\,dt = \int_{-\infty}^{\infty} u(t)\,p_x(t)\,dt \leq \int_{-\infty}^{\infty} v_\lambda(t)\,p_x(t)\,dt = E\{v_\lambda(x)\}$$

so

$$P\{X \geq 0\} \leq E\{e^{\lambda x}\} \quad \text{for} \quad \forall \lambda \geq 0$$

In this expression $\lambda \geq 0$ is a parameter which should be optimised looking for $E\{e^{\lambda x}\}$ minimisation. This relation is known as Chernoff Bound.

The Chernoff bound may be further narrowed for random variables that have pdf obeying the relation

$$p_X(-x) \le p_X(x)$$

In this this case we can write

$$E\{e^{\lambda x}\} = \int_{-\infty}^{\infty} e^{\lambda t}\, p_X(t)\,dt = \int_{-\infty}^{0} e^{\lambda t}\, p_X(t)\,dt + \int_{0}^{\infty} e^{\lambda t}\, p_X(t)\,dt \ge \ \dots$$

$$\dots \int_{0}^{\infty} [e^{\lambda t} + e^{-\lambda t}]\, p_X(t)\,dt = \int_{0}^{\infty} 2\cosh(\lambda t)\, p_X(t)\,dt \ge 2\int_{0}^{\infty} p_X(t)\,dt = 2\,P(X \ge 0)$$

so

$$P\{X \ge 0\} \le \frac{1}{2} E\{e^{\lambda x}\}$$

Again $\lambda$ is a parameter which should be optimised looking for $E\{e^{\lambda x}\}$ minimisation but now without any restriction on $\lambda$. In particular this relation can be used for symmetric pdf functions (very usual case)

$$p_X(-x) = p_X(x)$$

(return)

# Why use Power Control in the Reverse Channel?

- We will consider the reverse channel for a CDMA system with N users;

- The transmitted power and rate are described by two vectors $\mathbf{P}=[P_1,P_2,...,P_N]$ and $\mathbf{R}=[R_1,R_2,...,R_N]$. Each user's bandwidth is W, so the chip rate is constant for the system. The various users' rates are obtained with different processing gains as a function of individual data rate;

- Each user has a minimum QoS (quality of service) specified by its BER which can be obtained with a vector that specifies the required $(SNR)_i$ and will be expressed as $\Gamma=[\gamma_1,\gamma_2,..., \gamma_N]$;

- For each user we will also specify a minimum value for its data rate and a maximum power that can be used. These individual values are expressed by the vectors $\mathbf{r}=[r_1,r_2,...,r_N]$ and $\mathbf{p}=[p_1,p_2,...,p_N]$;

- The channel's attenuation is specified by a vector $\mathbf{h}=[h_1,h_2,...,h_N]$ where the product $h_iP_i$ represents the signal's level at BS-Base Station's input;

- The background noise is considered an AWGN with unilateral PSD $N_0$.

With these statements we can express the ratio (SNR) for each user as (reference [16])

$$(SNR)_i = \frac{W}{R_i} \frac{h_i P_i}{\sum_{\substack{j=1 \\ j \neq i}}^{N} h_j P_j + N_0 W}$$

For a given configuration we have to determine the solution (vectors **P** and **R**) for the following set of inequalities for i=1, 2, ..., N

$$\frac{W}{R_i} \frac{h_i P_i}{\sum_{\substack{j=1 \\ j \neq i}}^{N} h_j P_j + N_0 W} \geq \gamma_i$$ and $$0 < P_i \leq p_i$$ and $$R_i \geq r_i$$

If this set has no solution we need to relax some restriction or to exclude some users. If this set has more than one solution some additional restriction criteria can be imposed. One natural choice is to minimise the total transmitted power (with this criterion we will minimise the MAI for other adjacent cells and also save batteries).

Obviously the solution will be the equality in the first and third expressions so we can write

$$\frac{W}{r_i}\frac{h_iP_i}{\sum_{\substack{j=1\\ j\neq i}}^{N} h_jP_j + N_0W} = \gamma_i \qquad i = 1,\dots,N$$

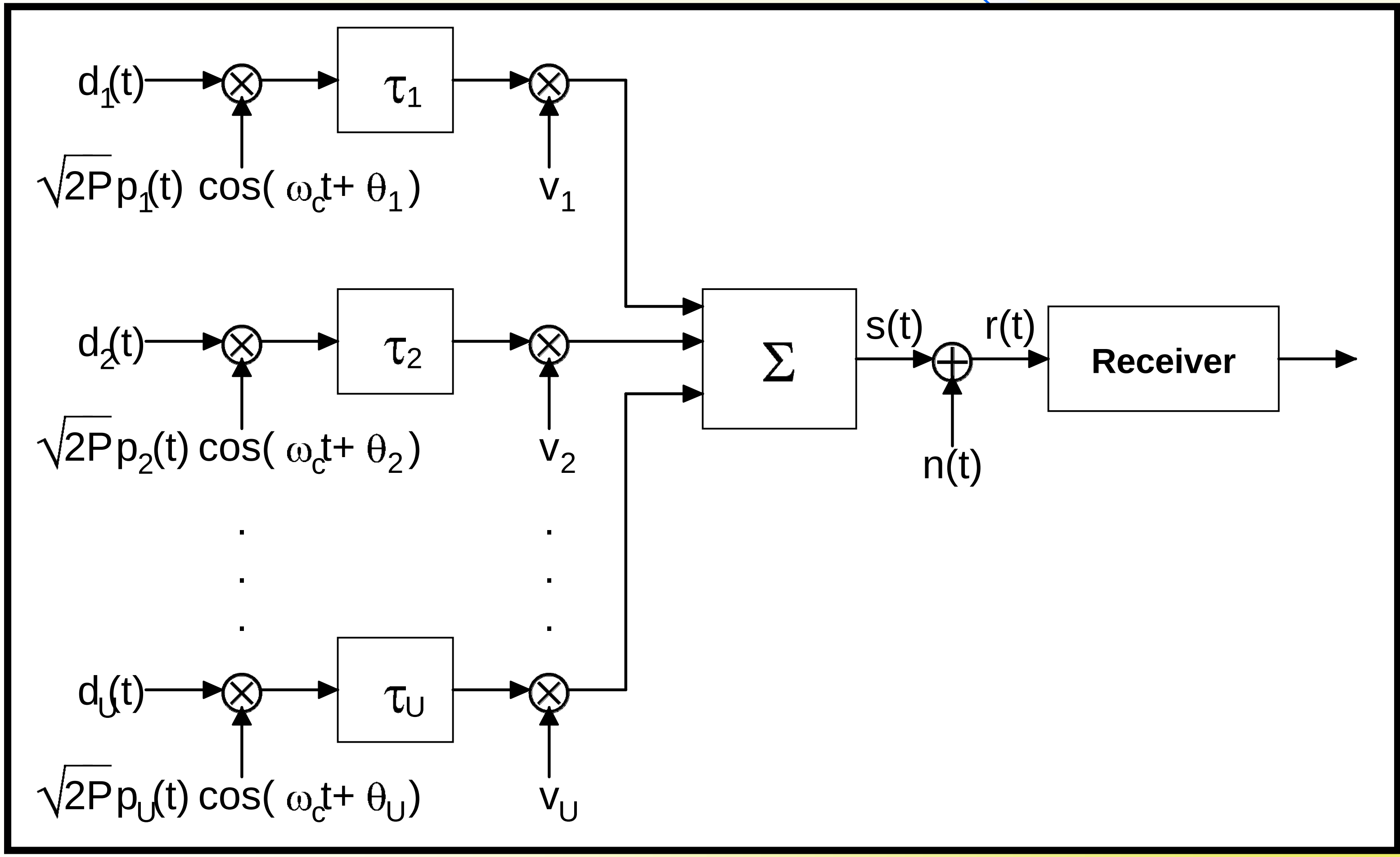
And solving for a particular $P_i$ (of course the same solution is valid for every i)

$$\left(\frac{W}{r_i\,\gamma_i} + 1\right) h_iP_i \left[1 - \sum_{j=1}^{N} \frac{1}{\left(\frac{W}{r_j\,\gamma_j} + 1\right)}\right] = N_0W$$

So our general solution, which was based on total power minimisation, when applied to a system with constant bit rate and QoS requirements for all users state that the product $h_iP_i$ needs to be constant for all users and this statement implies a perfect power control.

# Imperfect Power Control in the Reverse Channel

In a similar way to our previous developments we will include imperfect power control effects introducing a new random variable $v_i$ as shown below (reference [10])



We can write the receiver's input signal as

$$r(t) = s(t) + n(t) = \sum_{i=1}^{U} s_i(t) + n(t)$$

where for the $i^{th}$ generic user we have

$$s_i(t) = \sqrt{2P}\, v_i\, d_i(t-\tau_i)\, p_i(t-\tau_i) \cos\omega_c(t-\tau_i) + \theta_i)$$

with $$v_i = \sqrt{\frac{P_i}{P}}$$ denoting the power control's imperfection.

The procedure for calculation is similar and denoting with j the desirable user we can write

$$Z_j = \int_0^T r(t)\, p_j(t) \cos\omega_c t\, dt = Inf_j + Interf_j + \eta_j$$

(return)

Where we know that

$$\mathrm{Inf}_j = d_0^{(j)} \sqrt{\frac{PT^2}{2}}\, v_j$$

and

$$\mathrm{Interf}_j = \sum_{\substack{i=1 \\ i \neq j}}^{U} \sqrt{\frac{P}{2}}\, v_i \int_0^T d_i(t-\tau_i)\, p_i(t-\tau_i)\, p_j(t) \cos\varphi_i \, dt$$

with $\varphi_i = \theta_i - \omega_0 \tau_i$; additionally we have admitted that $v_i$ can be different from one user to another but it is constant during bit the interval (slow fading characteristics). With similar procedures as with the former, we can write

$$\mathrm{Interf}_j = \sqrt{\frac{PT^2}{2}} \sum_{\substack{i=1 \\ i \neq j}}^{U} v_i\, I_{i,j}(d_i, \tau_i, \varphi_i)$$

where $I_{i,j}(.)$ was defined on Nazari and Ziemer’s approach. (compare)

For variance determination we can state the following expressions

$$\mathrm{var}[Z_j] = \mathrm{var}[\mathrm{Inf}_j] + \mathrm{var}[\mathrm{Interf}_j] + \mathrm{var}[\eta_j]$$

$$\mathrm{var}[\mathrm{Inf}_j] = \frac{PT^2}{2}\mathrm{var}[v_j]$$

$$\mathrm{var}[\mathrm{Interf}_j] = \frac{PT^2}{2}\sum_{\substack{i=1 \\ i\neq j}}^{U} \mathrm{var}[v_i\, I_{i,j}(d_i, \tau_i, \varphi_i)]$$

$$\mathrm{var}[\eta_j] = \frac{N_0 T}{4}$$

Additionally for random sequences we know that

$$\mathrm{var}[I_{i,j}(d_i, \tau_i, \varphi_i)] = E[I_{i,j}(d_i, \tau_i, \varphi_i)^2] = \frac{1}{3N}$$

Now, it is well known that the variance of the product of n random variables can be determined by (reference [9])

$$\operatorname{var}\left[\prod_{j=1}^{n} y_j\right]\Bigg|_{\substack{E[y_1]=0 \\ \text{and/or} \\ E[y_2]=0 \\ \ldots\text{ and/or} \\ E[y_n]=0}} = \prod_{j=1}^{n} E[y_j^2]$$

if, at least one of that variables has zero mean. With this property in mind we can write

$$\operatorname{var}[\mathrm{Interf}_j] = \frac{PT^2}{2}\frac{1}{3N}E[v_j^2](U-1)$$

and

$$\operatorname{var}[Z_j] = \frac{PT^2}{2}\left[\operatorname{var}[v_j] + \frac{1}{3N}E[v_j^2](U-1) + \frac{N_0}{2PT}\right]$$

Now, considering the various mean values we can state that

$E[I_{i,j}(d_i, \tau_i, \varphi_i)] = 0$ and $E[\eta_j] = 0$ so $E[Z_j] = \sqrt{\frac{PT^2}{2}}\, E[v_j]$

For a site with a great number of users using random sequences it is possible to invoke the Central Limit Theorem and to estimate the BER with a gaussian assumption

$$P_{e,j} = Q\left(\frac{E[Z_j]}{\sqrt{var[Z_j]}}\right) = Q\left(\frac{E[v_j]}{\sqrt{var[v_j] + \frac{U-1}{3N}E[v_j^{\,2}] + \left(\frac{N_0}{2E_b}\right)}}\right)$$

(return)

Note that if we impose $v_j$=1 (constant) we obtain the previous perfect power control result as expected (compare).

For a numerical result, we need to specify $v_i$'s characterisation. One very useful approach is to consider a log-normal distribution for the bit's energy (as a consequence also for $v_j$). With this assumption we can write

$$E_j = E10^{(\xi_j/10)}$$

or

$$v_j = 10^{(\xi_j/20)}$$

Where $\xi_j$ is a gaussian random variable with zero mean and standard deviation $\sigma_\xi$. Note that $\sigma_\xi$=0 means perfect power control. With this assumption we can express the probability density function of the random variable $v_j$ as

$$f_{v_j}(v_j) = \frac{20\log(e)}{\sqrt{2\pi}\,\sigma_\xi v_j}\exp\left[-\frac{(20\log(v_j))^2}{2\sigma_\xi^2}\right]$$

So we are able to determine the mean and variance of $v_j$

$$E[v_j] = e^{\sigma^2/8}$$

$$\mathrm{var}[v_j] = e^{\sigma^2/2} - e^{\sigma^2/4}$$

where

$$\sigma = \frac{\sigma_\xi}{10\log(e)}$$

If we impose the same performance for a system with perfect power control (with U’ users) with another system now with imperfect power control (and U users) we can write (imposing the same argument for the Q function)

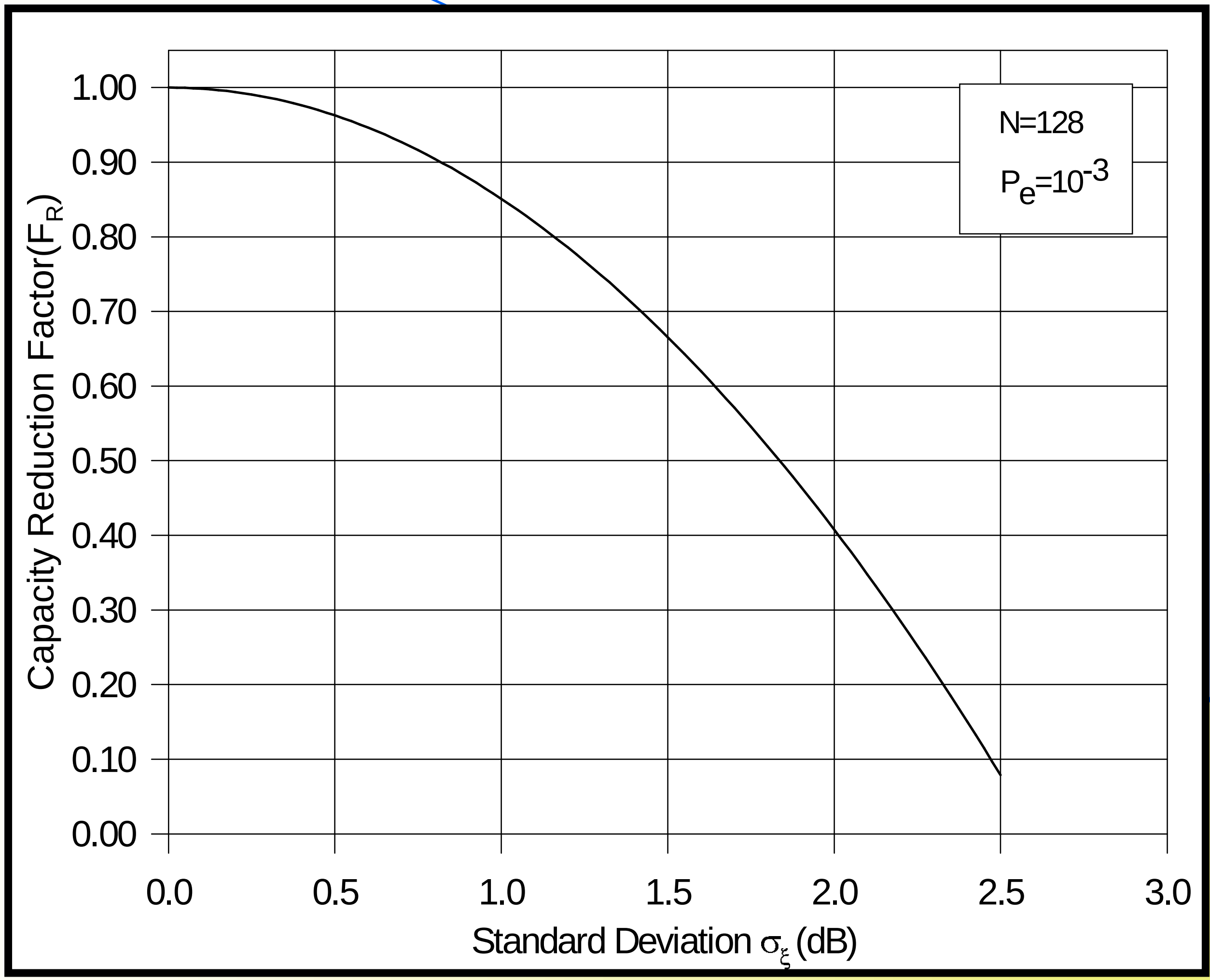
$$\frac{E[v_j]}{\sqrt{var[v_j] + \frac{U-1}{3N} E[v_j^2] + \left(\frac{N_0}{2E_b}\right)}} = \frac{1}{\sqrt{\frac{U'-1}{3N} + \left(\frac{N_0}{2E_b}\right)}}$$

And from this equation we can define the reduction factor as

$$F_R = \frac{U}{U'} = \frac{E[v_j^2] + (U'-1)E[v_j]^2 - 3N\,var[v_j]}{U'\,E[v_j^2]}$$

(return)

The next figure shows us an example where we have adopted N=128 and a desirable performance specified by $P_e=10^{-3}$ (reference [17])

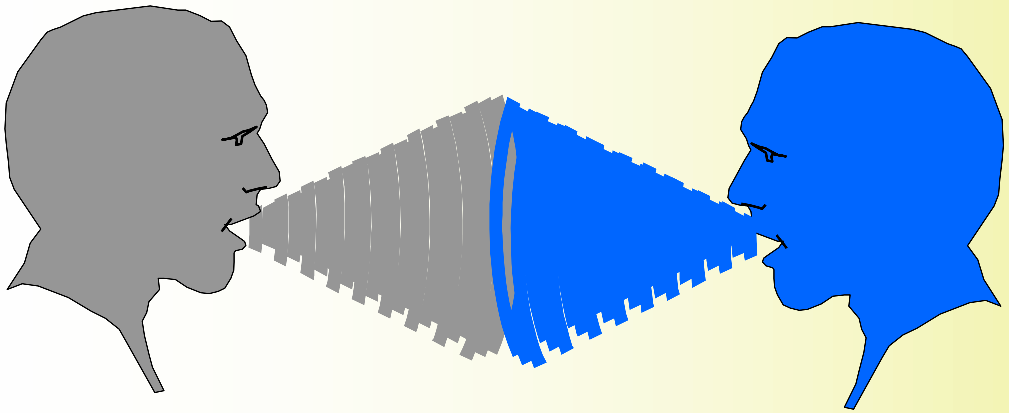


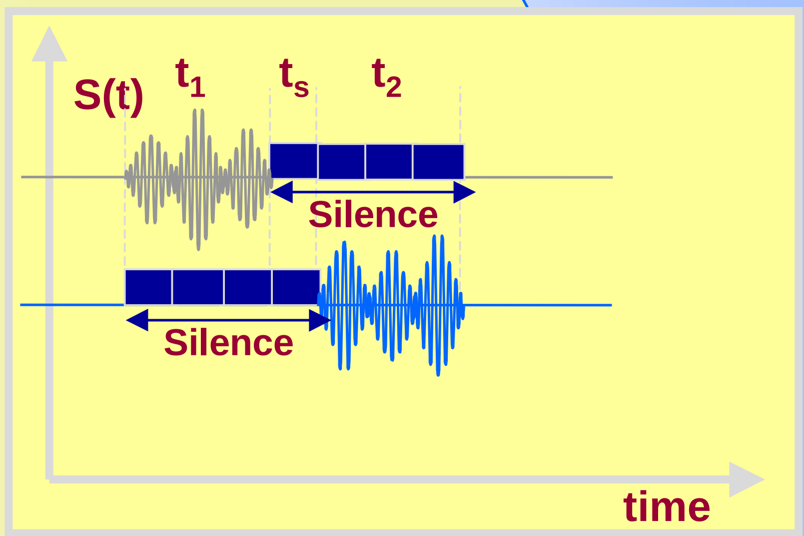

(return)

As mentioned in reference [19] experimental results show us that $\sigma_\xi$ can vary between 1 e 2 dB. From the graph we can see that for these values the loss will be 15% to 60% of original capacity (perfect power control case).

# Voice Activity Factor

Voice activity means that we have silence periods in a normal conversation. Approximately, 50% to 65% of time the transmission is unnecessary and no transmission means less interference to other users. A CDMA system can use this property with simplicity in order to increase capacity; here it is a natural thing to take advantage of this fact in contrast with other type of multiplexing systems (FDMA for instance).





Admitting a model that includes this no transmission case as equivalent a full rate transmission with a smaller bit energy (affected by a coefficient $\alpha$) we can write

$$E_b = PT_b\alpha$$

Where P is the signal's power in a perfect power control scenario, $T_b$ is the information bit duration and $\alpha$ is a discrete random variable that, for the IS-95 case, depends on effective transmission rate that can be 1200, 2400, 4800 or 9600 bps, and in correspondence we can use 0.125, 0.25, 0.5 or 1 for $\alpha$, respectively. Ignoring AWGN we can write the detector's output signal as

$$Z_j = Inf_j + Interf_j$$

where

$$Inf_j = b_0^{(j)}\sqrt{\frac{P}{2}}T_b v_j$$

and

$$Interf_j = \sum_{\substack{k=1 \\ k\neq j}}^{U}\sqrt{\frac{P}{2}}v_k\alpha_k\int_0^{T_b} b_k(t-\tau_k)c_k(t-\tau_k)c_j(t)\cos(\varphi_k)dt$$

Analogously as we have treated the imperfect power control case and also considering random sequences for users, we can resume the calculation procedure as

$$\mathrm{var}[\mathrm{Inf}_j] = \frac{PT_b^{\,2}}{2}\,\mathrm{var}[v_j]$$

$$\mathrm{var}[\mathrm{Interf}_j] = \frac{PT_b^{\,2}}{2}\sum_{\substack{k=1\\k\neq j}}^{U}\mathrm{var}[v_k\,\alpha_k\,I_{k,j}(b_k,\tau_k,\varphi_k)] = \frac{PT_b^{\,2}}{2}\,\frac{U-1}{3N}\,E[v_j^{\,2}]\,E[\alpha^2]$$

$$\mathrm{var}[Z_j] = \frac{PT_b^{\,2}}{2}\left[\mathrm{var}[v_j] + \frac{U-1}{3N}\,E[v_j^{\,2}]\,E[\alpha^2]\right]$$

and

$$E[Z_j] = \sqrt{\frac{P}{2}}\,T_b\,E[v_j]$$

and finally

$$P_e = Q\left(\frac{E[Z_j]}{\sqrt{\mathrm{var}[Z_j]}}\right) = Q\left(\frac{E[v_j]}{\sqrt{\mathrm{var}[v_j] + \frac{U-1}{3N}\,E[v_j^{\,2}]\,E[\alpha^2]}}\right)$$

(compare)

(return)

For a system with perfect power control and no voice activity control we have U' users determined by the required performance

$$P_e = Q[(\frac{1}{3N}(U^{'}-1))^{-1/2}]$$

For the same performance in both systems we should impose the same argument for the Q functions and expressing this gain as $G_{VA}$

$$G_{VA} = \frac{U}{U^{'}} = \frac{E[v_j^{\,2}]\,E[\alpha^2] + (U^{'}-1)\,E[v_j]^2 - 3N\mathrm{var}[v_j]}{U^{'}E[v_j^{\,2}]\,E[\alpha^2]}$$

(compare)

The complete characterisation depends on statistical properties of $v_j$ and $\alpha$. The first one was studied on imperfect power control item. The second term need some additional considerations. Admitting that we have transmission during a fraction $\beta$ of time we can assume the following (arbitrary) cases

1) Good voice quality: all frames with voice activity are transmitted with 9600 bps. No voice activity corresponds to 1200 bps. For this case we can write the probability density function for $\alpha$ as

$$P_1(\alpha) = \beta\delta(\alpha - 1) + (1 - \beta)\,\delta(\alpha - 0{,}125)$$

and for this case we can express $E[\alpha^2]$ as

$$E[\alpha^2] = \frac{63\beta + 1}{64}$$

2) Median voice quality: frames with voice activity are transmitted with 9600 bps in 50% of the time, 4800 bps in 25% of the time and 2400 in 25% of the time. No voice activity corresponds to 1200 bps. For this case we can write the probability density function for $\alpha$ as

$$P_2(\alpha) = 0{,}5\,\beta\delta(\alpha - 1) + 0{,}25\,\beta\delta(\alpha - 0{,}5) + 0{,}25\,\beta\delta(\alpha - 0{,}25) + (1 - \beta)\,\delta(\alpha - 0{,}125)$$

and express $E[\alpha^2]$ as

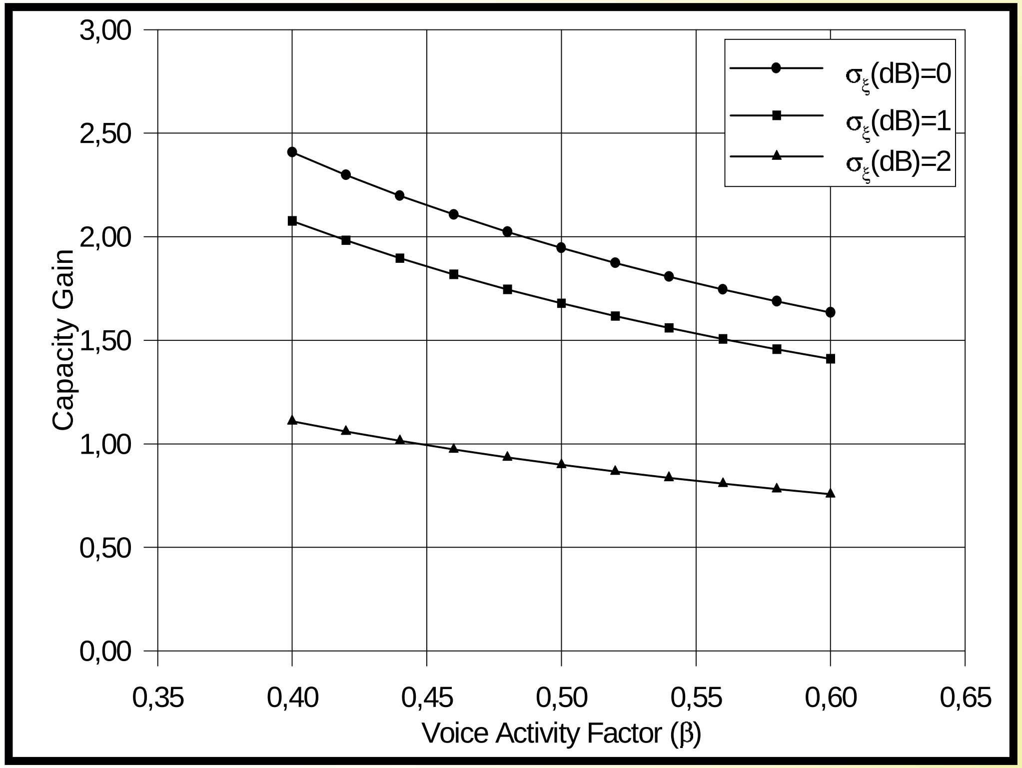
$$E[\alpha^2] = \frac{36\beta + 1}{64}$$

3) Acceptable voice quality: frames with voice activity are transmitted with 9600 bps in 25% of the time, 4800 bps in 50% of the time and 2400 in 25% of the time. No voice activity corresponds to 1200 bps. For this case we can write the probability density function for $\alpha$ as

$$P_3(\alpha) = 0{,}25\,\beta\delta(\alpha-1) + 0{,}5\,\beta\delta(\alpha-0{,}5) + 0{,}25\,\beta\delta(\alpha-0{,}25) + (1-\beta)\,\delta(\alpha-0{,}125)$$

and express $E[\alpha^2]$ as $$E[\alpha^2] = \frac{24\beta+1}{64}$$

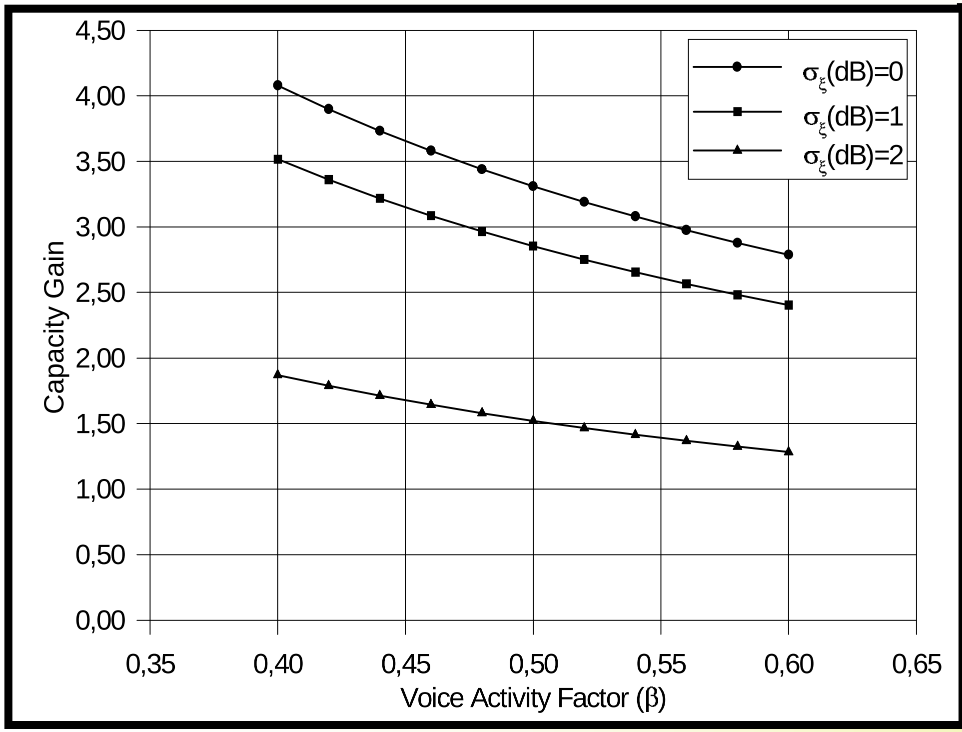
The next three figures show us these results for good, median and acceptable voice quality and imperfect power control modelled with a log-normal distribution (see reference [17] for additional details).



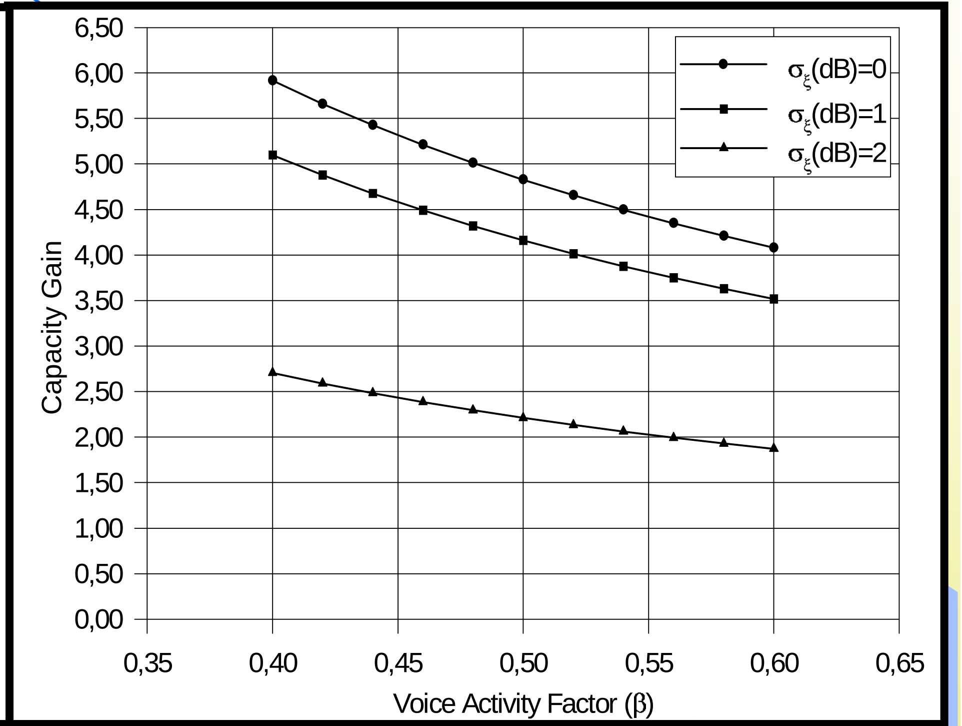

Good voice quality

(return)

Median voice quality



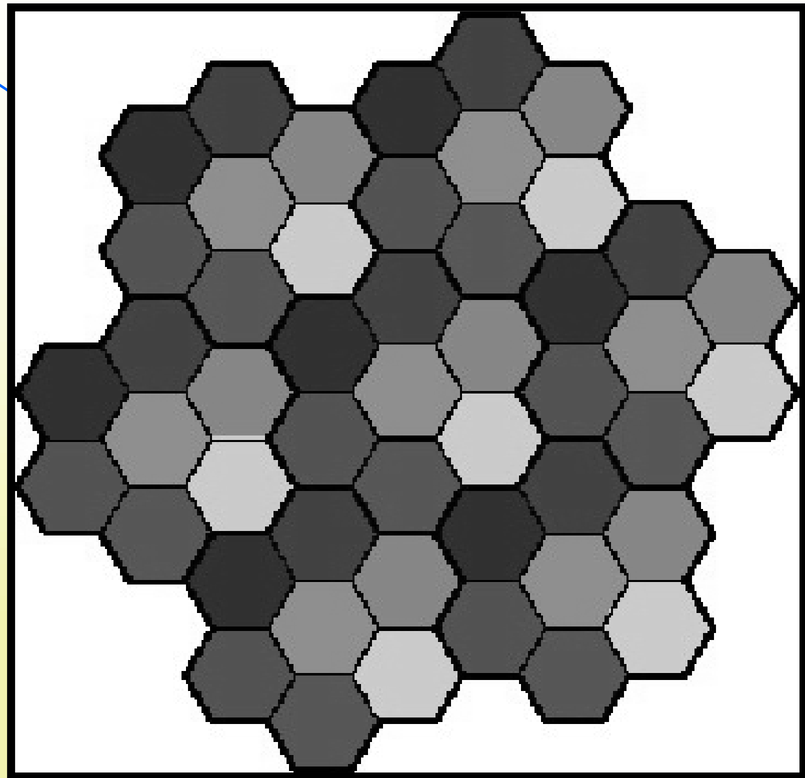

Acceptable voice quality




From the graphs we can see that considering a DS/CDMA system with an imperfect power control characterised by $\sigma_\xi$=1dB and with a voice activity factor $\beta$= 0.5 the capacity gain can be estimated as 4.16, 2.85 and 1.69 for acceptable, median and good voice quality, respectively.

# Inter - Cell MAI / Reuse Factor

In the next slides we will examine the additional interference caused in the system's central cell by users in other cells. This type of interference is named inter-cell MAI. With frequency reuse in all neighbour cells the total interference in the central cell will increase and the main goal here is to determine this increase considering central cell's interference as a reference. We define the reuse factor as



$$F = \frac{P_{int}}{P_{int} + P_{ext}}$$

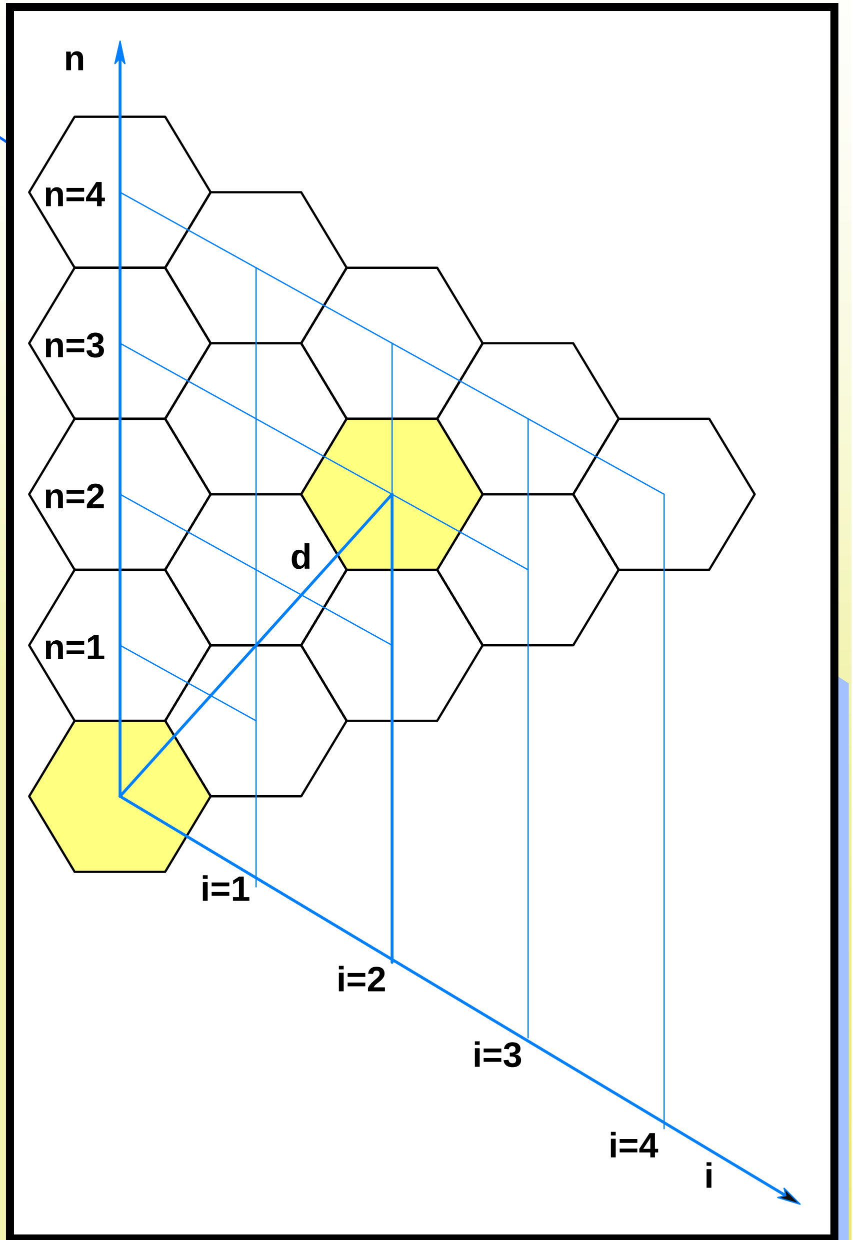
Where

$P_{int}$ : is the central cell's interference caused by its own users

$P_{ext}$ : is the interference caused by all other users in all other cells (inter-cell).

(obs.: for a isolated cell F=1)

First of all we need to establish an axis system for the cell's representation. In the figure we represent the usual solution. From this figure we can write the distance between the hexagon's centre with co-ordinates (n,i) and hexagon's centre with co-ordinates (0,0) as




$$d(n, i) = 2R\sqrt{n^2 + i^2 - ni} = kR$$

where R represents the inscribed circle's radius (references [11] and [17]).

The relation between the user's transmitted power ($P_t$) and the received power at its base station ($P_r$) can be written as

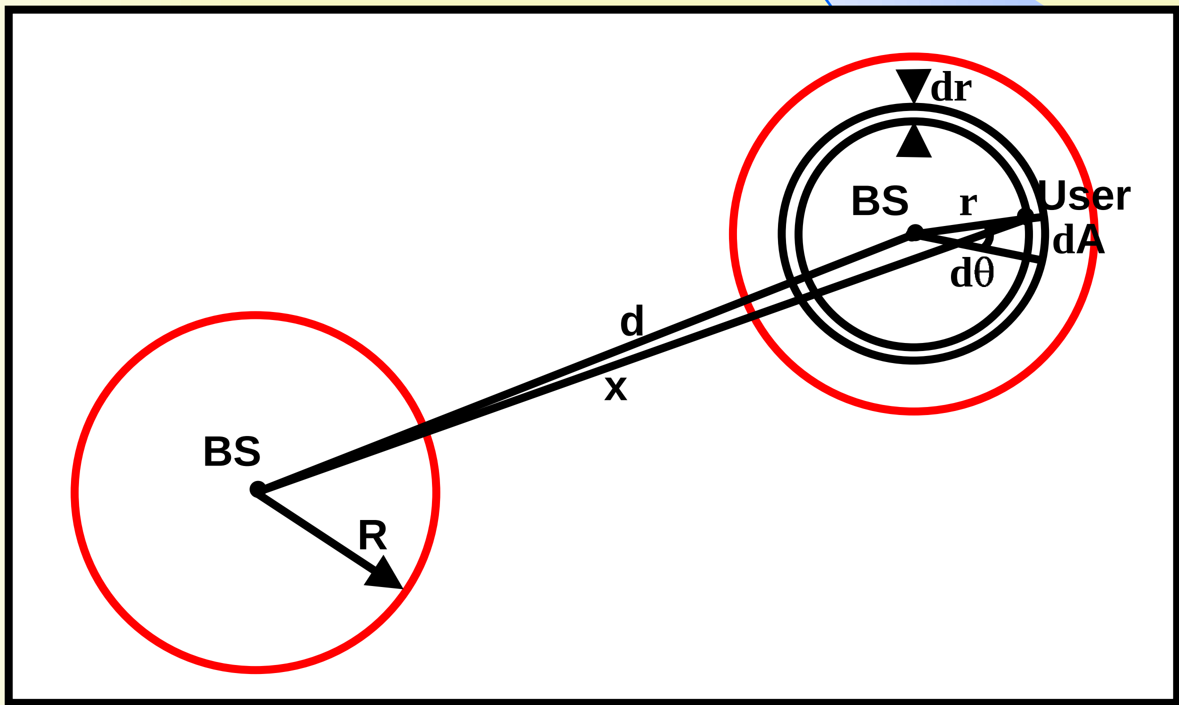
$$P_r = C\frac{P_t}{r^{\gamma}}$$

where C is a constant which depends on antennas and frequency and $\gamma$ is the propagation loss exponent (reference [20]).

This user will cause an interference in other base stations (see figure below) and this interference can be calculated by

$$I = S_c\left(\frac{r}{x}\right)^4$$

Where $S_c$ is the signal's level received in its own base station. Additionally it was admitted that the propagation loss exponent is $\gamma$=4 (this detail will be explained later).



The user's density can be expressed as

$$\rho = \frac{dU}{dA} = \frac{U}{\pi R^2}$$

The total interference imposed by U users in a distant cell in the central base station's receiver can be calculated as

$$P(d) = \int S_c \left(\frac{r}{x}\right)^4 \rho \, dA$$

Where the integral should be evaluated in all distant cell areas. Assuming a circular cell (instead of a hexagonal) from the relation

$$x = \sqrt{d^2 + r^2 + 2\, r\, d \cos\theta}$$

we can write

$$P(d) = 2 \int_0^{\pi} \int_0^{R} S_c \left(\frac{r}{x}\right)^4 \frac{U}{\pi R^2} \, r \, dr \, d\theta$$

After substitution and simplification

$$P(d) = \frac{2\,U\,S_c}{\pi R^2} \int_0^R dr\, r^5 \int_0^{\pi} \frac{d\theta}{\left(d^2 + r^2 + 2\,r\,d\cos\theta\right)^2}$$

And after evaluation

$$P(d) = \frac{2\,U\,S_c}{R^2} \left[ 2\,d^2 \ln\left( \frac{d^2}{d^2 - R^2} \right) - \frac{R^2 (4\,d^4 - 6\,d^2 R^2 + R^4)}{2\,(d^2 - R^2)^2} \right]$$

Expressing distance d as a function of co-ordinates (n,i) we have

$$P(k) = 2\,U\,S_c \left[ 2\,k^2 \ln\left( \frac{k^2}{k^2 - 1} \right) - \frac{4\,k^4 - 6\,k^2 + 1}{2\,(k^2 - 1)^2} \right]$$

For the reuse factor determination we can observe that

$$P_{int} = (U-1)S_c \approx U S_c$$

$$P_{ext} = \sum_{n=1}^{N} \sum_{i=1}^{n} 6P\left(k = 2\sqrt{n^2 + i^2 - ni}\right)$$

where N denotes the number of rings with active cells in the system. So finally we obtain

$$F = \frac{P_{int}}{P_{int} + P_{ext}} = \frac{1}{1 + 12 \sum_{n=1}^{N} \sum_{i=1}^{n} G(L)}$$

where

$$G(L) = 2L \ln\left(\frac{L}{L-1}\right) - \frac{4L^2 - 6L + 1}{2(L-1)^2}$$

and

$$L = k^2 = 4(n^2 + i^2 - ni)$$

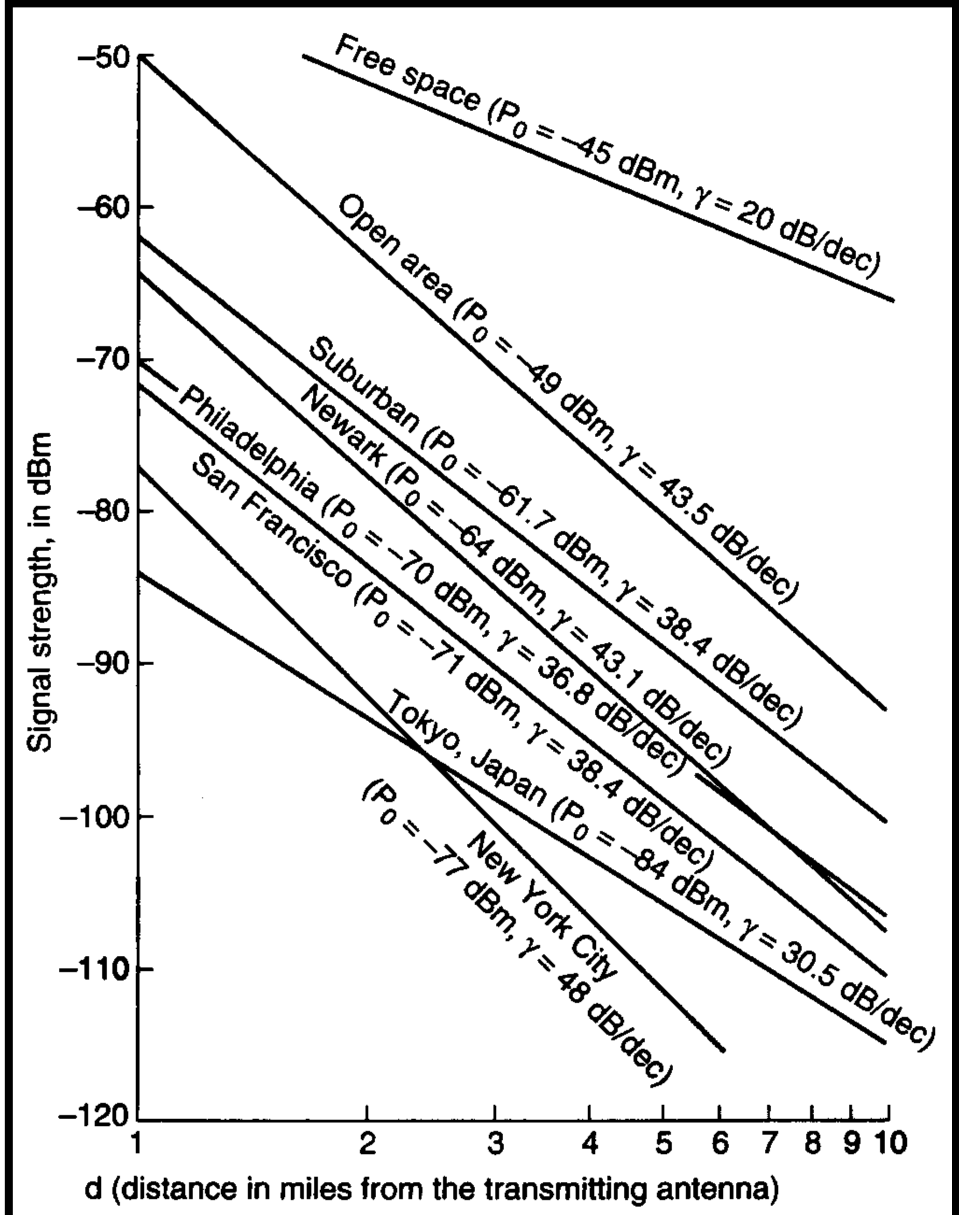
The table below shows us the evaluation of the final expression for various rings (N) in the considered system and also the individual percentage of that particular ring participation.

| N | F | $P_{ext\ ring}/P_{int}(\%)$ |
|---|---|---|
| 1 | 0,7787 | 28,42 |
| 2 | 0,7625 | 2,73 |
| 3 | 0,7582 | 0,74 |
| 4 | 0,7565 | 0,30 |
| 5 | 0,7556 | 0,15 |
| ... | ... | ... |
| 100 | 0,7539 | $\cong 0$ |

(return)

This table show us that the practical use of F=0,75 is a good choice for this factor and also that with N=2 we have a good approximation. This procedure can be adapted for non homogeneous traffic (but homogeneous in each cell). All that we need to do is to consider an appropriate factor for each cell density in P(d) evaluation.

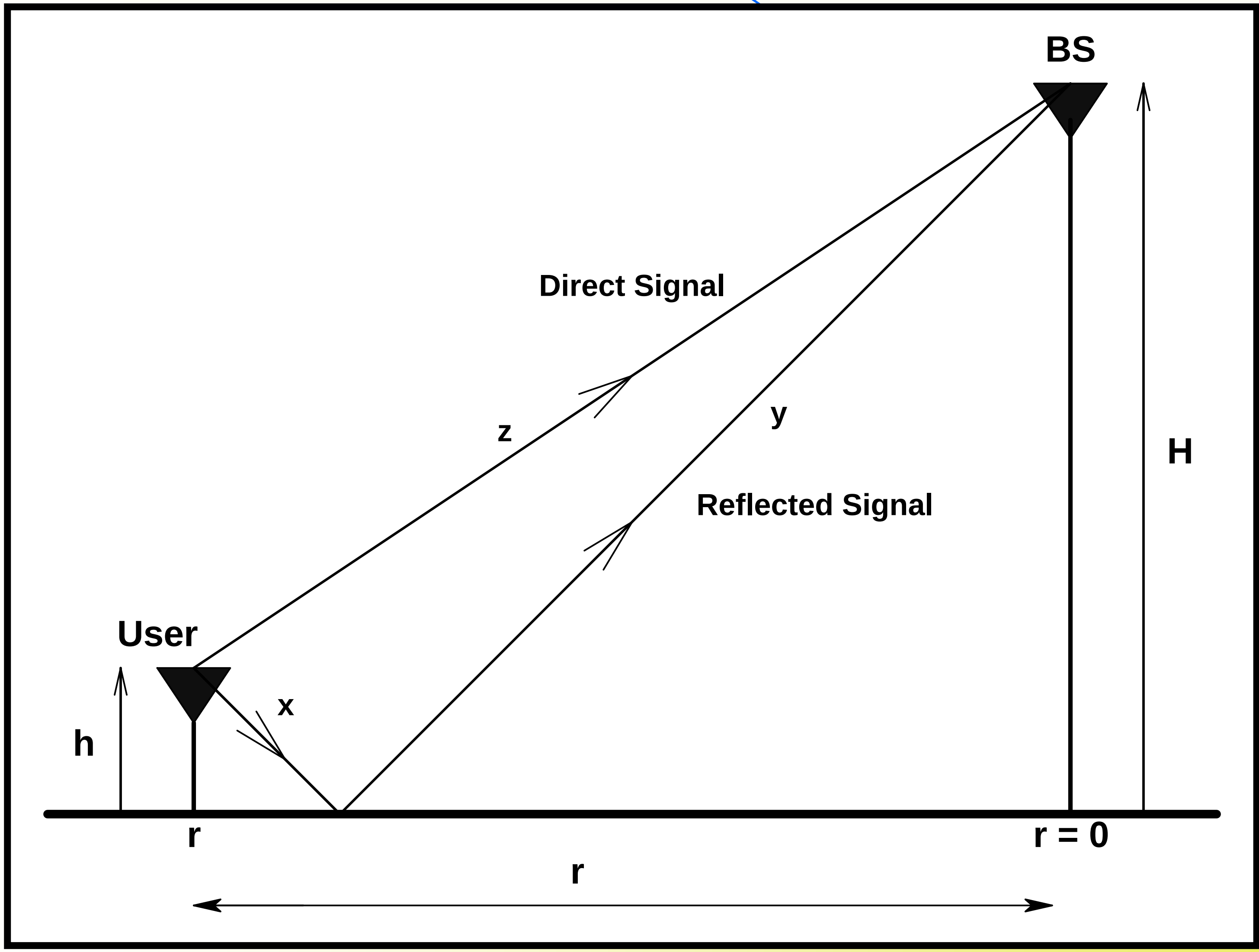
# Why use $\gamma$ = 4 for the propagation loss exponent?



Propagation path loss in different areas

First of all because it works. Practical measurements show that this value is a good approximation, at least as a first choice in a preliminary project as we can see in the figure (from reference [20]).

But despite “this strong” argument we “can prove” it with a very simple two rays model.



$$E = \frac{A}{r}\cos\left(2\pi f\, t\right)$$

Generic expression for the direct electric field at distance r from the source.

So the electric field at BS considering the soil reflection is

$$E = E_{direct} + E_{reflected} = \left(\frac{A}{r}\right)\cos(2\pi f\, t) + \left(\frac{A}{r}\right)\cos(2\pi f\, t + \varphi)$$

where

$$\varphi = \pi + \left(\frac{2\pi}{\lambda}\right)(x + y - z) = \pi + \theta$$

is the phase of reflected wave with

$$x + y - z = d_{reflected} - d_{direct}$$

and after some algebraic manipulations we obtain the combined electric field envelope as

$$|E| = \left|\left(\frac{2A}{r}\right)\mathrm{sen}\ \frac{\theta}{2}\right|$$

where $\theta$ is given by

$$\theta = \left(\frac{2\pi}{\lambda}\right)\left(\sqrt{(H+h)^2 + r^2} - \sqrt{(H-h)^2 + r^2}\right)$$

Now remembering that the received power is proportional to the squared value of electric field magnitude we can write

$$P \propto |E|^2$$

And with the previous result considering $r^2 \gg Hh$ we obtain

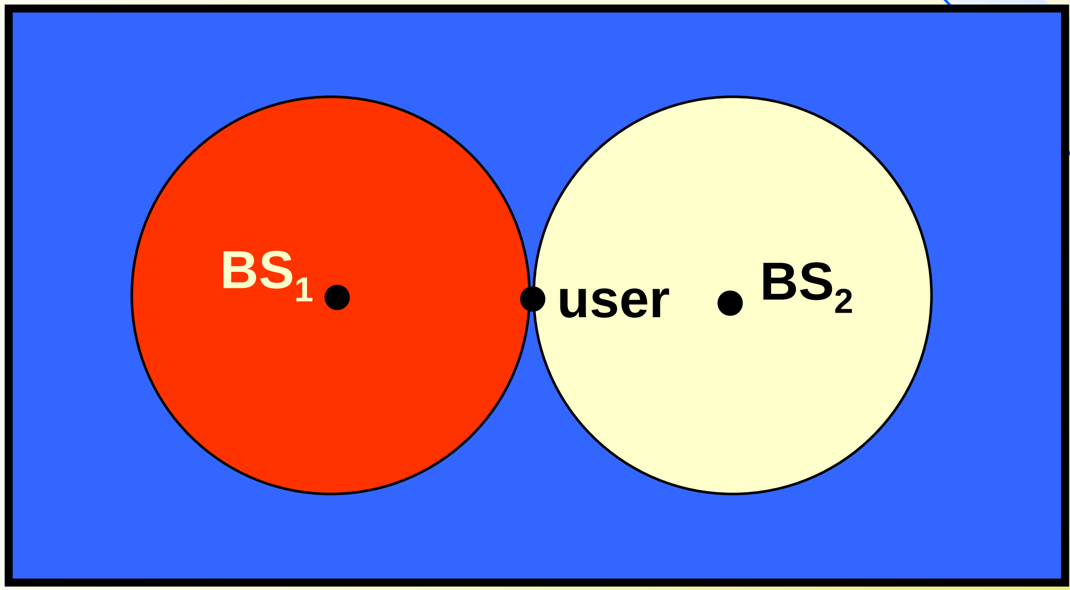
$$P \propto \left(\frac{4A^2}{r^2}\right)\left(\frac{1}{2} - \frac{\cos\theta}{2}\right) \cong \frac{16\pi^2 A^2 H^2 h^2}{r^4 \lambda^2} \propto r^{-4}$$

Which shows us that the received power is proportional to the distance raised to minus four between transmitter and receiver.

Again, this is a simple model that coincides with the measured values and not a physical phenomenon description.

# Imperfect Power Control in the Forward Channel

For the forward channel we will adopt a worst case analysis. If we have a user at mid point between two BS we have a maximum interference from the undesirable BS (see reference [12] for more details).




If we assume that we have U users in each cell we can write the input signal at user's receiver as

$$r(t) = \sum_{i=1}^{U} A\alpha_1 d_i(t-\tau_i)\, p_i(t-\tau_i)\cos(\omega_0 t + \theta_i) + \sum_{i=U+1}^{2U} A\alpha_2 d_i(t-\tau_i)\, p_i(t-\tau_i)\cos(\omega_0 t + \theta_i) + n_w(t)$$

Where almost all terms have known interpretation and additionally

A : is the nominal received amplitude for all users in the desirable user's input

$\alpha_i$ : indicates two random variables (i=1 or 2) which represent a different fading in each channel (from $BS_1$ and from $BS_2$)

For long codes it is easy to show that the conditioned BER is given by

$$P(e|\alpha_1, \alpha_2) = Q\left[\frac{\alpha_1}{\sqrt{\frac{N_o}{2E_b} + \frac{U}{3N}\left(\alpha_1^2 + \alpha_2^2\right)}}\right]$$

Considering $E_b/N_o >> 1$

$$P(e \mid \alpha_1, \alpha_2) = Q\left(\left[\sqrt{\frac{U}{3N}\left(1+\frac{\alpha_2^2}{\alpha_1^2}\right)}\right]^{-1}\right)$$

Define $u=\alpha_2/\alpha_1$ and $v=u^2$. Knowing $\alpha_i$ statistical characteristics we are able to determine the probability density function for the random variables u and v. For instance admitting that $\alpha_i$ has Rayleigh statistics we can write (see reference [9])

$$f_u(u) = \frac{2u}{(u^2+1)^2}$$

for $u \geq 0$ and zero otherwise. So in the next step we can write that

$$f_v(v) = \frac{1}{(1+v)^2}$$

for $v \geq 0$ and zero otherwise. From this result we can finally write

$$P_e = \int_0^{\infty} \frac{1}{(v+1)^2} Q\left(\sqrt{\frac{3N}{U(1+v)}}\right) dv$$

This Rayleigh type channel integral should be evaluated numerically.

Another very useful description for $\alpha_i$ is the log-normal distribution (as we have used in the reverse channel). In this case we have

$$f_{\alpha_i}(\alpha_i) = \frac{\beta}{\sqrt{2\pi}\,\sigma\,\alpha_i} \exp\left[ -\frac{1}{2\sigma^2} \left( \beta \ln(\alpha_i) - d \right)^2 \right]$$

Where $\beta$=20/ln10 and d and $\sigma$ are parameters of log-normal distribution (mean and standard deviation respectively). So for $u=\alpha_2/\alpha_1$ we obtain

$$f_u(u) = \frac{\beta}{2\sqrt{\pi}\,\sigma\,u} \exp\left[ -\frac{\beta^2}{4\sigma^2} \ln^2(u) \right]$$

for $u \geq 0$ and zero otherwise.

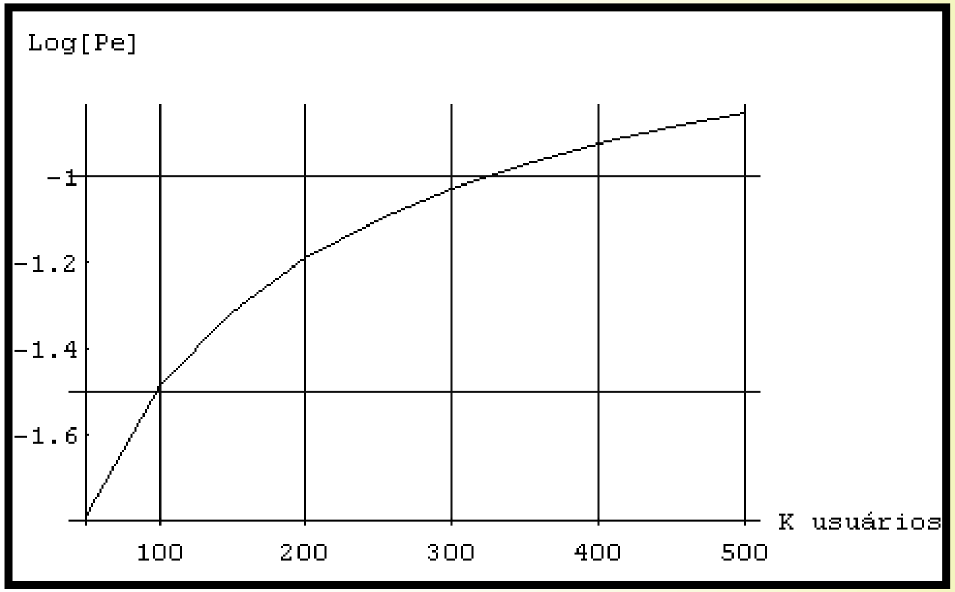
And finally all that we need to do is to evaluate the integral

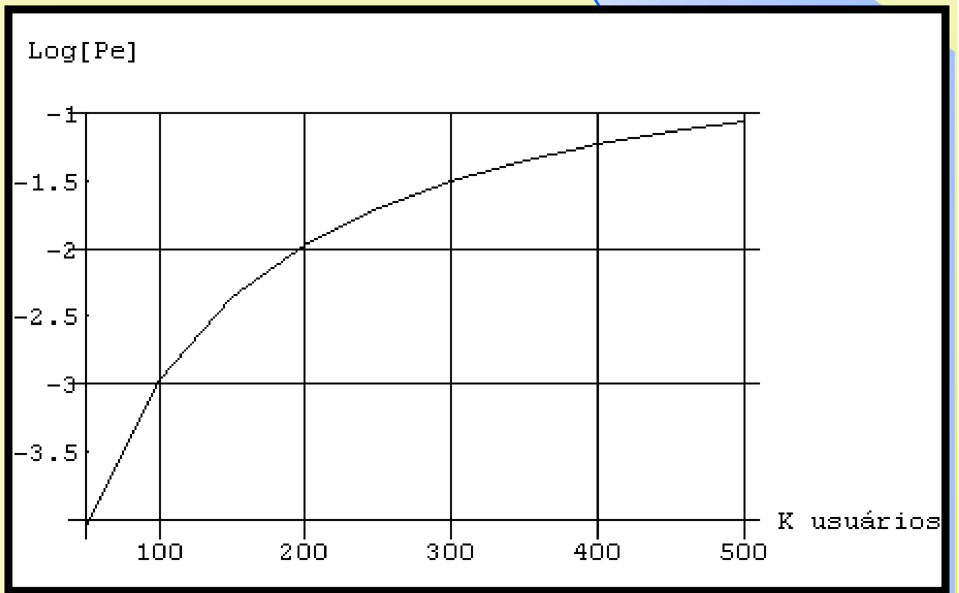
$$P_e = \int_0^{\infty} f_u(u)\, Q\left( \sqrt{\frac{3N}{U(1+u^2)}} \right) du$$

This log-normal type channel integral should be evaluated numerically.

A CDMA system should be balanced at least for vocal applications. This means that we have to have the same capacity in both directions. Thereupon we need to calculate these two capacities carefully and also design appropriate error correcting codes for both sides to guarantee this balance and the desirable performance. We will return to this point later when we will consider the general capacity for a CDMA system.

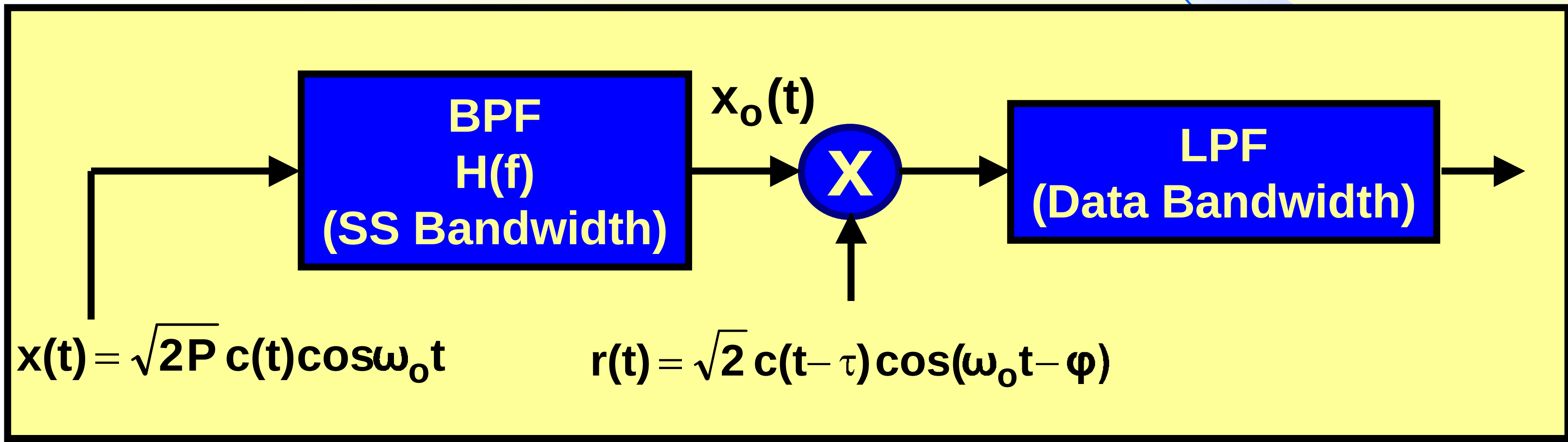
The figures below represent the BER calculated for the forward channel in a Rayleigh environment and with N=511 and without and with error correction (Golay block code (23,12) with t=3 errors correction capability), respectively.



# The Influence of Bandwidth Limitation

As we know with rectangular chip waveforms the Spread Spectrum signal's bandwidth is unlimited and its PSD follows a $\text{sinc}^2$ type function. Our main objective in this section is to establish this bandwidth limitation influence. For this determination we start with the model




Where x(t) represents the unlimited SS signal and r(t) the local generator used for de-spreading purpose. Next we will calculate the correlation's loss as a function of bandwidth's limitation.

Initially we truncate the c(t) sequence to length 2T sec. Assuming that c(t) code is very long, we may consider each chip random and statistically independent from other chip. We may write the input signal as

$$x(t) = \sqrt{2P}\,\mathrm{Re}\{e^{j\omega_0 t}\frac{1}{2\pi}\int_{-\infty}^{\infty} A_T(j\omega)e^{j\omega t}d\omega\}$$

Where $A_T(j\omega)$ denotes the Fourier transform of a 2T sec. segment of c(t) and Re{Z} the real part of complex number Z. After filtering by the BPF (assumed to be centred at $\omega_o$) and denoting $H'(j\omega)$ as the baseband equivalent of $H(j\omega)$, i. e. $H(j\omega)=H'[j(\omega-\omega_o)]$ for $\omega>0$, we obtain

$$x_o(t) = \sqrt{2P}\,\mathrm{Re}\{e^{j\omega_0 t}\frac{1}{2\pi}\int_{-\infty}^{\infty} A_T(j\omega)H'(j\omega)e^{j\omega t}\,d\omega\}$$

In general the reference r(t) will be shifted in time and rf phase from $x_o(t)$ so that is modelled as

$$r(t) = \sqrt{2}\,\mathrm{Re}\{e^{j(\omega_0 t-\varphi)}c(t-\tau)\}$$

In analogy with $x_o(t)$ representation we may write

$$r_o(t) = \sqrt{2}\,\mathrm{Re}\{e^{j(\omega_0 t - \varphi)} \frac{1}{2\pi} \int_{-\infty}^{\infty} A_T(j\omega) e^{j\omega(t-\tau)} d\omega\}$$

Next we should calculate the cross correlation function between the two random functions $x_0(t)$ and $r_0(t)$ given by

$$R(\tau,\varphi) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \overline{x_o(t)\, r_o(t)}\, dt$$

where the overbar denotes the ensemble average. For evaluation details see reference [22]. The final result is

$$R(\tau,\varphi) = \frac{\sqrt{P}}{2\pi} \int_{-\infty}^{\infty} S_p(\omega) H'(j\omega) e^{j(\omega\tau+\varphi)} \cos\varphi\, d\omega$$

Where $S_p(\omega)$ is the PSD of c(t) signal under the assumption of random, independent NRZ symbols with transition density 1/2. Additionally it was assumed that the data bandwidth can be neglected in comparison with the SS bandwidth (processing gain >>1).

We know that h(t) is real when H'(jω) is complex conjugate symmetric. In this case is easy to show that R(τ, φ) is real and maximised when φ=0. So we can express this results as

$$R(\tau,0)=\sqrt{P}\int_{-\infty}^{\infty} S_p(\omega)H'(j\omega)e^{j\omega\tau}\frac{d\omega}{2\pi}$$

In other words: when the bandpass filter preceding the correlator is symmetric around the carrier frequency, the correlated output signal maximisation occurs when the reference carrier is in phase with the input carrier.

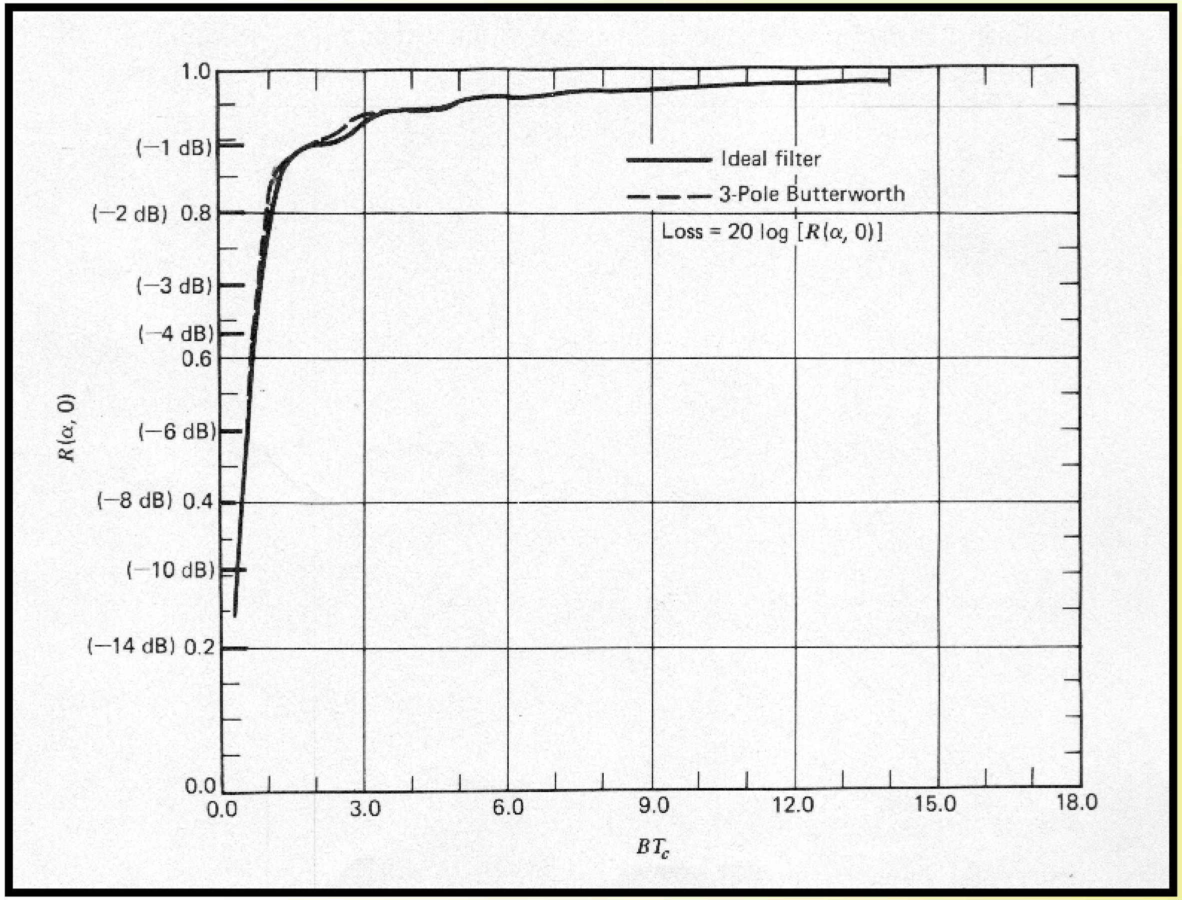
In order to obtain an estimate of correlation loss due to pre-filtering the c(t) sequence consider the value for R(0,0) for a symmetric multi-pole filter modelled as an ideal filter, with linear phase across the band -B/2 to B/2

$$H'(j\omega)=\begin{cases} e^{-j\omega\alpha} & \text{for} \quad |f|<B/2 \\ 0 & \text{for} \quad |f|>B/2 \end{cases}$$

When $\tau=\alpha$ (the time delay introduced by the filter) we have

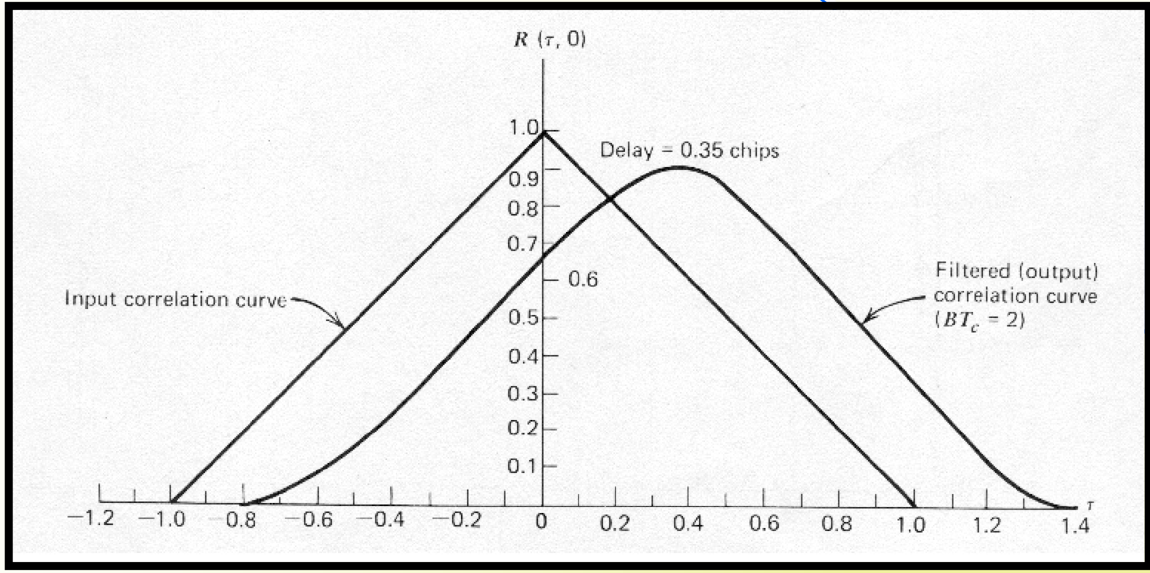
$$R(\alpha,0)=\sqrt{P}\,T_c\int_{-\pi B}^{\pi B}\operatorname{sinc}^2(fT_c)\,\frac{d\omega}{2\pi}$$

In the next figure we show this result for various values of $BT_c$, the channel bandwidth to chip rate ratio.




For the commonly used value $BT_c$=2 the loss is 0,9 dB. Also plotted in figure, the maximum correlation value for a three pole Butterworth filter with B denoting the 3 dB bandwidth (notice that the two filters have almost identical peak correlation values).

In the next figure, the correlation curve with prefiltering of $BT_c=2$ (three pole Butterworth) is shown. Note that the peak value is diminished by filtering and the curve is somewhat broadened, and further that the correlation curve is delayed by about 0,35 chips.

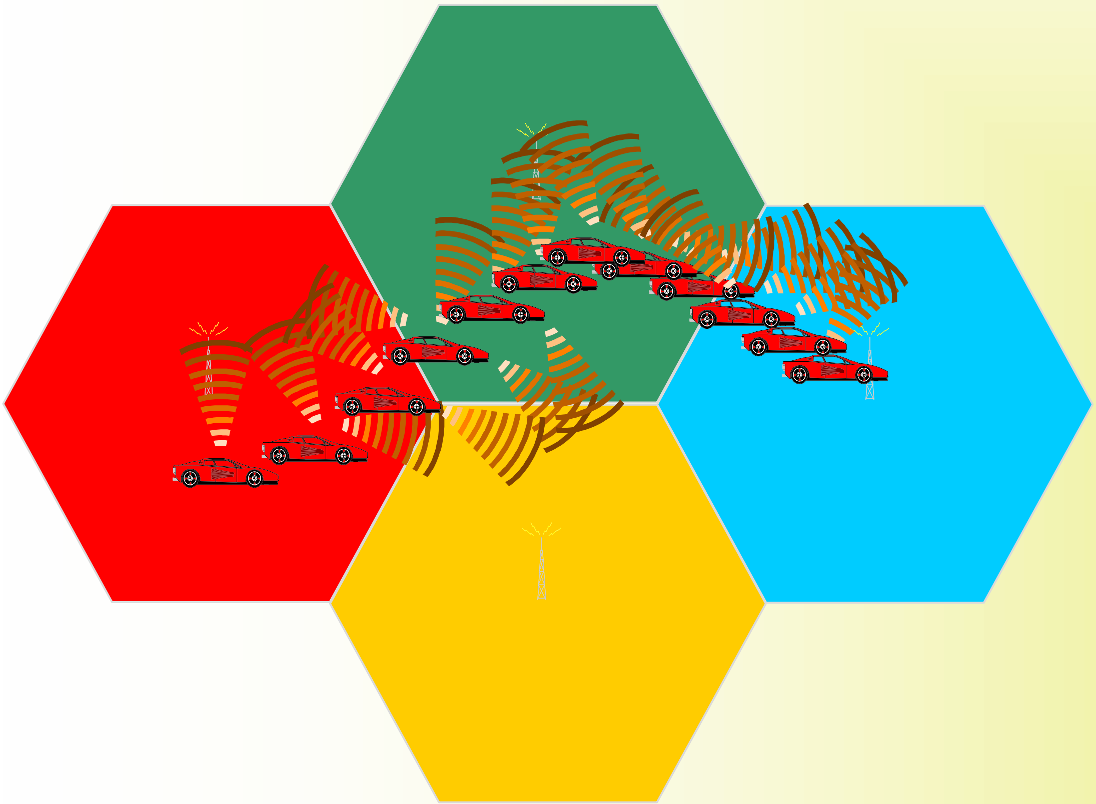


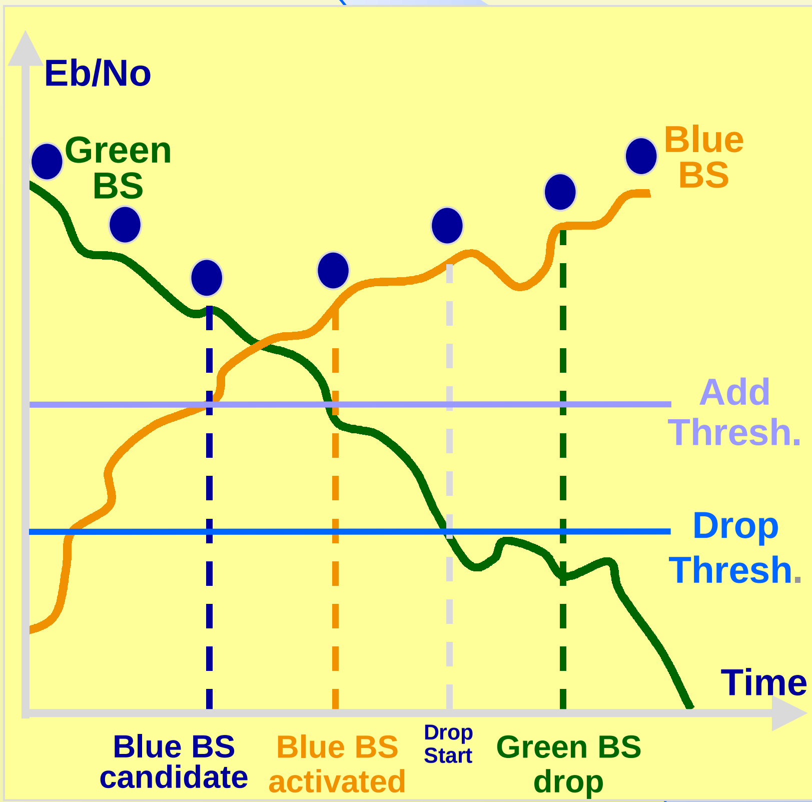

With additional insight purpose these results can also be obtained with a general theorem about linear systems with stochastic inputs (see reference [9] for details) that establish the auto correlation of the output of a linear system, with impulse response h(t), in terms of the cross correlation between input x(t) and output y(t) signals

$$R_{y,y}(t_1, t_2) = \int_{-\infty}^{\infty} R_{x,y}(t_1 - \alpha, t_2)\, h(\alpha)\, d\alpha$$

# Capacity Reduction by Soft Handoff

Soft handoff is a system's mechanism that allows us to switch users from one cell to another neighbour cell which could be better for the current conversation. During this transition the user can be simultaneously connected to two (or three) BS - Base Stations. Obviously, this facility reduces the system's capacity because we are using two (or three) physical channels for only one conversation.





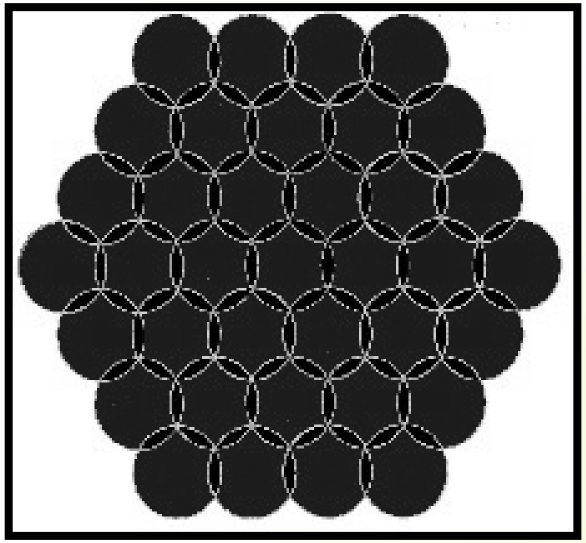
This capacity reduction can be geometrically calculated considering the areas covered by for one, two or three BS. This coverage depends though, on the imposed confidence for signal's strength but here we will focus only on the geometrical problem.The capacity loss for an uniform distribution of users can be defined as

$$P_{SH} = \frac{N_U}{N_C} = \frac{SA_1 + SA_2 + SA_3}{SA_1 + 2SA_2 + 3SA_3}$$

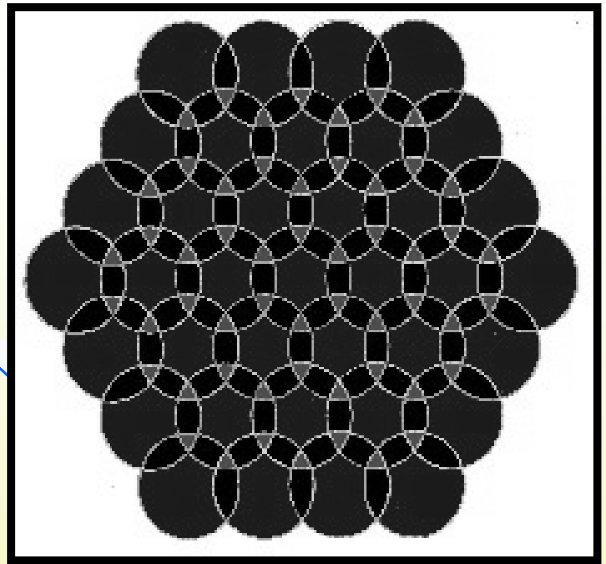
where $N_u$ and $N_c$ represent the numbers of active users and the number of active channels, respectively, and $SA_i$ (i=1,2,3) represents the total area covered by i BS. Imposing that all areas will be covered by at least one BS and that we have no regions with more than three BS we obtain the following restriction

$$\frac{\sqrt{3}}{3} \le \frac{R}{D} \le \frac{\sqrt{3}}{2}$$

where R is the cell radius and D the distance between two adjacent cell's centres. The next two figures illustrate this concept: for the first figure we have $SA_3$=0 (so R/D has its minimum value) and for the next R/D=0,65.



In these figures the blue regions (area of type $A_1$) are attended by only one BS, black regions (area of type $A_2$) by two BS and red regions (area of type $A_3$) by three BS. It can be shown that



$$A_3 = D^2\left[\frac{\sqrt{3}}{4}\left(1-\sqrt{12f^2-3}\right)+3\beta f^2\right]$$

$$A_2 = D^2\left(2f^2\theta - f\sin\theta\right) - 2A_3$$

$$A_1 = \pi R^2 - 6(A_3 + A_2)$$

where

$$\theta = \cos^{-1}\left(\frac{1}{2f}\right)$$ and $$\beta = \sin^{-1}\left(\frac{\sqrt{12f^2-3}-1}{4f}\right)$$ and $$f = \frac{R}{D}$$

Considering now that we have N≥2 rings (N includes the central cell), we can express the areas $SA_i$ as

$$SA_3 = 6\sum_{n=1}^{N}(2n-1)A_3$$

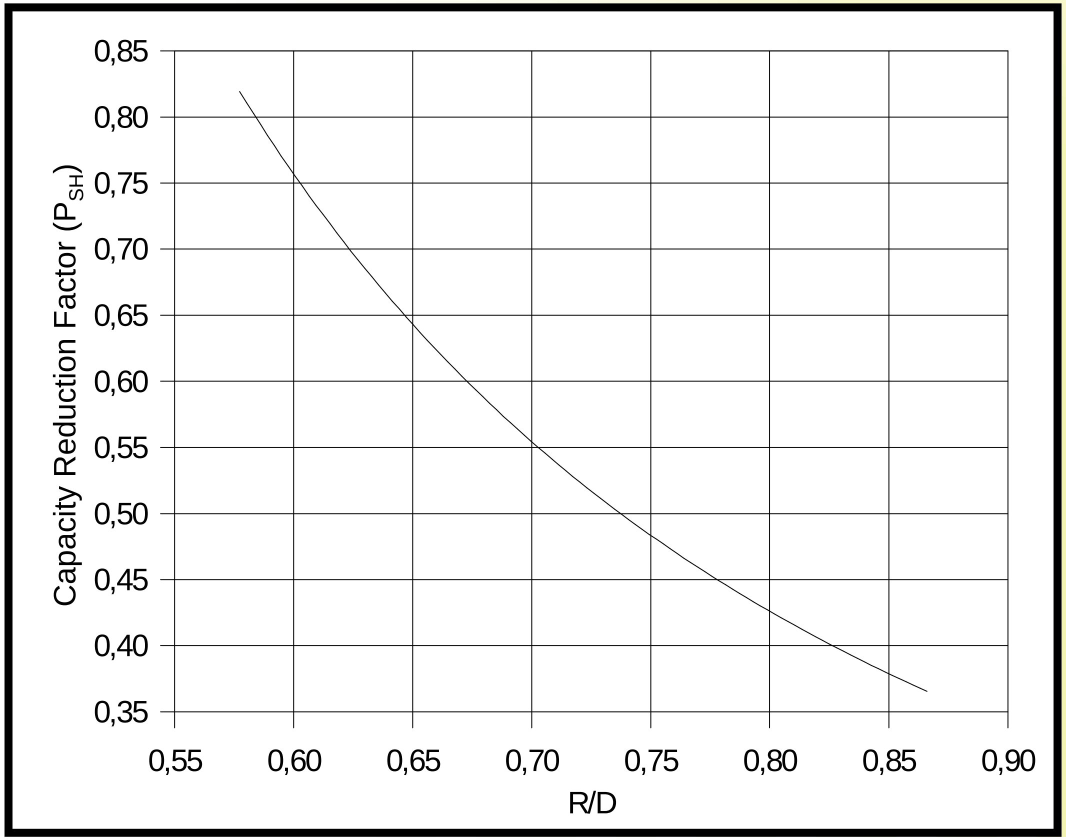
and

$$SA_2 = 6\left[\sum_{n=1}^{N}(2n-1) + \sum_{n=1}^{N-1} n\right]A_2$$

and

$$SA_1 = \left(1 + 6\sum_{n=1}^{N-1} n\right)A_1$$

and from these we obtain the graph for $P_{SH}$, as a function of R/D



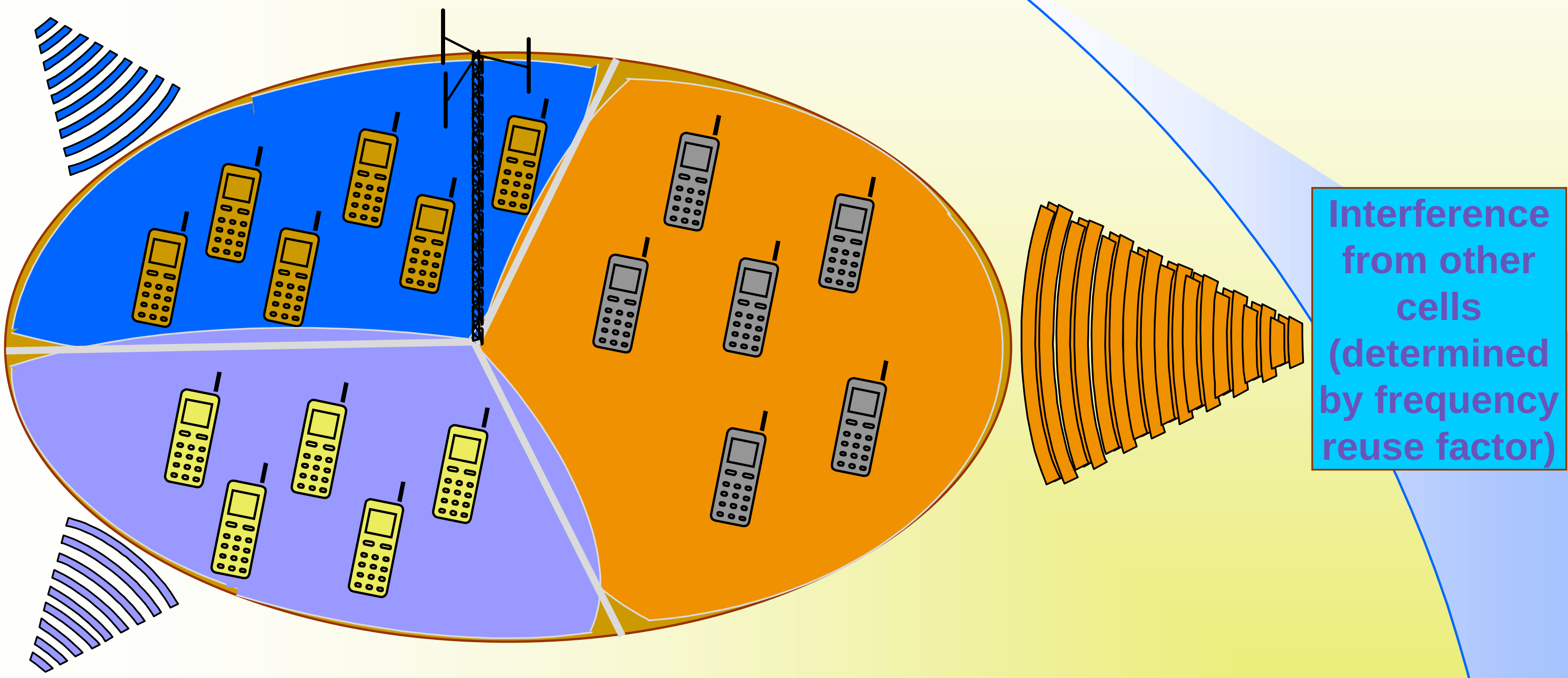

The graph was plotted for N=10, but it can be shown that increasing N the result is very close to the presented.

This graph show us that if we impose R/D=0,65 35% of system's capacity is lost by the soft handoff mechanism.

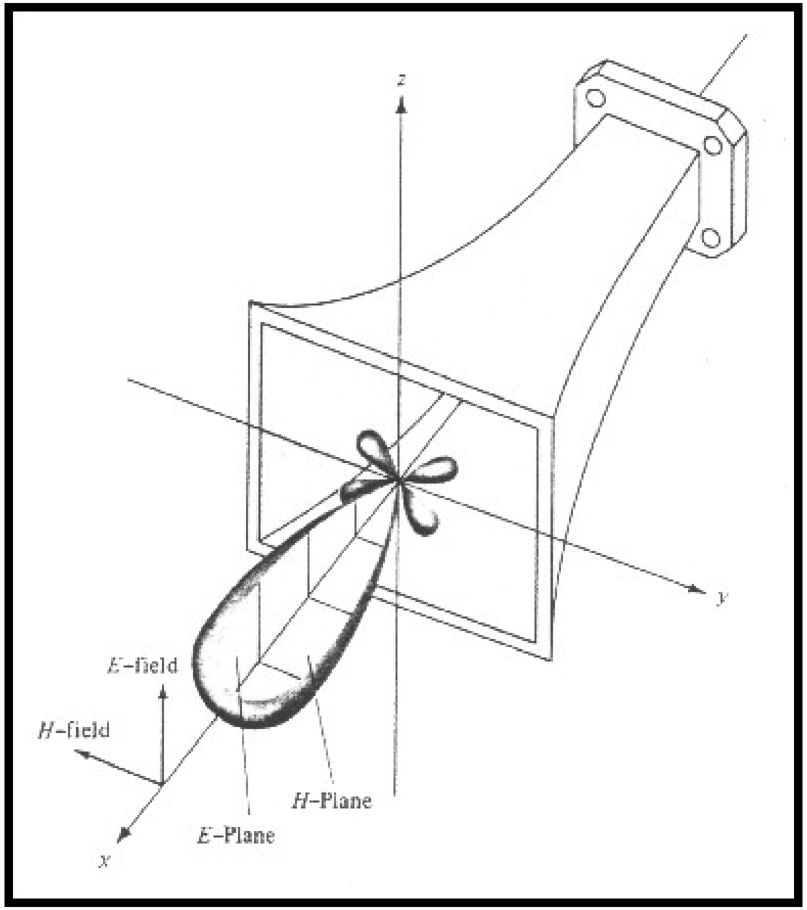
(return)

# Sectoring

In CDMA systems the capacity is interference limited. The sectoring reduces the MAI (greater capacity) but introduces erlang losses (smaller capacity).




First of all we need to know the antenna’s radiation diagram used in the BS. In reality this is a three dimensional diagram.




Three dimensional radiation diagram for a pyramidal horn antenna, reference [28].

In general, it is very hard to work with three dimensional diagrams, so it is more usual to consider only two dimension: the horizontal and the vertical radiation diagrams.

These two diagrams correspond to the intercession of one x-z vertical plane (elevation plane, $\varphi=0$) and one x-y horizontal plane (azimutal plane, $\theta=90^o$) with the three dimensional radiation diagram.

For many practical cases it is sufficient to consider only one dimension. For instance, it is easy to show that for the pyramidal horn type antenna these two diagrams will be equivalent. So now we need to model our radiation diagram. We will represent it by $G(\theta)$

$$G(\theta) = AF(\theta)$$

Where A is a constant that represents antenna gain relative to an ideal isotropic antenna (which means equal radiation in all directions) and $F(\theta)$ is a polar function with the following characteristics

- has maximum value 1 for $\theta=0$ and
- has value 0,5 for $\theta=\theta_{ref}=\pm\pi/3$ when considering sectors with $2\pi/3$ aperture.

(all developments ahead can be easily adapted for other antenna apertures).

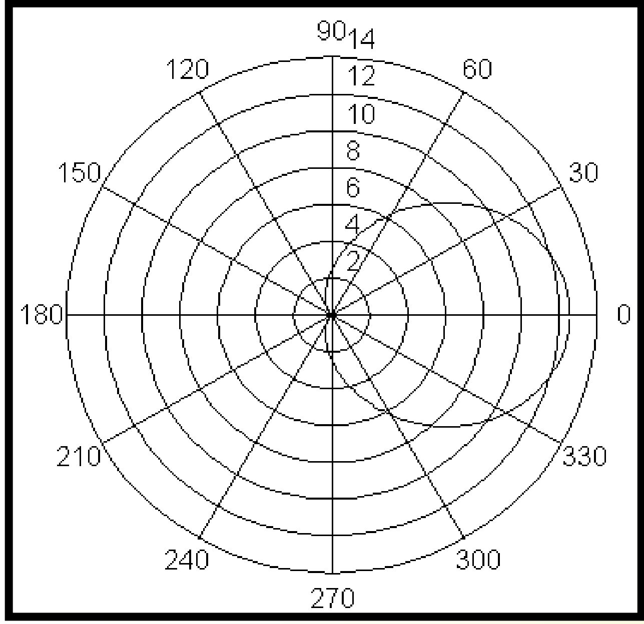
With some minor adaptations we can consider a Pascal's function for our objective

$$F(\theta) = \left(\frac{f+\cos\theta}{f+1}\right)^{\alpha}$$

where

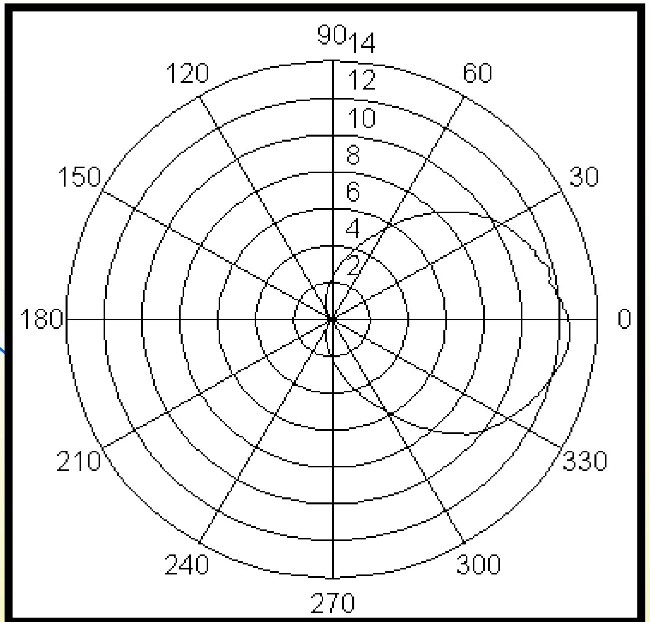
$$\alpha = \frac{\log(0,5)}{\log\left(\frac{f+\cos\theta_{ref}}{f+1}\right)}$$

This last equation guarantees the second required characteristic. The parameter $f \geq 1$ should be determined in order that the model matches with the real antenna's radiation diagram. In the next two figures we compare an real antenna's radiation diagram with one obtained with the described model.




<u>Model</u>
with f=1,1

<u>Real</u>
model DB874H120
(Decibels Products)




For imperfect sectorization analyses we will consider

-three sectors in a cell, numbered as 1, 2 and 3 and located in the intervals ]-π/3, π/3], ]π/3, π] and ]π,5π/3], respectively;

- user's power is controlled which means equal power (P) in the BS's receiver input;

- the three cell's antennas are very close so we can write for the i<u>th</u> user, located in sector 1, that the power $P_i$ that reaches the antennas is

$$P_i = \frac{P}{G(\theta_i)}$$

Where $G(\theta)$ describes the antenna's horizontal radiation diagram and $\theta_i$ is the signal's arrival angle formed by the arrival direction and the sector 1 central ray.

Considering that all cell's antennas have the same radiation diagram we can restrict our analyses to sector 1. Therefore for the signal that has an arrival angle $\theta_i$ we can express the power that reaches sector 1's receiver as

$$I_i(\theta_i) = \begin{cases} P & -\pi/3 < \theta_i \le \pi/3 \\ \dfrac{P}{G(\theta_i - 2\pi/3)} G(\theta_i) & \pi/3 < \theta_i \le \pi \\ \dfrac{P}{G(\theta_i - 4\pi/3)} G(\theta_i) & \pi < \theta_i \le 5\pi/3 \end{cases}$$

Admitting that all users are uniformly distributed in the cell the arrival angle $\theta_i$ can be considered uniformly distributed in the interval $]0, 2\pi]$. So the mean interference caused by the $i^{th}$ user can be calculated as

$$E[I_i(\theta_i)] = \int_0^{2\pi} I_i(\theta_i) f(\theta_i)\, d\theta_i = \frac{1}{2\pi}\int_0^{2\pi} I_i(\theta_i)\, d\theta_i$$

and considering all other (U-1) users as independent the total interference is

$$I = \sum_{i=1}^{U-1} E[I_i(\theta_i)] = \frac{(U-1)}{2\pi}\int_0^{2\pi} I_i(\theta_i)\, d\theta_i$$

From previous equations we can establish the following relations

$$I = \frac{P(U-1)}{3}(1+m)$$

with

$$m = \frac{3}{2\pi}\int_{\pi/3}^{5\pi/3} \frac{I_i(\theta)}{P}\, d\theta$$

On the other hand, for a system without sectoring the total interference is I=P(U-1) so we can express the sectoring gain as

$$G_S = \frac{3}{1+m}$$

(return)

Note that m represents the interference's integral outside the antenna's aperture. Of course, for ideal conditions m approaches 0 and $G_S$ approaches to 3.

All calculations were done for the former modelled antenna and we have obtained $G_S$=2,25.

In the next tables we show some additional results that were obtained with described methodology.

# Three sectors results

## Real data

| Model | DB871H120 | DB872H120 | DB874H120 | DB878H120 |
|---|---|---|---|---|
| Gain (dB) | 05 | 08 | 11 | 14 |
| $FBR_{mean}$(dB) | 15,85 | 17,14 | 18,98 | 20,21 |
| $G_s$(824 MHz) | 2,32 | 2,27 | 2,28 | * |
| $G_s$(835 MHz) | 2,34 | 2,25 | 2,27 | 2,26 |
| $G_s$(837 MHz) | 2,30 | 2,26 | 2,26 | * |
| $G_s$(849 MHz) | 2,31 | 2,25 | 2,26 | * |

## Model

| Model | f=1,1 | f=2,3 | *Flat-top* | *Flat-top* | *Flat-top* |
|---|---|---|---|---|---|
| Gain (dB) | 11 | 11 | 0 | 0 | 0 |
| $FBR_{mean}$ (dB) | 33,70 | 17,06 | 6 | 10 | 100 |
| $G_s$ | 2,25 | 2,15 | 1,99 | 2,49 | 3,00 |

Notes
1) FBR denotes front to back ratio (in dB) and
2) * denotes unavailable data

## Six sectors results

### Real data

| Model | DB881H60 | DB882H60 | DB884H60 |
|---|---|---|---|
| Gain (dB) | 08 | 11 | 14 |
| $FBR_{mean}$(dB) | 24,20 | 27,33 | 29,94 |
| $G_s$(824 MHz) | 4,22 | 4,40 | 4,46 |
| $G_s$(835 MHz) | 4,29 | 4,50 | 4,50 |
| $G_s$(837 MHz) | 4,30 | 4,47 | * |
| $G_s$(849 MHz) | 4,29 | 4,50 | 4,49 |

### Model

| Model | f=1,1 | f=2,3 | *Flat-top* | *Flat-top* | *Flat-top* |
|---|---|---|---|---|---|
| Gain (dB) | 11 | 11 | 0 | 0 | 0 |
| $FBR_{mean}$ (dB) | 139,02 | 67,66 | 6 | 10 | 100 |
| $G_s$ | 4,43 | 4,40 | 2,65 | 3,98 | 6,00 |

# Smart Antennas

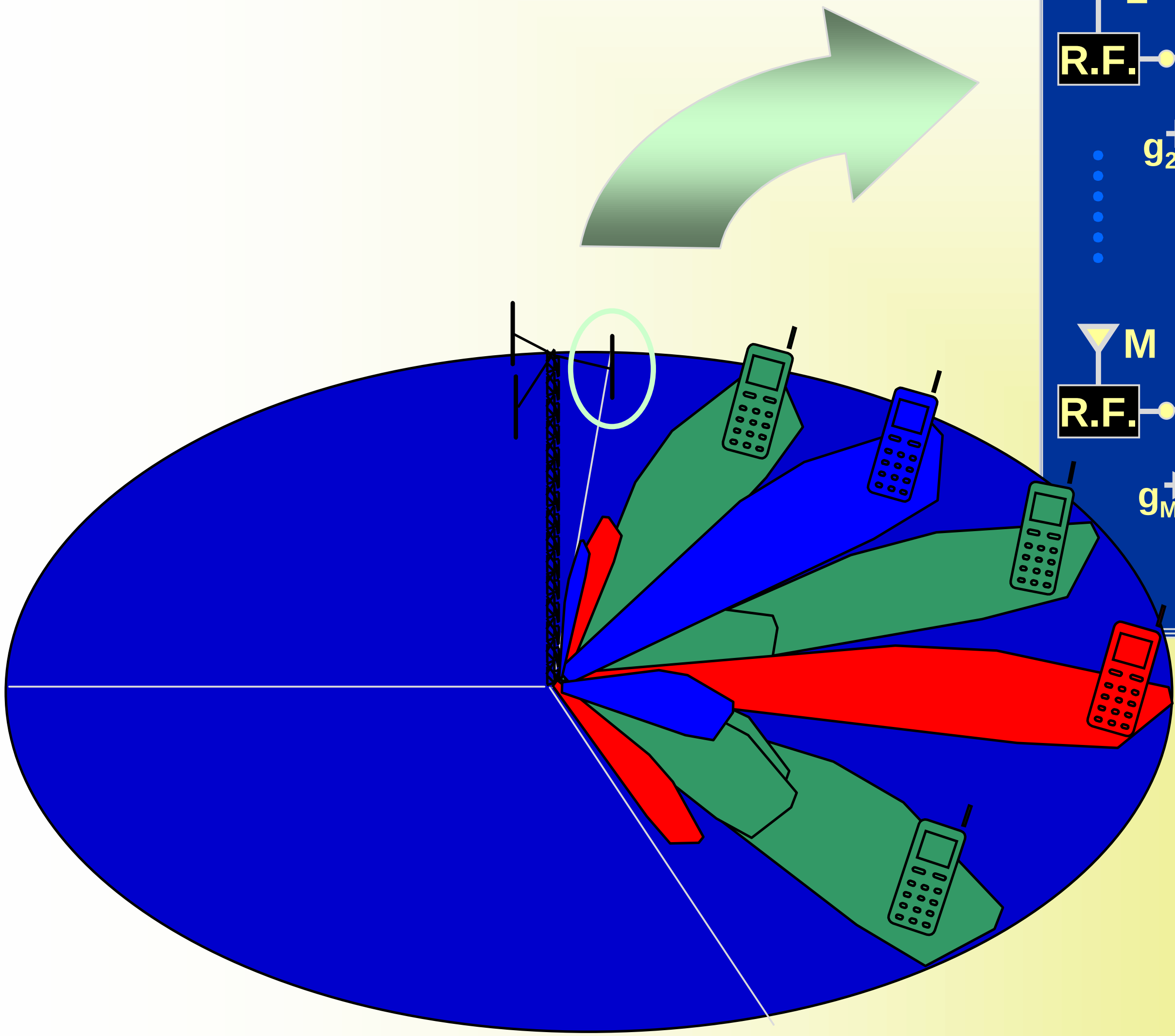
From the previous section we can conceive that with smaller antenna's aperture we are able to diminish MAI's effects more and more. In fact the general idea behind smart antennas is to build antennas for each individual user with a very small aperture.

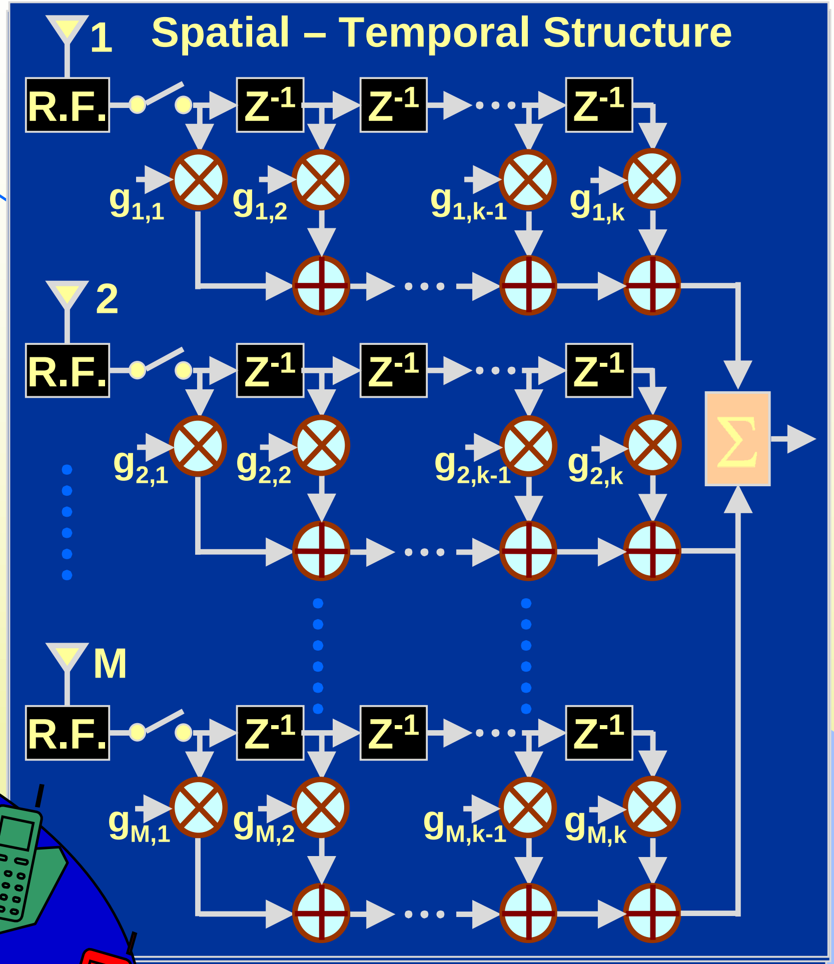
Next we present some general ideas and a very simple formulation with an introductory objective. State of the art in this area is well beyond the scope of these notes. To anyone interested in more details we recommend reference [31] as starting point.

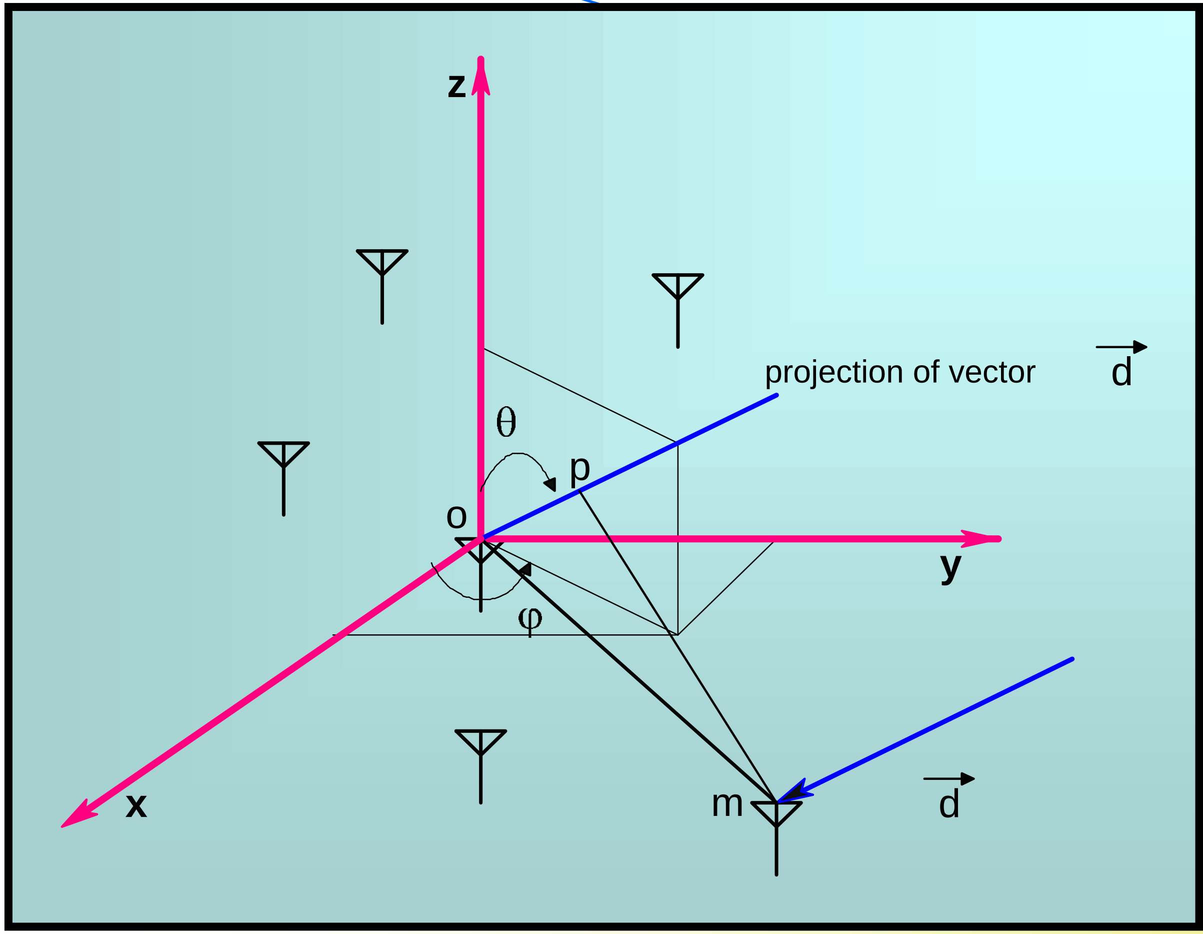
Smart Antennas can be considered as a spatial separation among signals and this fact usually is referred to as SDMA-Spatial Division Multiple Access technique.

Frequently it uses spatial (detailed in sequel) and also temporal (as used in circuits with diversity to combat multi-path fading and ISI) techniques for signal enhancement/detection as illustrated in next slide.

## Smart Antennas Structures








In this figure we present a set of antennas with an incident plane wave following vector d direction. We wish to determine the phase difference $\Delta\Psi_m$ between signals that arrive in antenna “m” and our reference antenna “0”.

In order to simplify our task we will assume that

1) All antennas are close enough to ensure that we have no amplitude variations for the received signal in each antenna;
2) We have no mutual coupling among the antennas;
3) The incident wave’s bandwidth is small when compared with carrier’s frequency.

The difference between the two trajectories is given by $\vec{op}$ vector's module denoted here by $\Delta d_m$. Let ($x_m$, $y_m$, $z_m$) the co-ordinates of point "m" . It is easy to show that

$$\Delta\psi_m = \beta\,\Delta d_m = \beta\,(x_m \cos\varphi \sin\theta + y_m \sin\varphi \sin\theta + z_m \cos\theta)$$

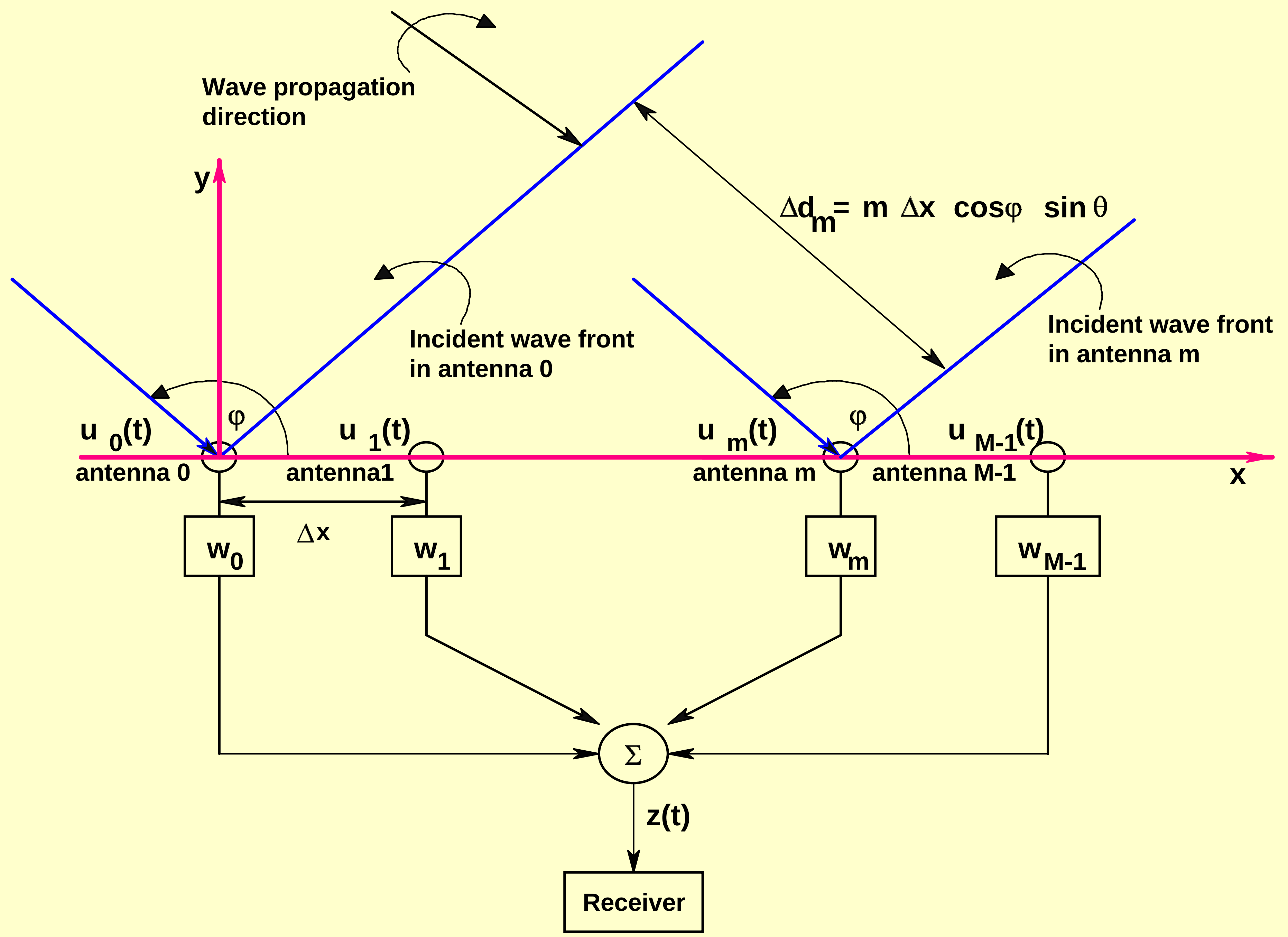
with $\beta=2\pi/\lambda$ and where $\lambda$ is the carrier's wavelength.

More usually smart antennas are constituted by LES-Linear Equally Spaced antennas. In the next figure we show an example with M (numbered from 0 to M-1) radiation elements spaced by $\Delta$x and represented in x-y plane.

Let s(t), the baseband representation of received signal in each antenna. We can express the signal received by antenna "m" as (remember that with our representation y=z=0)

$$u_m(t) = As(t)\, e^{-j\beta\,\Delta d_m} = As(t)\, e^{-j\beta\, m\, \Delta x \cos\varphi \sin\theta}$$

where A is a constant for all signals $u_i(t)$, i=0, 1, ..., M-1.


Wave propagation direction
y
x
$\Delta d_m = m\ \Delta x\ \cos\varphi\ \sin\theta$
Incident wave front in antenna 0
Incident wave front in antenna m
φ
φ
$u_0(t)$
$u_1(t)$
$u_m(t)$
$u_{M-1}(t)$
antenna 0
antenna1
antenna m
antenna M-1
Δx
$w_0$
$w_1$
$w_m$
$w_{M-1}$
Σ
z(t)
Receiver

With these M signals defined we can write the receiver's input signal as

$$z(t) = \sum_{m=0}^{M-1} w_m u_m(t) = As(t) \sum_{m=0}^{M-1} w_m e^{-j\beta m \Delta x \cos\varphi \sin\theta} = As(t) f(\theta, \varphi)$$

The signal's power can be evaluated as

$$P_r = \frac{1}{2}|z(t)|^2 = \frac{1}{2}|As(t)|^2 |f(\theta, \varphi)|^2$$

Manipulating $W_m$ (smart antennas are also known as adaptive antennas) it is possible to adjust the radiation diagram's maximum to any desired direction ($\theta_0$, $\varphi_0$). For instance, putting $W_m$ in the form

$$w_m = e^{j\beta m \Delta x \cos\varphi_0}$$

we obtain for $|f(\theta, \varphi)|^2$

$$|f(\theta, \varphi)|^2 = \left[ \frac{\sin\left( \pi M \left( \frac{\Delta x}{\lambda} \right) (\cos\varphi \sin\theta - \cos\varphi_0) \right)}{\sin\left( \pi \left( \frac{\Delta x}{\lambda} \right) (\cos\varphi \sin\theta - \cos\varphi_0) \right)} \right]^2$$

And if we are interested in the direction $\theta_0=\pi/2$ we can rewrite the equation as

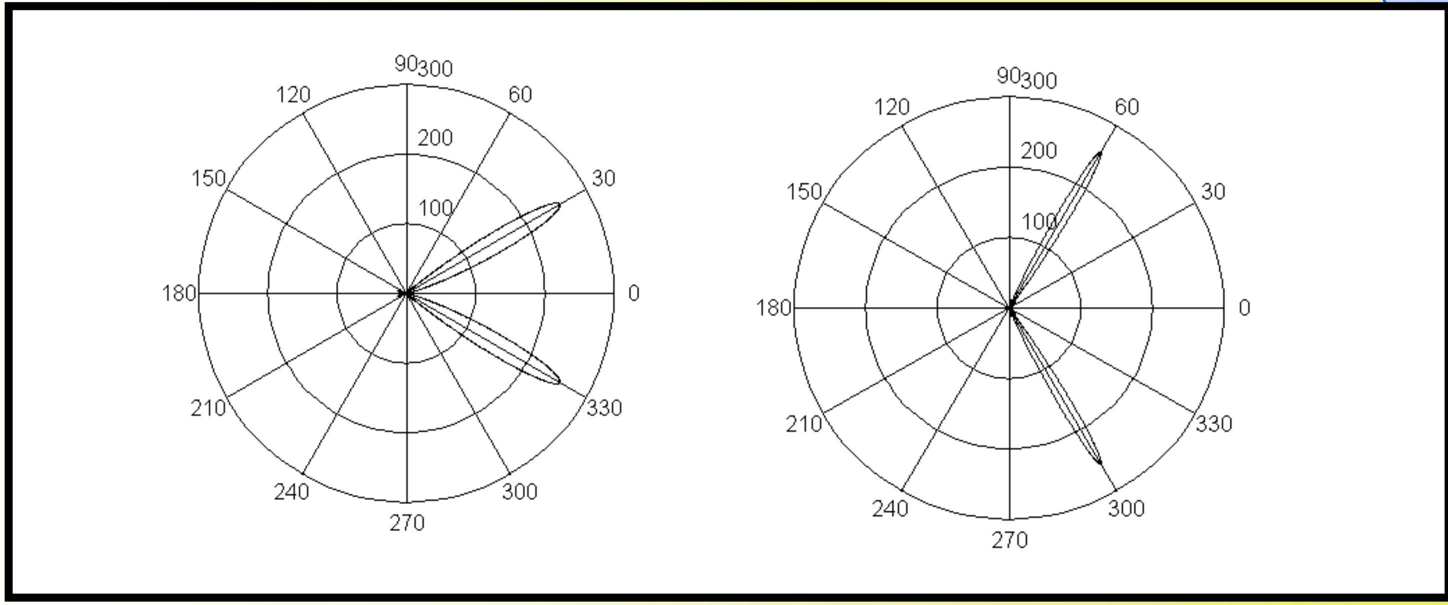
$$\left|f\left(\pi/2,\varphi\right)\right|^2 = \left[\frac{\sin\left(\pi M\left(\frac{\Delta x}{\lambda}\right)(\cos\varphi-\cos\varphi_0)\right)}{\sin\left(\pi\left(\frac{\Delta x}{\lambda}\right)(\cos\varphi-\cos\varphi_0)\right)}\right]^2$$

Which represents the horizontal radiation diagram for a set of LES antennas.

The antenna's gain can be evaluated by

$$G = \lim_{\varphi\to\varphi_0}\left|f\left(\pi/2,\varphi\right)\right|^2 = M^2$$

Example: a set of M=16 antennas with $\Delta x/\lambda=1/2$ allows us to calculate the above expression and in the next figures we show the results for two values of $\varphi_0$: $\pi/6$ and $\pi/3$



The interference for a user with direction $\varphi_0$ caused by other users in the cell can be estimated considering, firstly, that all users reach the BS's receiver with perfect power control and consequently the i^th user's signal with direction $\varphi$ reaches the BS with a power $P_i$ given by

$$P_i = \frac{P}{G}$$

Where P is the power level which corresponds to the perfect power control case and G the gain of LES antennas.

From our previous results we can write that

$$I_i(\varphi) = \frac{P}{G}\left|f(\pi/2, \varphi)\right|^2$$

And considering all users in the cell

$$I = \iint_{A_T} I_i(\varphi)\rho\, dA_T$$

where $\rho$ is the user's density and $A_T$ the total cell's area. Admitting that users are uniformly distributed in the cell we obtain

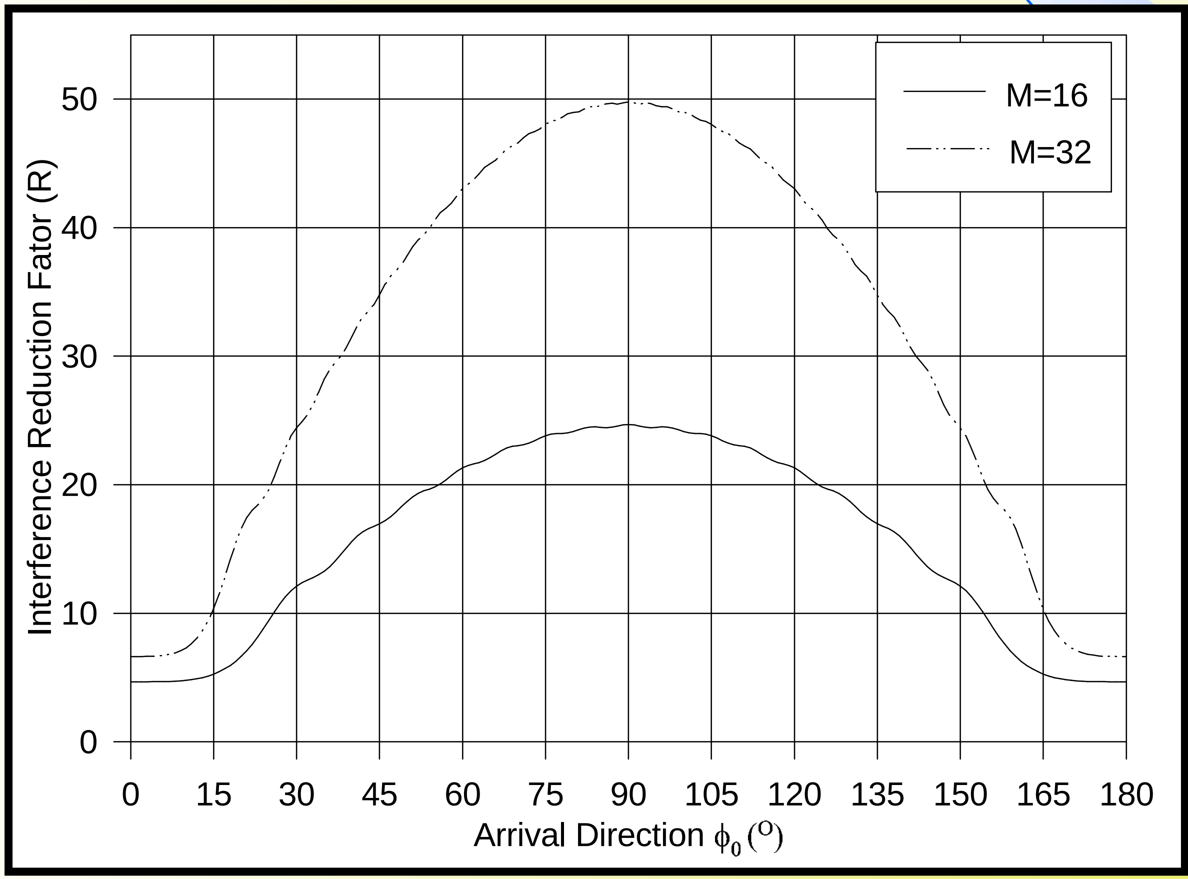
$$I = \frac{PU}{2\pi G}\int_0^{2\pi}\left|f(\pi/2, \varphi)\right|^2 d\varphi$$

where $U = \rho A_T$ is the total number of users in the cell. In this expression the PU product represents, approximately, the total interference for a perfect power control case in a isolated cell.

This last observation allows us to write an interference reduction factor due to the set of LES antennas use as

$$R = \frac{2\pi G}{\int_0^{2\pi} \left| f(\pi/2, \varphi) \right|^2 d\varphi}$$

The figure below represents this reduction factor for two values of M. As we can observe the factor is more significant for signals which reach the set of antennas in a direction perpendicular to the antennas' plane.



# General Capacity Expression for the Reverse Channel

Assuming an isolated cell, with perfect power control in the reverse channels and omnidirectional antenna at its BS we can write the SNR for desirable user as

$$(SNR) = \frac{C}{I} = \frac{P_0}{(U-1)P_0} \rightarrow U = \frac{W}{R} \times \frac{1}{\frac{E_b}{N_i}} + 1$$

where

W : bandwidth of all SS signals

R : data rate

$E_b$ : bit's energy

$N_i$ : equivalent power spectral density of all other (U-1) users (MAI)

Note that we have ignored thermal noise (the system is assumed interference limited) and the $E_b/N_i$ relation should be interpreted as the ratio between bit's energy to total equivalent noise density required by the detector (determined by modulation, FEC and so on), for a fixed BER performance.

From our previous sections we need to add some correction factors for this number of active channels in the cell

$$U \approx \frac{W}{R} \times \frac{1}{\frac{E_b}{N_i}} \times G_{VA} \times F \times P_{SH} \times G_S$$

where (in the brackets we put some typical project values for the system defined in IS-95)

| | | | |
|---|---|---|---|
| $G_{VA}$ : | voice activity and power control | (2,20) | (compare) |
| F : | frequency reuse | (0,75) | (compare) |
| $P_{SH}$ : | soft handoff | (0,65) | (compare) |
| $G_S$ : | sectoring | (2,25) | (compare) |

All factors were assumed independent and with direct influence on system's capacity: the factors $G_{VA}$, F and $G_S$ act through MAI reduction/enhancement and the factor $P_{SH}$ acts through additional channels necessity.

With separated values for voice activity and power control this is the classic expression for capacity determination found in literature.

For the IS-95 system W/R=128, $E_b/N_i$=5 (7 dB) and with the above values we have U≈62 channels.

We can repeat this determination with another approach. Considering our previous developments and ignoring thermal noise we may write from slide 63

$$Z_x = Inf_x + Interf_x$$

Next, we will take into account that the interference is diminished by sectoring ($G_S$>1) and enhanced by the reuse factor (F<1). Remembering that these factors were calculated as power ratios we may write

$$Z_x = Inf_x + \frac{Interf_x}{\sqrt{FG_S}}$$

From this equation we are able to write (in a similar manner as we did in slide 73)

$$P_e = Q\left(\frac{\sqrt{FG_S}\,E[Z_x]}{\sqrt{var[Z_x]}}\right) = Q\left(\frac{\sqrt{FG_S}\,E[v_x]}{\sqrt{var[v_x] + \frac{U-1}{3N}E[v_x^2]E[\alpha^2]}}\right)$$

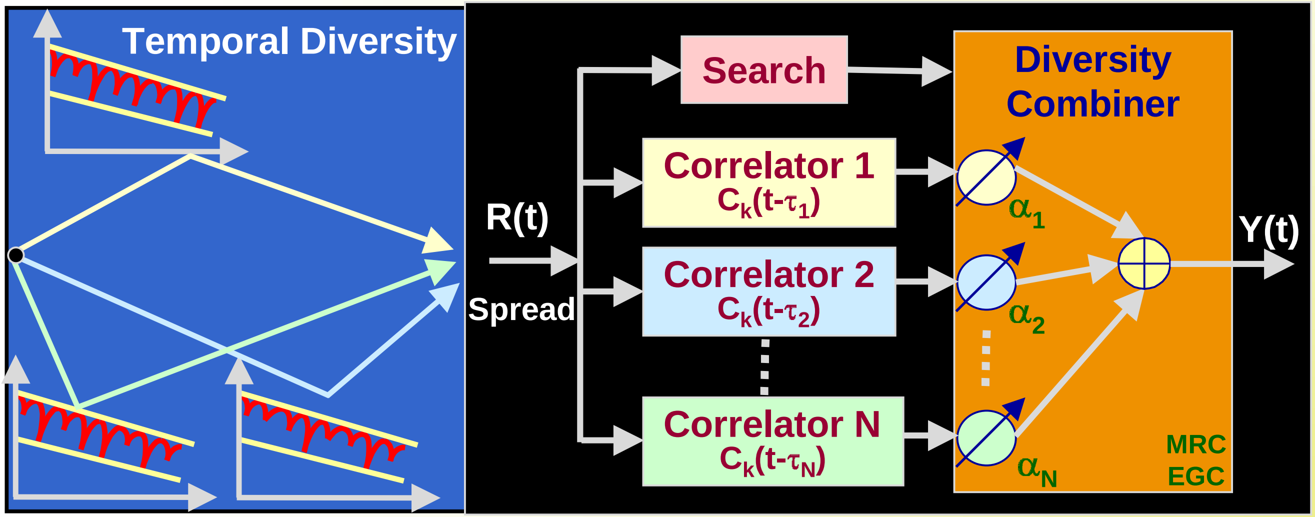
and finally from this equation

$$U = \left\{\frac{3N}{E[v_x^2]E[\alpha^2]}\left[\left(\frac{\sqrt{FG_S}\,E[v_x]}{Q^{-1}(P_e)}\right)^2 - var[v_x]\right] + 1\right\}P_{SH}$$

This expression may be evaluated with the former values used for IS-95 system and power control, voice activity and soft handoff imposed conditions.

# Rake Receiver

- Spreading codes with small cross-correlation ⇒ multipath with a delay greater than $T_c$ can be considered as no-correlated and seems as resolvable path;
- In order to avoid ISI the delay spread should be smaller than $T_b \Rightarrow R_b < Bw_{coh}$;
- Rake receiver needs to identify the strongest paths. The "Search" block follows the input and always uses the strongest available multipaths.




**IS-95 System**

Forward: 3 Fingers (mobile)

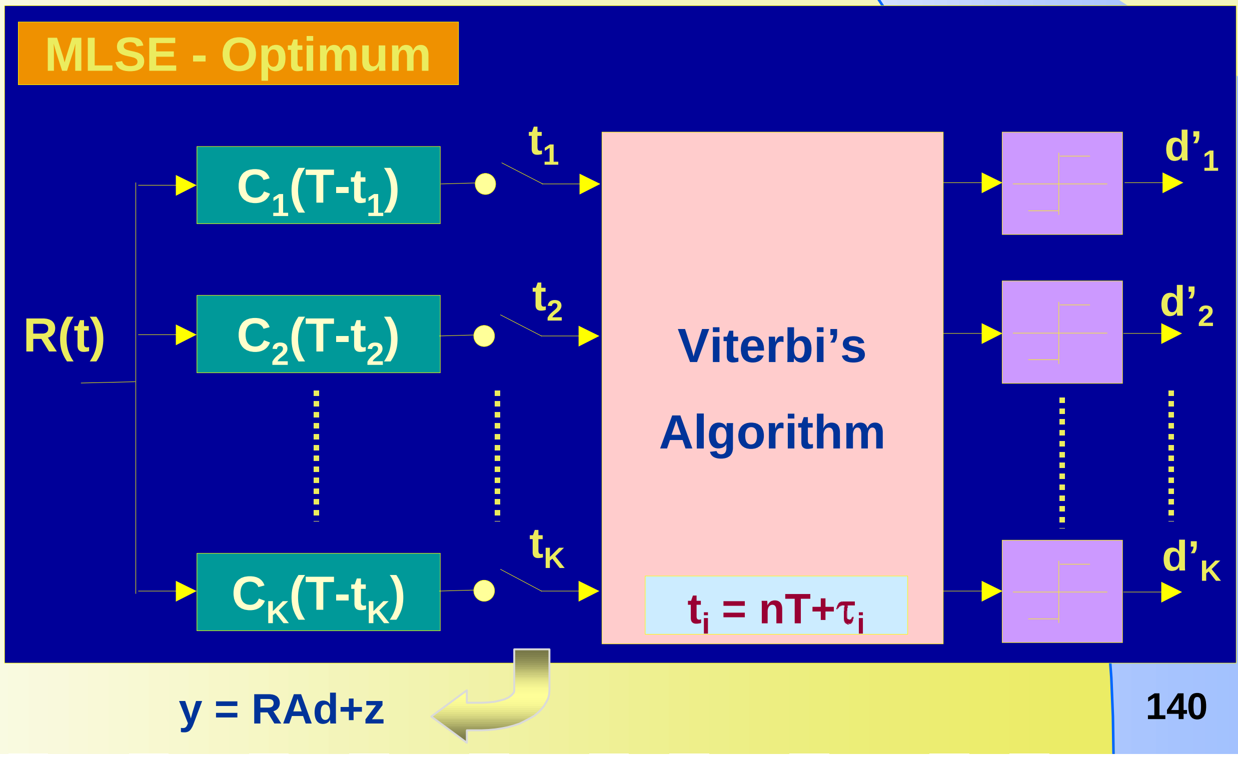
Reverse: 4 Fingers (BS)

**EGC**: Equal Gain Combining means that all coefficients are equal $\alpha_i$=1 for the final combination (same weight for all considered multipaths); **MRC**: Maximal Rate Combining, which is in fact optimum for AWGN type interference, means that coefficients $\alpha_i$ are selected according to their channel attenuation factors (multipaths are enhanced by its relative power).

# Multi-user Detection - Introduction and Linear Approach

Conventional Detectors (like Matched Filter or Rake Receiver) are inefficient because they treat the MAI as AWGN and additionally are sensitive to near far effect, which imposes a strong power control necessity (compare). The next natural step is to consider some type of detector that uses all users' information for the desirable signal detection. S. Verdú (MLSE - Maximum Likelihood Sequence Estimation-reference [23] and [24] for additional details) first introduced this type of detector. General idea: Chooses d which maximises the probability that d was transmitted given that r(t) was received $\Rightarrow$ max P[d|r(t)] for all t.

This type of detector requires an exhaustive search procedure and so is not very practical. So some sub-optimum variations will be explored in next slides.



Multi-user Detectors (MuD) can be classified as linear and non linear.The first type includes Decorrelator and MMSE (Minimum Mean Square Error). The second type includes SIC (Subtractive Interference Cancellation), PIC (Parallel Interference Cancellation) and ZF-DF (Zero Forcing Decision Feedback).

We begin this description with an introduction to matrix-vector notation for DS/CDMA signals. Before we generalise, we will consider a baseband CDMA system with three synchronous users. We can write the conventional detector's outputs as

$$y_1 = A_1 d_1 + \rho_{1,2} A_2 d_2 + \rho_{1,3} A_3 d_3 + z_1$$
$$y_2 = \rho_{2,1} A_1 d_1 + A_2 d_2 + \rho_{2,3} A_3 d_3 + z_2$$
$$y_3 = \rho_{3,1} A_1 d_1 + \rho_{3,2} A_2 d_2 + A_3 d_3 + z_3$$

Where $y_i$, $A_i$, $d_i$, $\rho_{i,j}$ and $z_i$ represent the $i^{th}$ user's detector output, the $i^{th}$ user's amplitude, the $i^{th}$ user's data, the cross-correlation between the sequences used by $i^{th}$ and $j^{th}$ users and the $i^{th}$ user noise component, respectively.

or in a matrix-vector representation

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & \rho_{1,2} & \rho_{1,3} \\ \rho_{2,1} & 1 & \rho_{2,3} \\ \rho_{3,1} & \rho_{3,2} & 1 \end{bmatrix} \begin{bmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

(return)

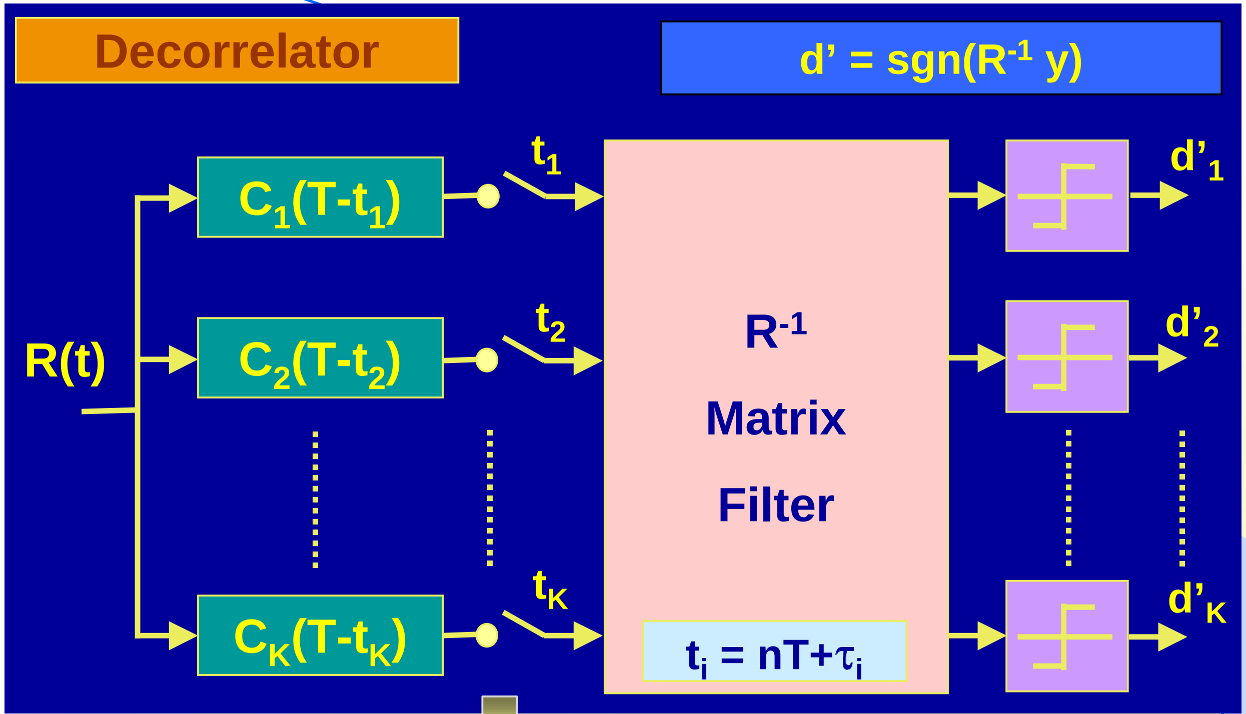
$$y = RAd + z = [I + Q]Ad + z = Ad + QAd + z$$

where R is known as correlation matrix. The first natural approach is to calculate $R^{-1}$ and perform the operation

$$R^{-1}y = R^{-1}RAd + R^{-1}z = Ad + R^{-1}z$$

Which is just the decoupled data plus a noise term (note that the operation is near far resistant).

This type of detector is known as decorrelator and the next figure show us its operation.




y = RAd+z

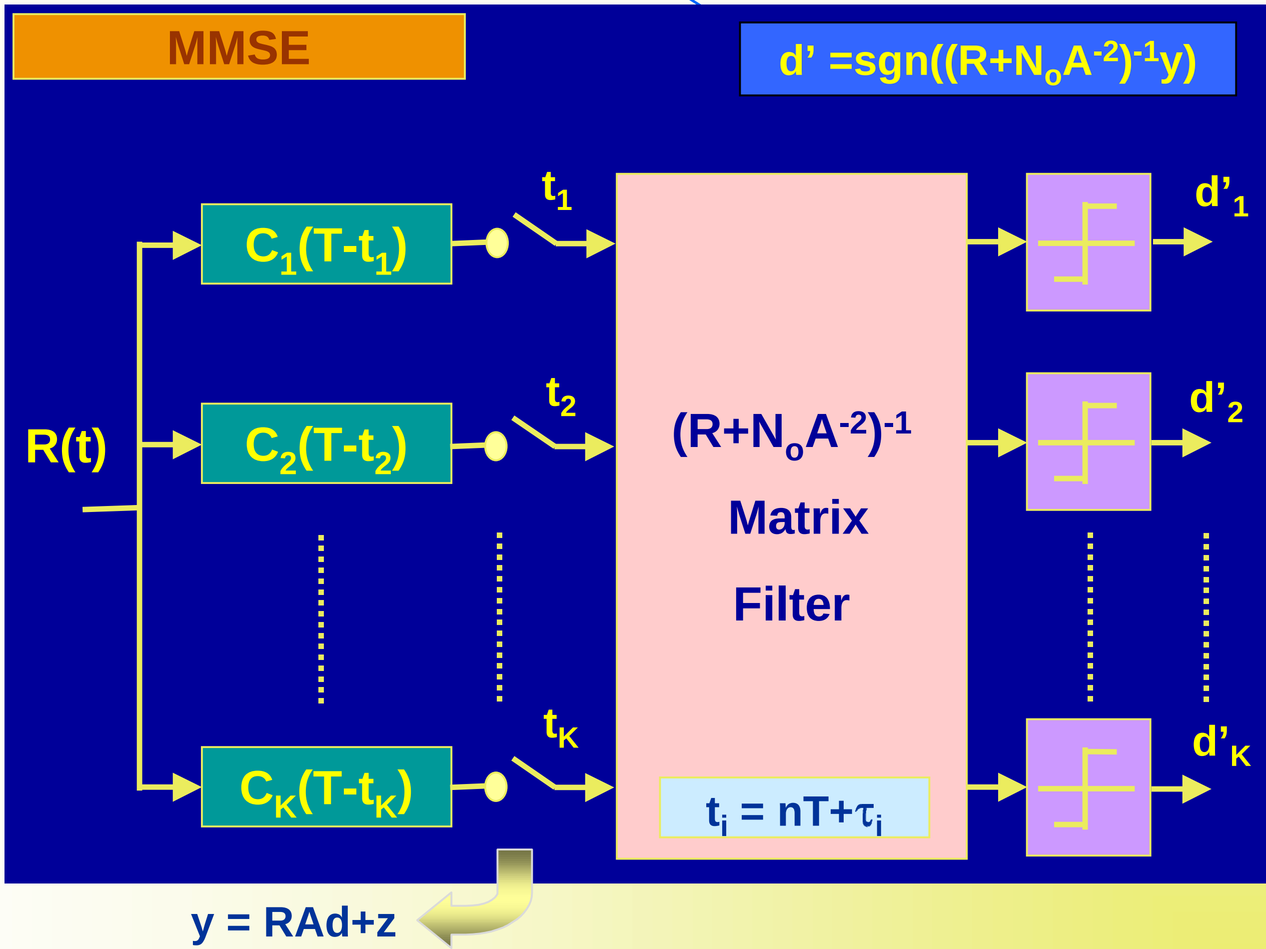
For correct decorrelator operation we need to know the cross-correlation between all sequences and invert the correspondent matrix (in real time!). The final result shows additionally that the noise is enhanced by the matrix operation ($R^{-1}z$). Considering this noise enhancement the next natural choice is to try to find a matrix F that minimises

$$E(\|Ad - Fy\|^2)$$

The solution for this minimisation problem, reference [25], can be expressed as

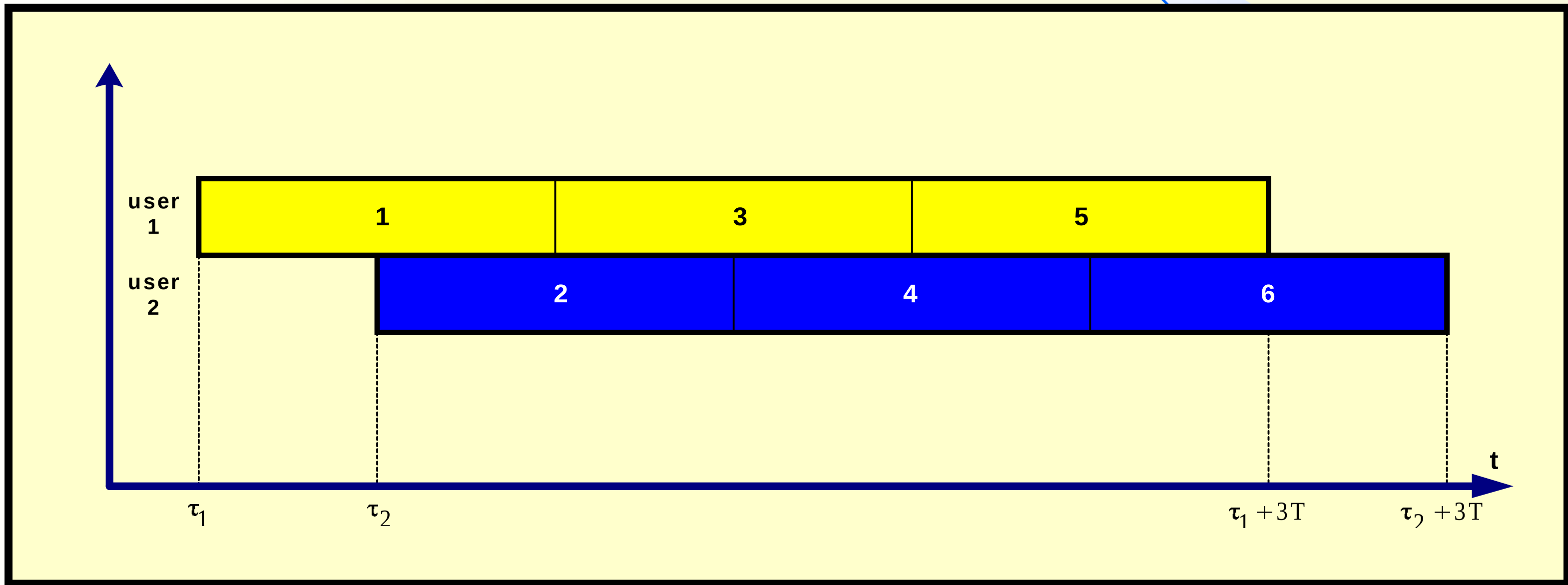
$$F = (R + N_0 A^{-2})^{-1}$$

This solution is known as MMSE (Minimum Mean Square Error) and the correspondent implementation is represented below




For a synchronous system the detection focus is a bit interval, independent of others. For an asynchronous system we have an overlap between bits of different intervals.

So any decision made on a particular bit ideally needs to take into account the decision on two overlapping bits of each user. Assuming a message with N bits and K users in the system the order of matrices is NK. The matrix R now contains the partial correlation that exists between every pair of the NK code words and is of size NK×NK. (return)




In the figure an example is presented with N=3, K=2. The conventional detector output can be described using the same previous equations where we treat the problem as if there were 6 users (each transmitting one bit over the interval $3T+\tau_2-\tau_1$)

Where now R can be written as

$$R = \begin{bmatrix} 1 & \rho_{2,1} & 0 & 0 & 0 & 0 \\ \rho_{1,2} & 1 & \rho_{3,2} & 0 & 0 & 0 \\ 0 & \rho_{2,3} & 1 & \rho_{4,3} & 0 & 0 \\ 0 & 0 & \rho_{3,4} & 1 & \rho_{5,4} & 0 \\ 0 & 0 & 0 & \rho_{4,5} & 1 & \rho_{6,5} \\ 0 & 0 & 0 & 0 & \rho_{5,6} & 1 \end{bmatrix}$$

and here the zero entries mean that the correspondent bits do not overlap. With this new representation for matrix R the general equation

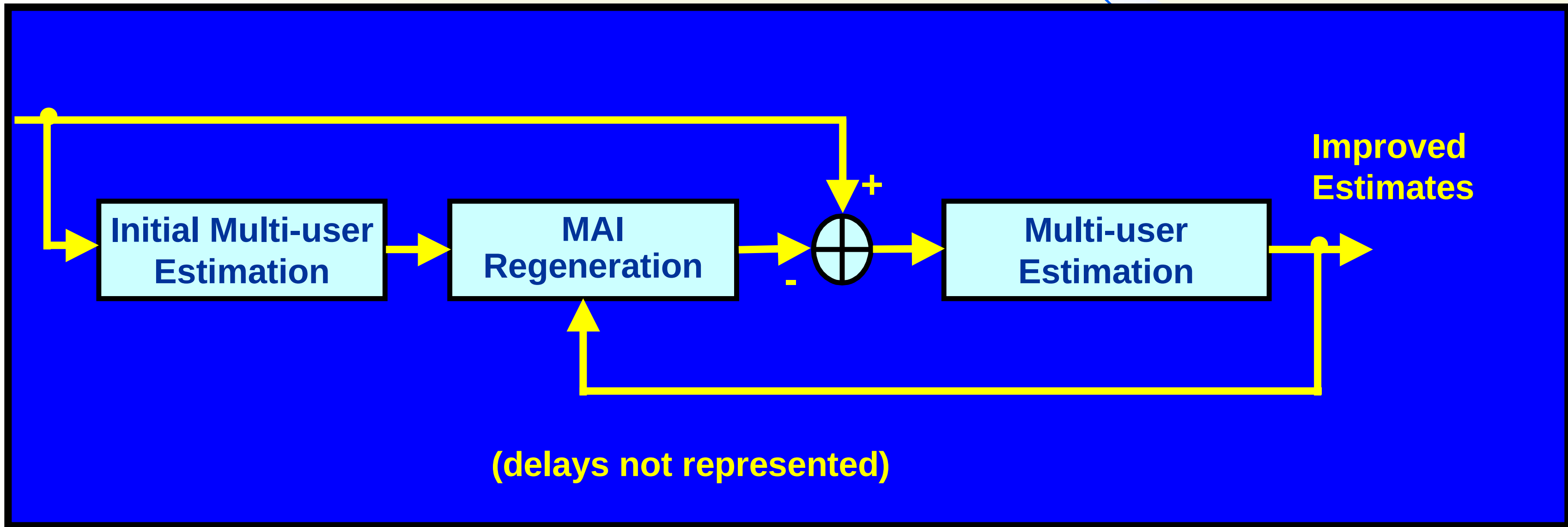
$$y = RAd + z$$

remains valid with new dimensions for matrices y, A, d and z, but with the same interpretation as for a synchronous case.

Additional details in reference [35], in Portuguese.

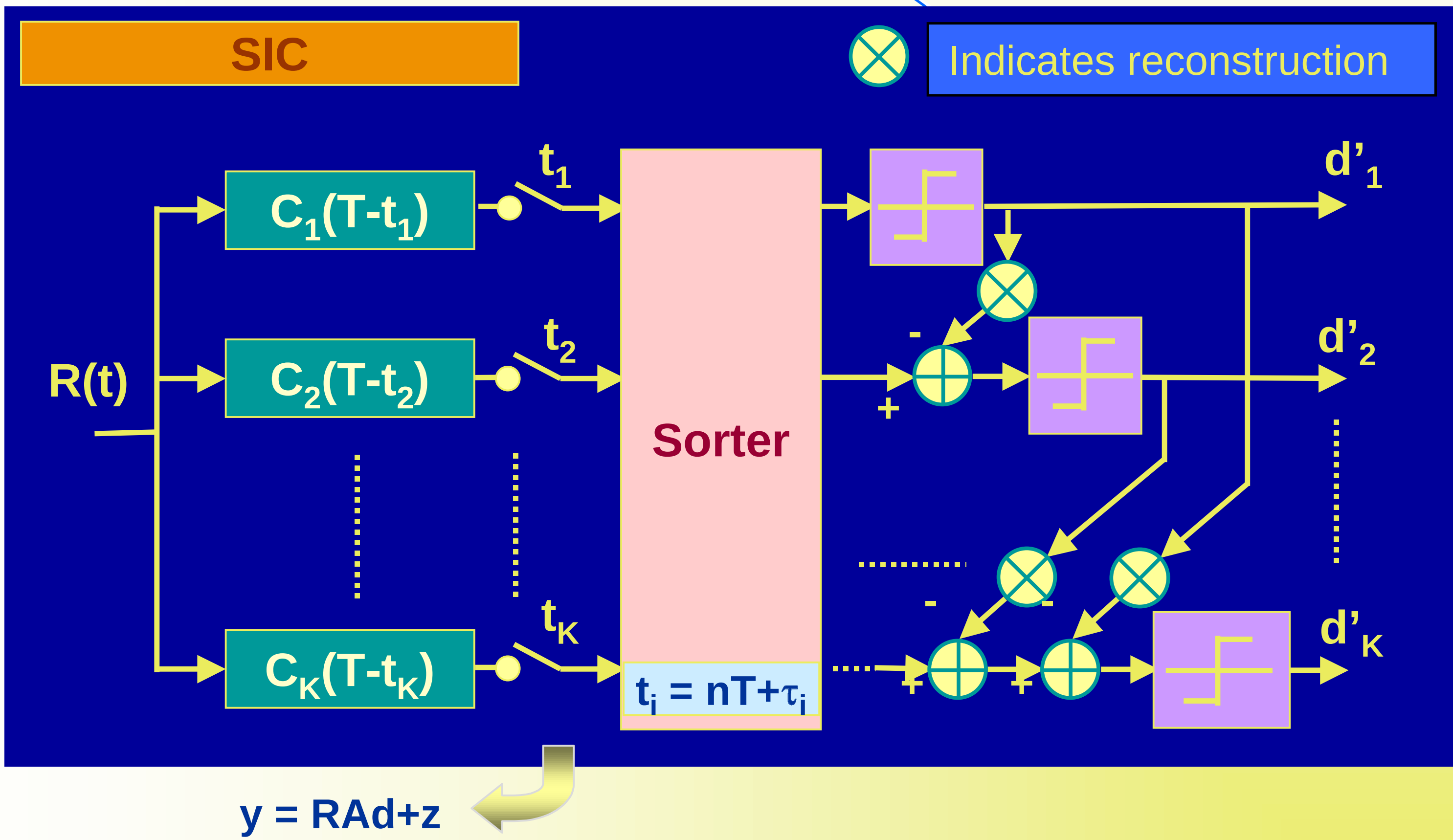
# Multi-user Detection - Non Linear Approach

The non linear approach or IC (Interference Cancellation) type MUD try to subtract the influence of others users (or most of them) from the desired user and after this operation we ideally have a “cleaned” version of the input signal for subsequent detection.




The first scheme considered from this class is known as SIC (Successive Interference Cancellation). In the SIC scheme initially we should order the signals in concordance of their power. The first signal (the strongest) is demodulated and reconstructed. From the delayed input version we subtract this reconstructed signal and we are ready to demodulate the second signal, and so on.

This type of detector eliminates all strongest MAI for each user. In the last step, for the last user, we have only AWGN type noise without any MAI. The next figure illustrates the process.




This type of detector has an inferior performance when compared with linear type multi-user detectors and also has a delay proportional to number of active users. Its advantage is the relative simplicity (avoids matrix inversion).

The SIC scheme is well adapted for users with some power disparities. Now, we can ask if there exists a power disparity distribution for system optimisation? The answer is yes and can be exploited with the argument that we want a uniform performance for all users. With this performance objective in mind we can impose SNR=constant for all K users. From the ordering and with the $k^{th}$ user's power nominated by $P_k$ we can write

$$P_k / (P_{k+1} + P_{k+2} + \ldots + P_K + I_0) = \gamma$$

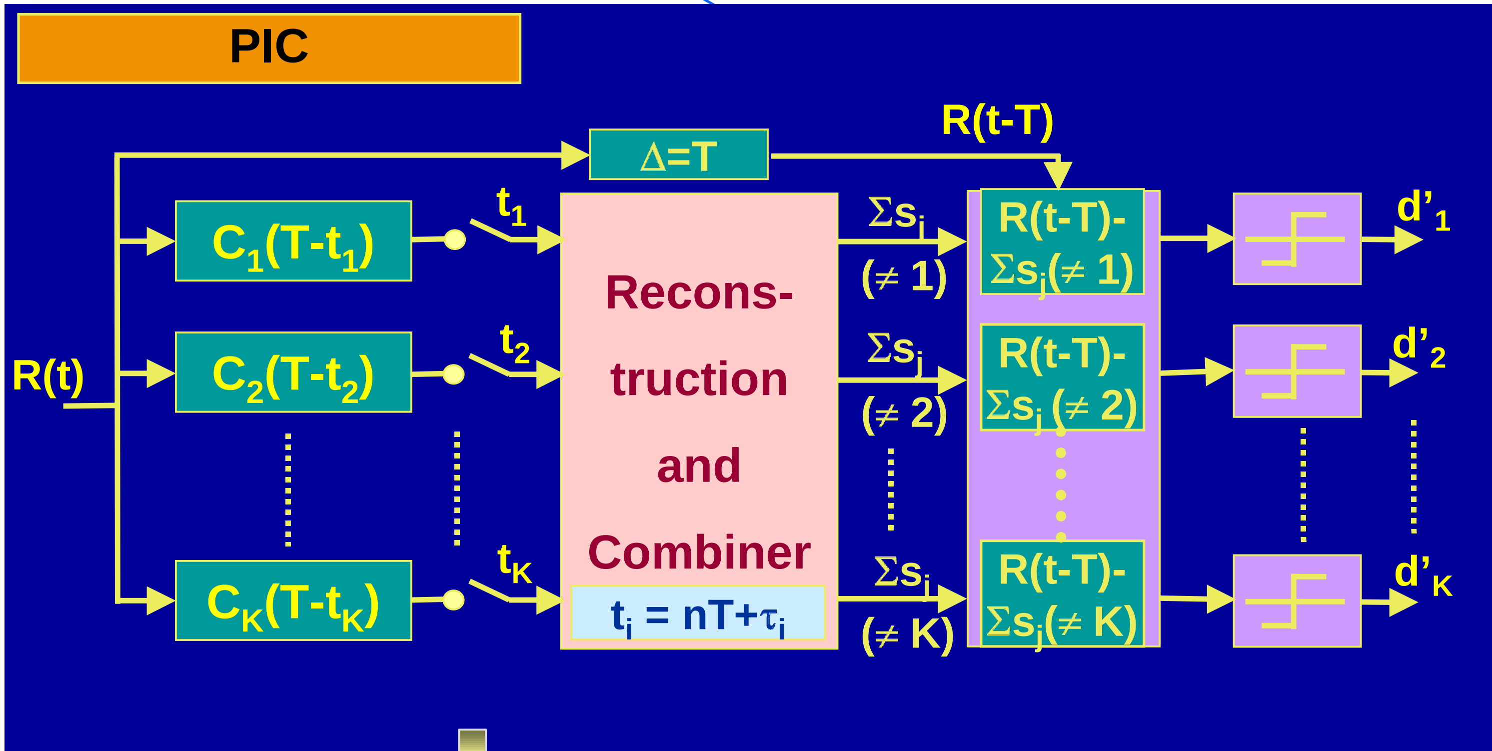
With k=1,..,K and where $I_0$ denotes the AWGN power.

Solving this we have $P_i = \gamma(\gamma+1)^{K-i} I_0$ or $P_i / P_{i+1} = \gamma + 1$

Which indicates that an exponential power distribution for the users' power is the ideal for the SIC scheme.

The second type of IC is known as PIC (Parallel Interference Cancellation).

Here after all signal reconstruction we subtract from the delayed input, R(t-T), all reconstructed signals, $\Sigma s_j$, except the desired, for each branch.




y = RAd+z

The PIC scheme has better performance than SIC scheme. In an ideal scenario the final signal for demodulation on each branch have no MAI from other users and, additionally, the total delay is only one bit interval. As we can see in the structure and based on our former conclusions about power disparities, we can infer that the perfect power control scheme is the best suited power control algorithm for PIC detectors. Despite this fact the scheme is near far resistant.

The ZF-DF (Zero Forcing - Decision Feedback) is another IC approach and is based on Cholesky decomposition, reference [25], for matrix R

$$R = F^T F$$

where F is a lower triangular matrix.

Applying $(F^T)^{-1}$ to the matched filter outputs we obtain

$$F^{-T} y = FAd + F^{-T} z$$

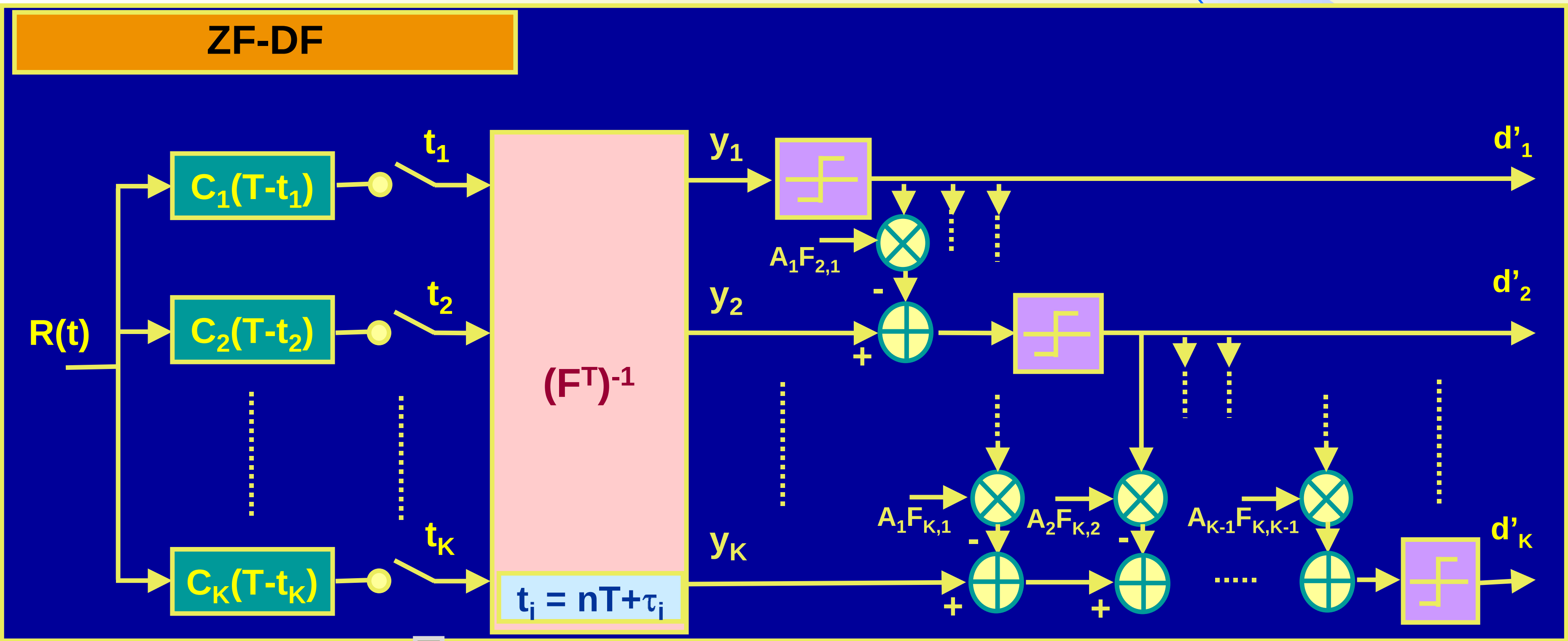
where we have used $F^{-T} = (F^T)^{-1}$ to simplify notation.

With this decomposition we can see that the signal for the first user is MAI free; the second user contains MAI only from the first; the third user contains MAI only from the first and second and so on.

Generically the $i^{\underline{th}}$ user contains MAI only from the previous users which was detected and can be subtracted.

The approach is analogous to the ZF-DF equaliser used to combat ISI, reference [25]. The next figures show us the FAd matrix and the ZF-DF implementation.

$$\mathbf{FAd}=\begin{bmatrix} F_{1,1} & 0 & 0 & 0 & 0 \\ F_{2,1} & F_{2,2} & 0 & 0 & 0 \\ F_{3,1} & F_{3,2} & F_{3,3} & 0 & 0 \\ \dots & \dots & \dots & \dots & 0 \\ F_{K,1} & F_{K,2} & F_{K,3} & \dots & F_{K,K} \end{bmatrix}\begin{bmatrix} A_1 & 0 & 0 & 0 & 0 \\ 0 & A_2 & 0 & 0 & 0 \\ 0 & 0 & A_3 & 0 & 0 \\ \dots & \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & 0 & A_K \end{bmatrix}\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \dots \\ d_K \end{bmatrix}$$




y = RAd+z

Additional details in reference [36], in Portuguese. A final remark: As we can see the basic problem in linear multi-user detection is to solve a linear equation

$$y = RAd + z$$

or in other form **y=Qu** (for decorrelating detector **Q=R** and for MMSE detector **Q=R+** $N_o$**A**$^{-2}$ where the two matrices are symmetric positive definite and block tridiagonal).

Instead of inverting the matrix **Q** using a direct method we can solve the linear equation using iterative methods. All iterative methods depend on a splitting of **Q**. Let **Q=D+L+U** where **D** is a diagonal matrix, **L** is strictly lower triangular and **U** is strictly upper triangular. Some of possible iterations are

Jacobi iteration $$u_{m+1} = D^{-1}[y - (L + U)u_m]$$ (leads to PIC)

Now the inversion is simple task because **D** is a diagonal matrix. It can be shown that Jacobi iteration converges iff the eigenvalues of **-D**$^{-1}$**(L+U)** all have magnitudes smaller than one and this fact allows us to explain why standard PIC does not converge.

Gauss-Seidel iteration $$u_{m+1} = D^{-1}[y - Lu_{m+1} - Uu_m]$$ (leads to SIC)

Other possibilities: Jacobi Over-Relaxation iteration, Successive Over-Relaxation iteration, Conjugate Gradient iteration and so on. See reference [30] for additional details.

The main problem in applying the above iterations for detection is the size of Q which is $N \times K$ (slide 130) where $K \to \infty$ for continuous transmission. The inherent detection delay is at least K which is unacceptable even for $K < \infty$.

In fact this challenging area is open for new contributions.

Finally, concluding this multi-user section, we highlight some new possible approaches for IC type detectors, reference [13]:

1- Using the decorrelating detector as the first stage;

2- Using the already detected bits at the output of the current stage to improve detection of the remaining bits in the same stage (multi-stage principles will be explored in the next section);

3- Doing a partial MAI cancellation at each stage, with the amount of cancellation increasing for each successive stage, reference [29].

We will return to this subject in the Multi-rate CDMA Systems section where the partial MAI cancellation will be a very usual approach for multi-stage multi-rate multi-user PIC type detectors.

# MPIC - Multi-stage Parallel Interference Cancellation

For this section we will assume a symbol synchronous system so that we need only consider one symbol interval i. The (chip rate) received signal for the $i^{th}$ symbol interval is

$$r(i) = S(i)\, d(i) + n(i)$$

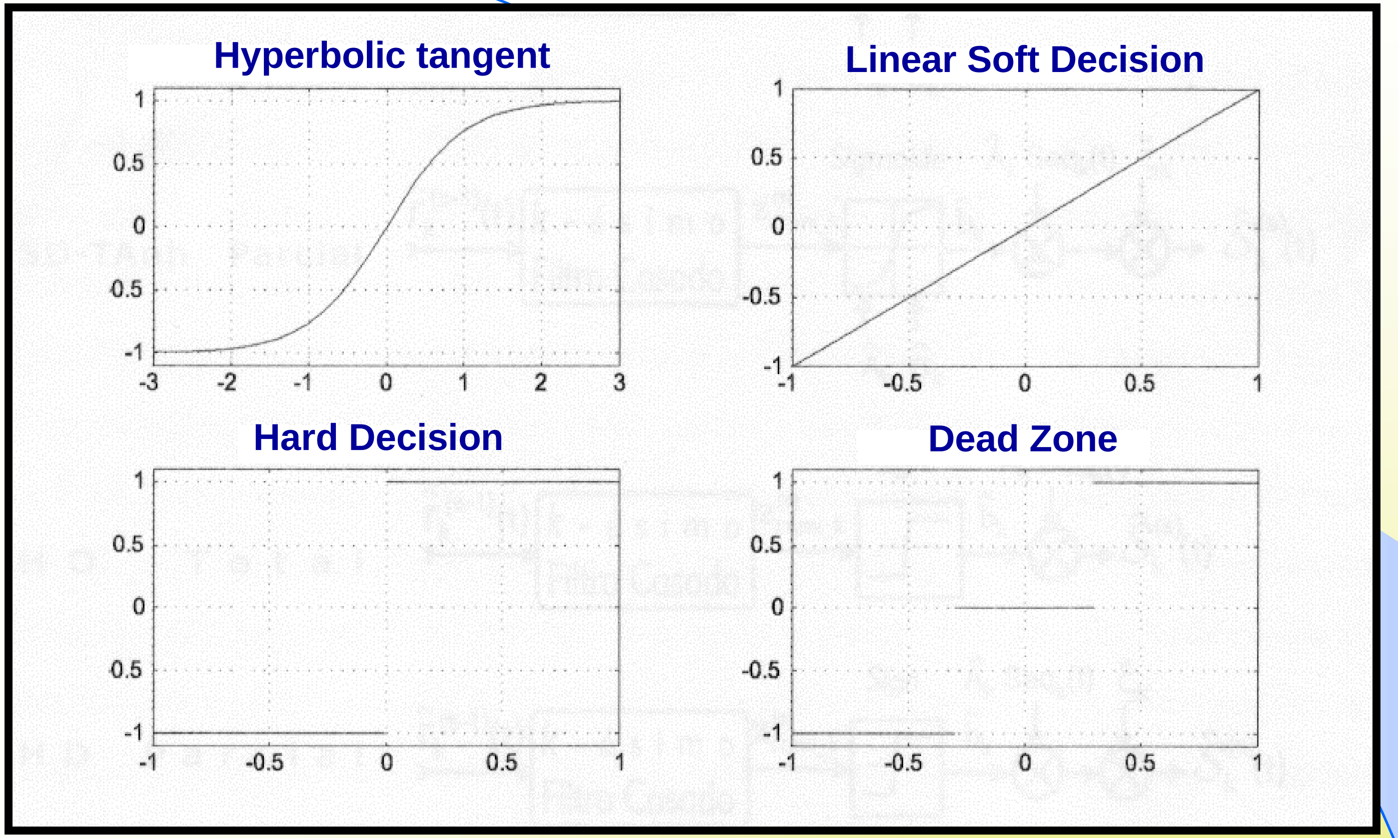
**S(i)** is the spreading code matrix where its $k^{th}$ column corresponds to the $k^{th}$ user; **d(i)** is the vector of symbols transmitted by all users and **n(i)** is an AWGN vector representing receiver's noise. To detect $d_1(i)$ (the first symbol in **d(i)**) in the first stage, we form the decision statistics

$$y_{1,1}(i) = s_1^H(i)\, r(i)$$

i. e., it is simply the conventional detector. **(compare)**

In our notation when there are two subscripts the first represents the stage index and the second the user index, i. e., $y_{m,k}(i)$ refers to the $k^{th}$ user in the $m^{th}$ stage.

Now $y_{m,k}(i)$, the decision signal, will be mapped with a function f(.) to obtain $z_{m,k}(i)$, the effective signal for cancellation. Some possible approach are presented in the next figure.




Using a Hyperbolic tangent decision circuit we are approaching the Hard decision circuit with strong signals and to the Linear Soft decision for weak signals (when we are not confident with the decision). On the other hand with Dead Zone type decision circuit we avoid making a decision when we are not sure (dead Zone's window can be optimised through BER minimisation, see reference [34] for details).

Now in the second stage, for the k$^{th}$ user, we want to cancel the detected interference for the K-1 users.

$$y_{2,k}(i) = s_k^H(i)[r(i) - \sum_{j=1, j\neq k}^{K} z_{1,j}(i) s_j(i)]$$

the signal in the brackets are used for the detection of k$^{th}$ user in the second stage.

$$= s_k^H(i)[r(i) - \sum_{j=1}^{K} z_{1,j}(i) s_j(i) + z_{1,k}(i) s_k(i)] = s_k^H(i) e_1(i) + z_{1,k}(i)$$

where

$$e_1(i) = r(i) - \sum_{k=1}^{K} z_{1,k}(i) s_k(i)]$$

denotes the residual signal with all estimated interference from the first stage cancelled.

The process can be easily generalised with the following expressions

$$y_{m,k}(i) = s_k^H(i) e_{m-1}(i) + z_{m-1,k}(i)$$

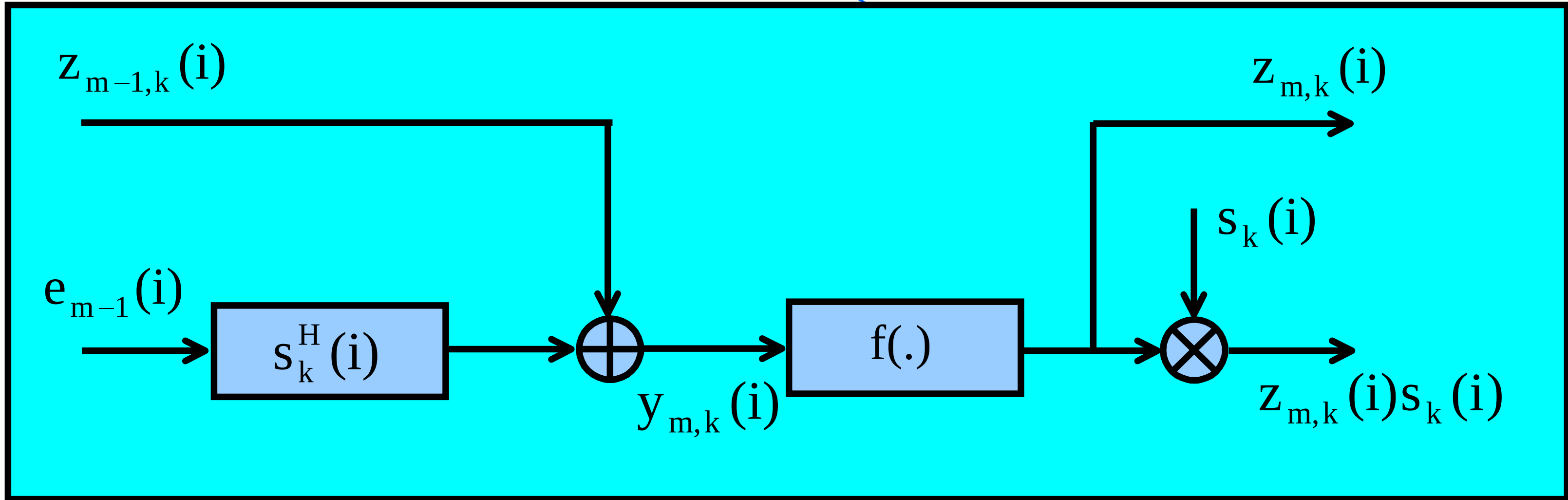
and

$$e_m(i) = r(i) - \sum_{k=1}^{K} z_{m,k}(i) s_k(i)$$

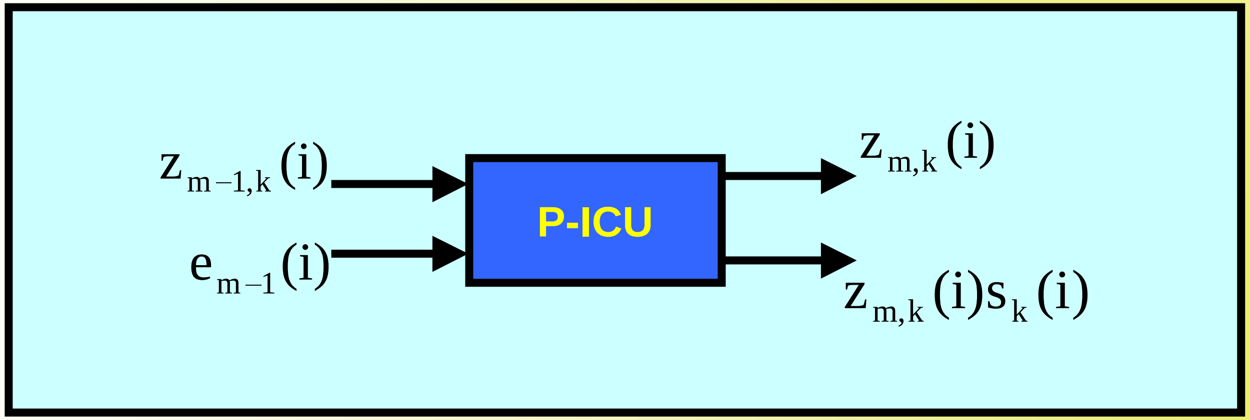
with

$$z_{m,k}(i) = f[y_{m,k}(i)]$$

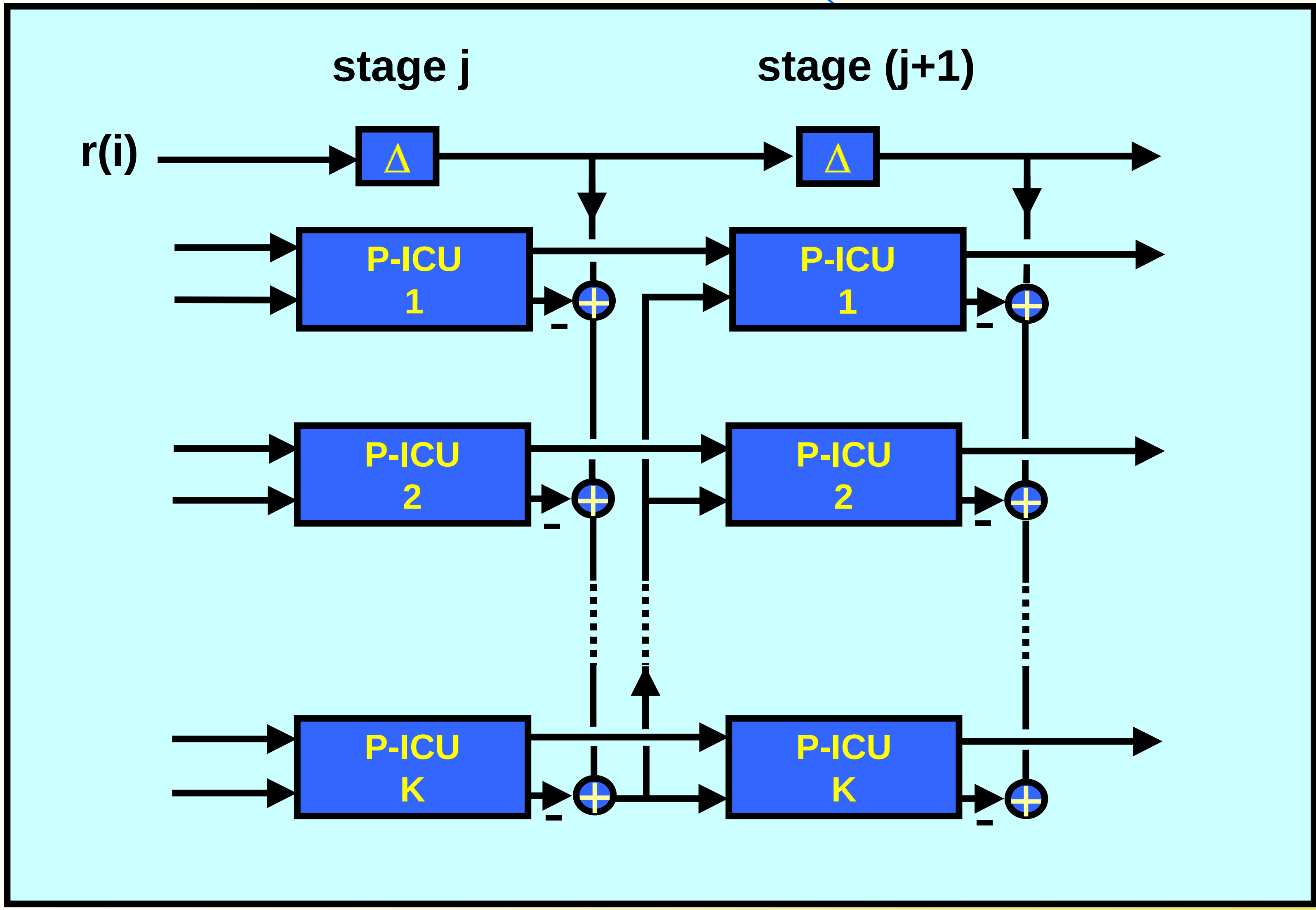
This representation allows us to implement the multi-stage PIC in a modular form with P-ICU (Parallel Interference Cancellation Unit)




For the first stage (m=1) we adopt the following initial values: $z_{m-1,k}(i)=0$ and $e_{m-1}(i)=r(i)$. This structure is represented in a modular form as



Finally with this representation we can implement a K user, multi-stage PIC detector as a set of interconnected P-ICUs (only two successive stages are represented below). Obviously, for the last stage we should use a hard decision circuit for final detection. See reference [33] for additional details.



# MSIC - Multi-stage Successive Interference Cancellation

We can repeat the previous procedure for multi-stage SIC scheme. For the detection signal of first user in the first stage we can write

$$y_{1,1}(i) = s_1^H(i)\, r(i)$$

For the second detection signal in the first stage we should subtract the first user estimate from the received signal

$$r_{1,2}(i) = r(i) - z_{1,1}(i)\, s_1(i)$$

where

$$z_{m,k}(i) = f[\, y_{m,k}(i)\,]$$

is the mapping function.

Now from $\mathbf{r}_{1,2}$ we can express $\mathbf{y}_{1,2}$ as

$$y_{12}(i) = s_2^H(i)\, r_{1,2}(i)$$

And we follow with this procedure until all K (first stage) symbols have been detected. For the first symbol in the second stage we use as input signal

$$r_{2,1}(i) = r(i) - \sum_{k=2}^{K} z_{1,k}(i)\, s_k(i)$$

Which is the input signal with all interference based on the estimate of first stage removed, except for the first user.

Of course, this signal should be “cleaner” than the signal in the first stage which did not benefit from cancelling interference from users 2 to K.

Therefore

$$y_{2,1}(i)=s_1^H(i)r_{2,1}(i)=s_1^H[r(i)-\sum_{k=1}^{K}z_{1,k}(i)s_k(i)+z_{1,1}(i)s_1(i)]=z_{1,1}(i)+s_1^H(i)r_{1,K+1}(i)$$

Where $\mathbf{r_{1,K+1}(i)}$ was defined as

$$r_{1,K+1}(i)=r(i)-\sum_{k=1}^{K}z_{1,k}(i)s_k(i)$$

and represents the “cleaned” input signal after the first stage, i. e., a signal with all estimated interference in the first stage removed. For the second symbol in the second stage we form

$$r_{2,2}(i)=r_{2,1}(i)-z_{2,1}(i)s_1(i)=r(i)-\sum_{k=2}^{K}z_{1,k}(i)s_k(i)-z_{2,1}(i)s_1(i)$$

Which is the input signal with all interference based on the estimate of the first stage removed, except for the first user which was removed with the second estimate. From this expression we can write the next decision signal as

$$y_{2,2}(i)=s_2^H(i)[r(i)-z_{2,1}(i)s_1(i)-\sum_{k=3}^{K}z_{1,k}(i)s_k(i)]=s_2^H(i)[r_{2,2}(i)+z_{1,2}(i)s_2(i)]$$

Therefore $$y_{2,2}(i)=s_2^H(i)r_{2,2}(i)+z_{1,2}(i)$$

Now we are able to generalise this procedure to other users in the second stage with the following expressions

$$y_{2,k}(i) = s_k^H(i)\, r_{2,k}(i) + z_{1,k}(i)$$

$$r_{2,k}(i) = r_{2,k-1}(i) - z_{2,k-1}(i)\, s_{k-1}(i)$$

With the definition

$$r_{2,1}(i) = r_{1,K+1}(i)$$

And for a generic user k in the $m^{\underline{th}}$ stage

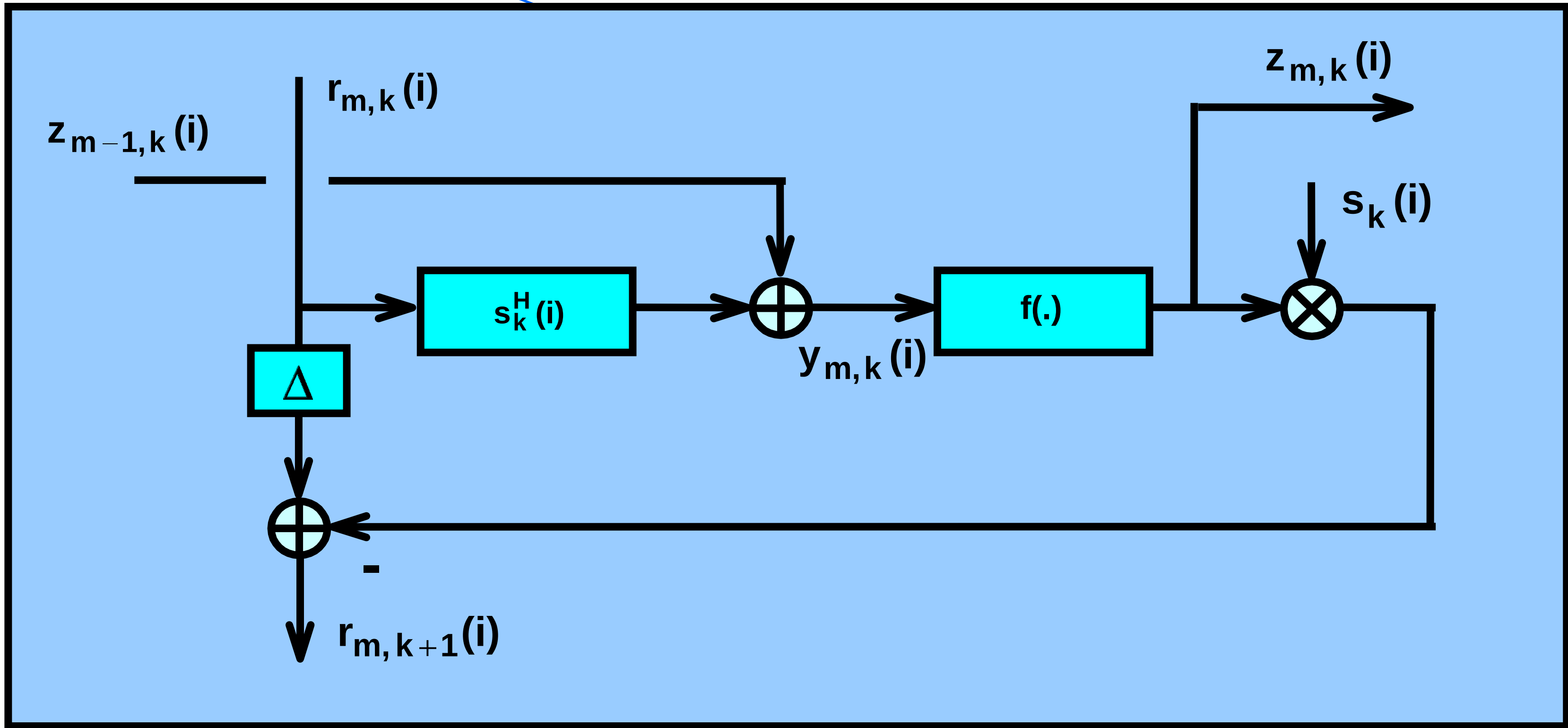
$$y_{m,k}(i) = s_k^H(i)\, r_{m,k}(i) + z_{m-1,k}(i)$$

$$r_{m,k+1}(i) = r_{m,k}(i) - z_{m,k}(i)\, s_k(i)$$

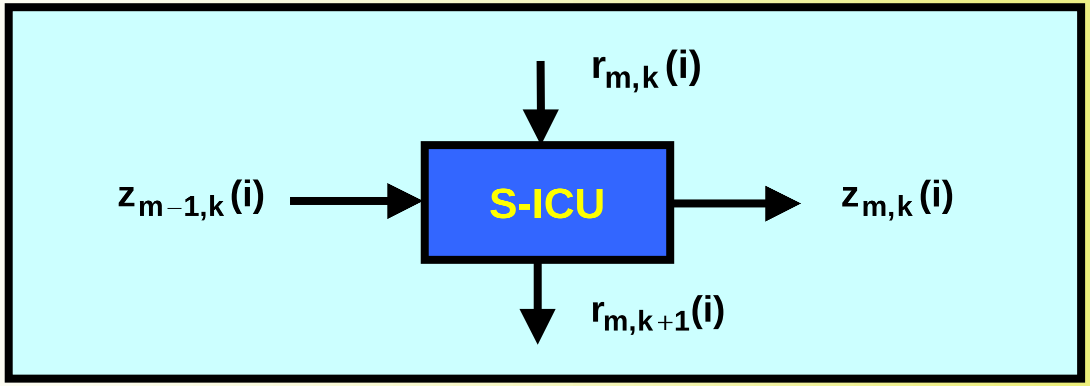
With the definition

$$r_{m,1}(i) = r_{m-1,K+1}(i)$$

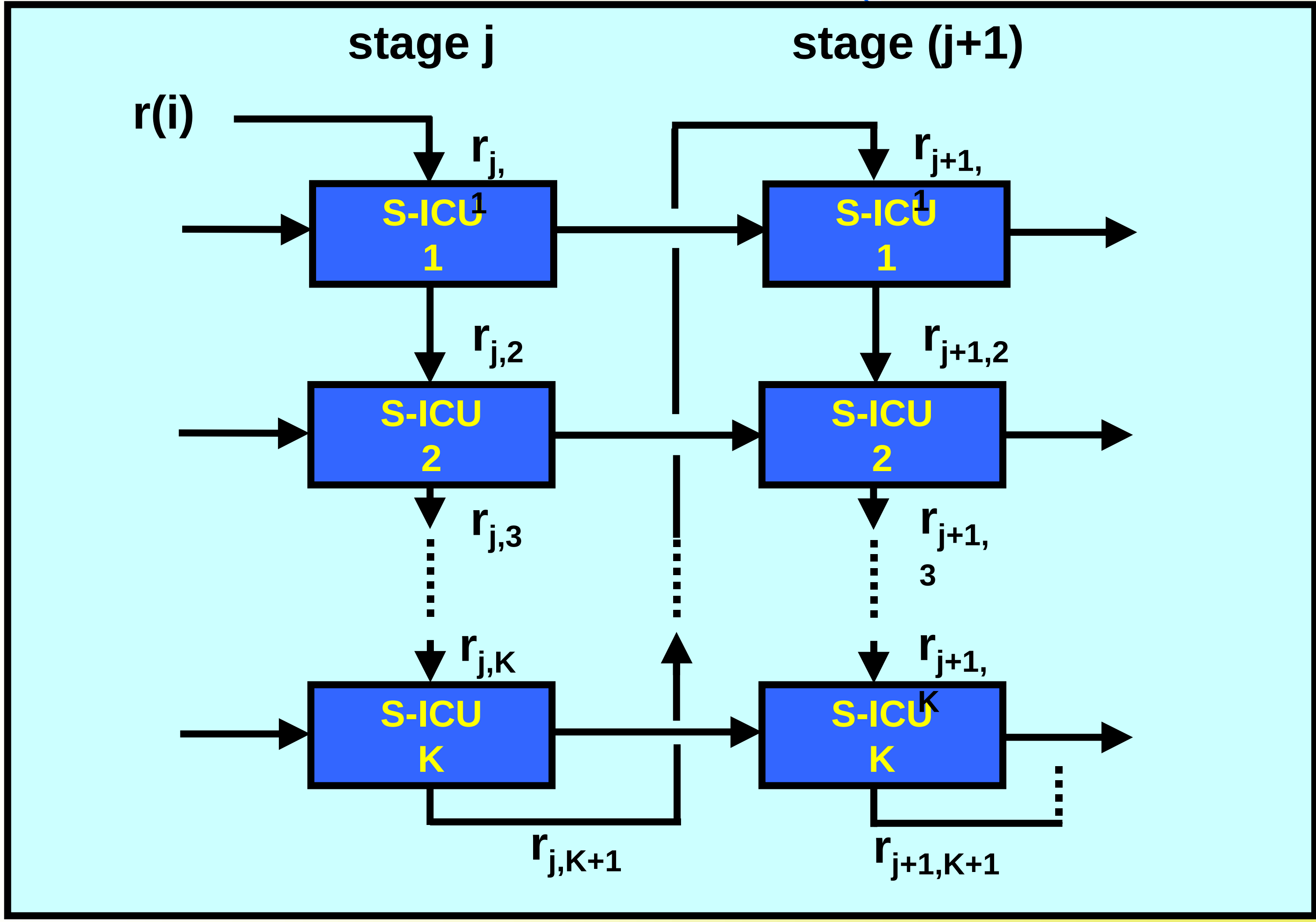
This representation allows us to implement the multi-stage SIC in a modular form with S-ICU (Serial Interference Cancellation Unit)




For the first stage (m=1) we adopt the following initial values: $z_{m-1,k}(i)=0$ and $r_{m,1}(i)=r(i)$. This structure is represented in a modular form as



Finally with this representation we can implement a K user, multi-stage SIC detector as a set of interconnected S-ICUs (only two successive stages are represented below). Obviously, for the last stage we should use a hard decision circuit for final detection. See reference [32] for additional details.



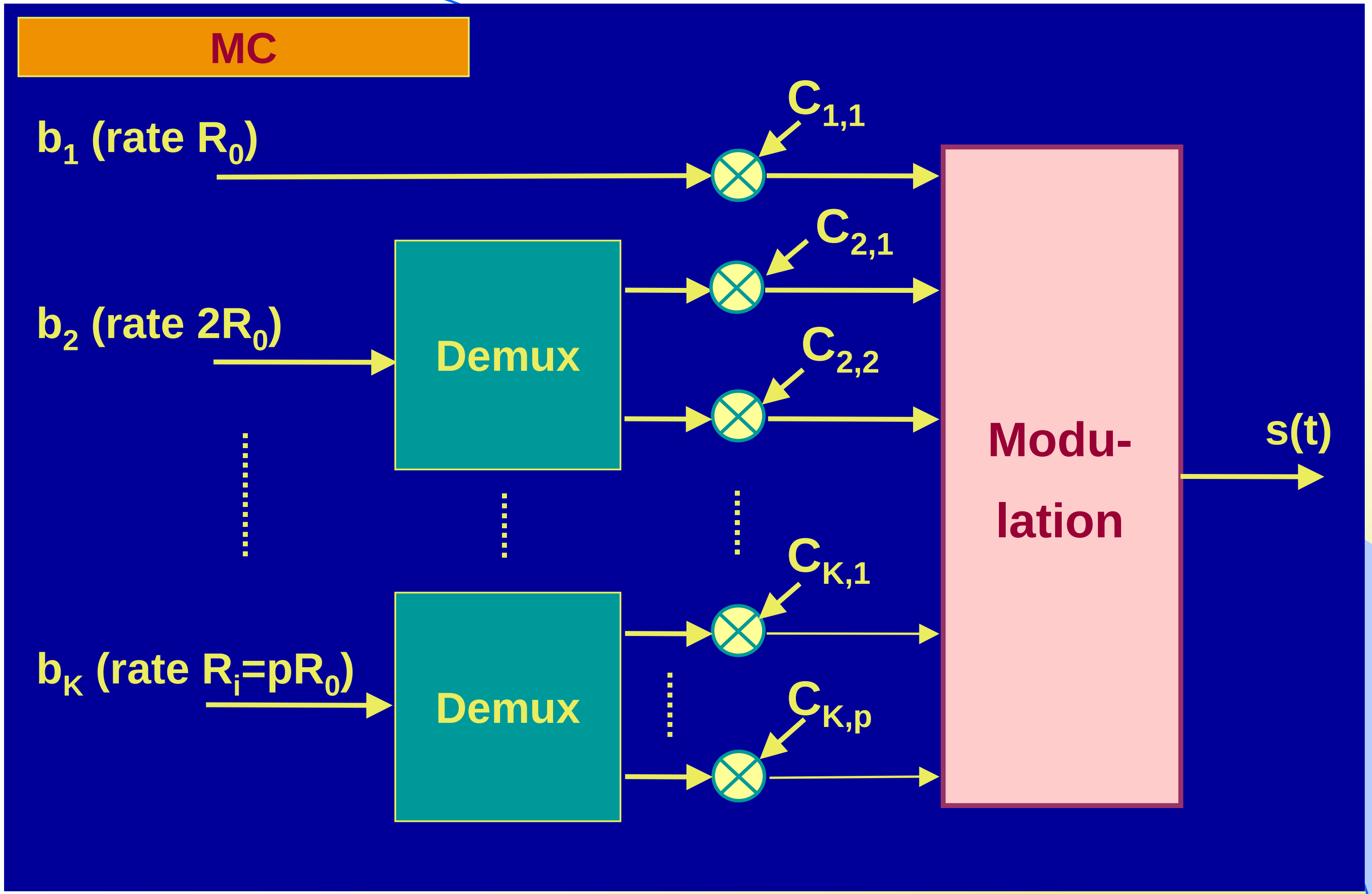
# Multi-rate CDMA Systems

We will explain in this section how to implement multi-rate CDMA systems through MC (multi-code) schemes, VPG (variable processing gain) schemes and MM (multi-modulation) schemes. Obviously hybrid schemes are also possible.

For the MC scheme $p=R_i/R_o$ parallel channels with orthogonal codes transmit the desired rate $R_i$. $R_o$ is the rate for each individual channel and the processing gain for each user, in each channel, is constant and given by $W/R_o$.

Using orthogonal codes we avoid self interference (interference caused by other channels of the same user). With this scheme each user with a rate $pR_o$ is equivalent to p users with the basic rate $R_o$ (equivalency here means system's resource use).

In the next figure we explain the basic MC scheme.




Another very usual scheme is the VPG also named VSG (variable spreading gain). Now all users use BPSK modulation (or QPSK) and the processing gains are given by

$$G_p = W/R_i$$

All users occupy the same bandwidth, use constant chip rate and a unique code ($C_i$) for each user. The processing gain varies with the data rate and users with high rate compensate its lower processing gain transmitting with higher power. The $i^{\underline{th}}$ user's signal can be written as

$$s_i(t) = \left\{ \sum_m \sqrt{2P_i}\, b_{i,m} \left[ \sum_{n=1}^{G_i} c_{i,m+n}\, h(t - mT_i - nT_c) \right] \right\} \cos(\omega_c t)$$

Where

$P_i$ : is the power of $i^{\underline{th}}$ user

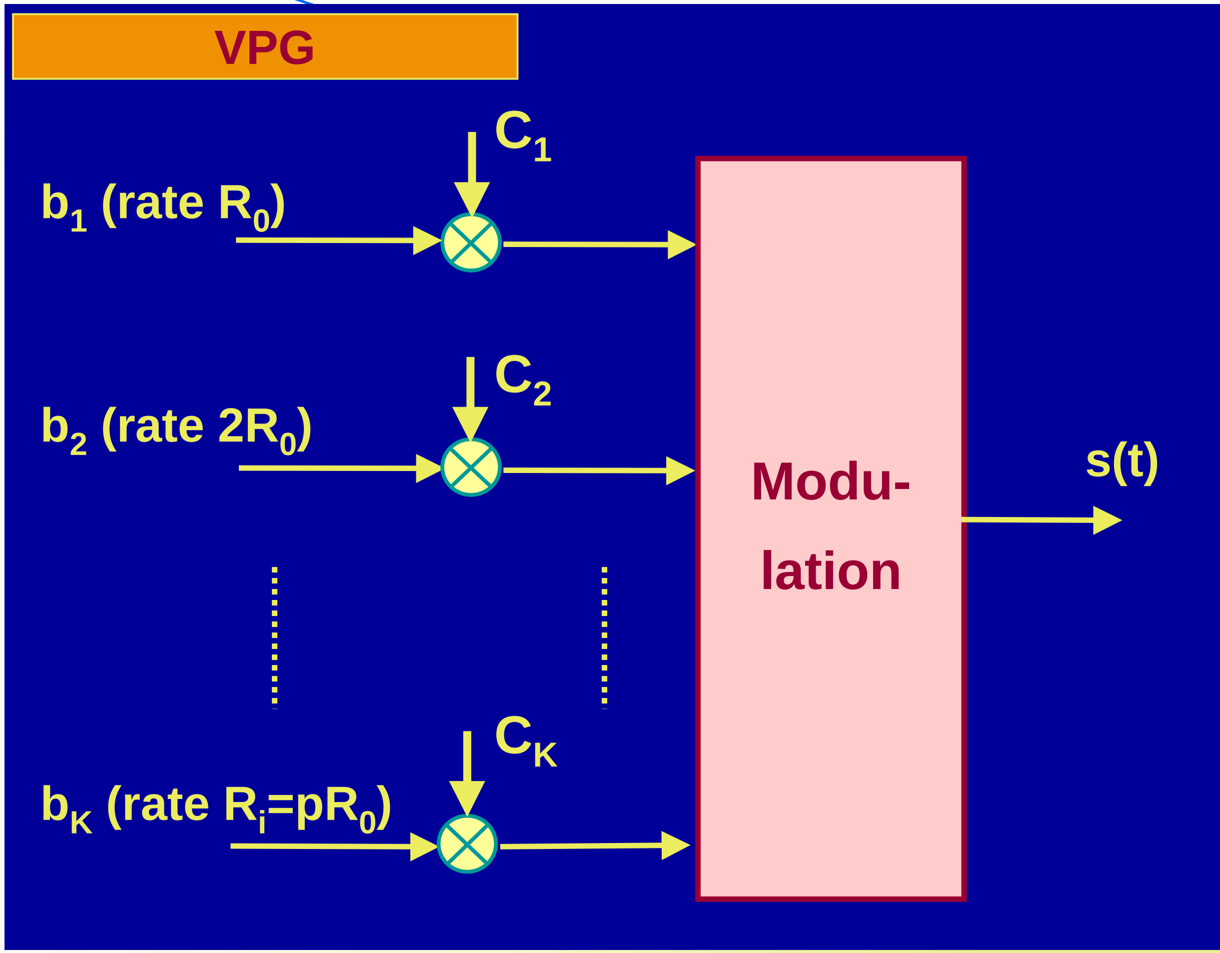
$\{b_i\}$ : is its data sequence, with period $T_i$ and such that $b_i \in \{-1,1\}$

$\{c_i\}$ : is the $i^{\underline{th}}$ spreading code, with period $T_c = 1/f_c$ and such that $c_i \in \{-1,1\}$

$h(t)$ : is a rectangular pulse with duration $T_c$ and normalised amplitude

$G_i = T_i/T_c$ : is the variable processing gain (or the number of chips per data bit)

In the next figure we explain the basic VPG scheme.



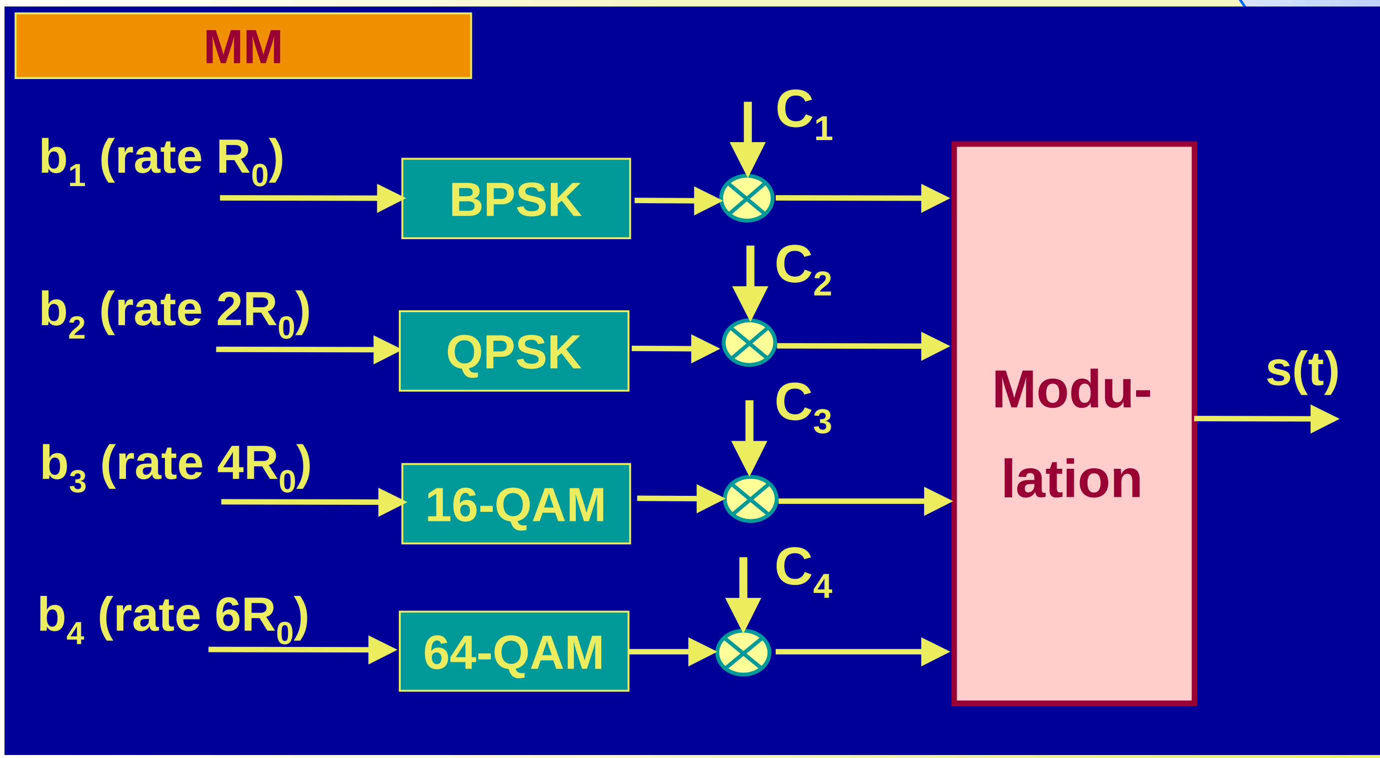

Finally we will consider the MM scheme. In this scheme each user's modulation format can be selected, for instance, from a BPSK (Binary Shift Keying) to 64-QAM (64-ary quadrature amplitude modulation) as a function of desired rate. Considering the minimum rate $R_o$ as a reference, the number of symbols for the i-th user using BPSK modulation and transmitting at rate $R_i$ can be written as

$$M = 2^{R_i/R_0} \qquad i = 1, 2, \ldots$$

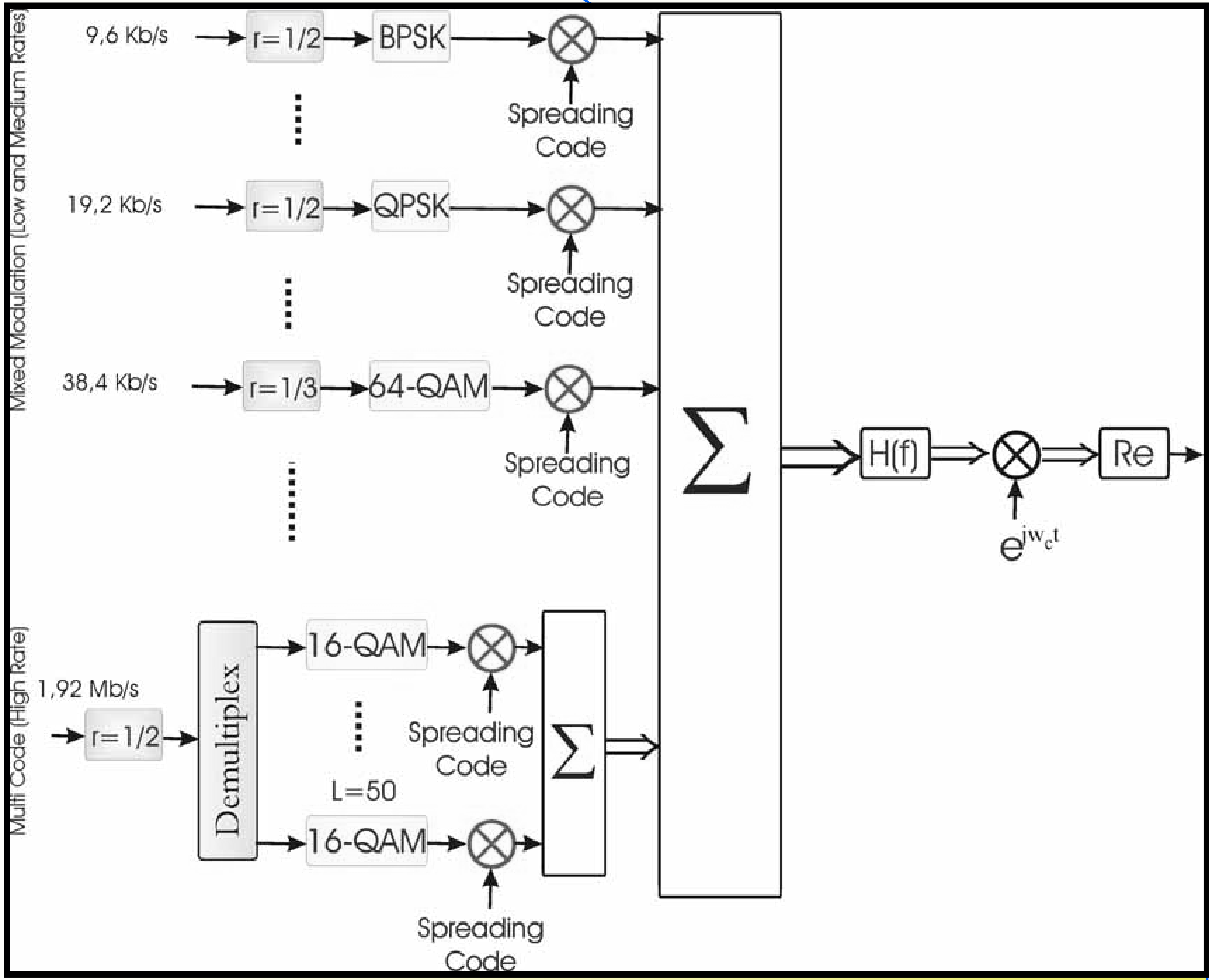
The processing gain expression is the same for all users and given by

$$GP = W/R_n$$

where W is the signals' bandwidth after multiplication by its spreading code (with the same chip rate for all users independently of its data rate) and $R_n$ the common symbol rate. With this type of modulation we can take advantage of different reception sensibilities for each type of modulation for, varying the QoS, to obtain the same BER. Below we have an example with data rates $R_0$, 2 $R_0$, 4 $R_0$ and 6 $R_0$ which can be easily generalised.



In this slide we present a possible multi-rate environment implemented with previous described strategies (all spreading codes with the same rate).



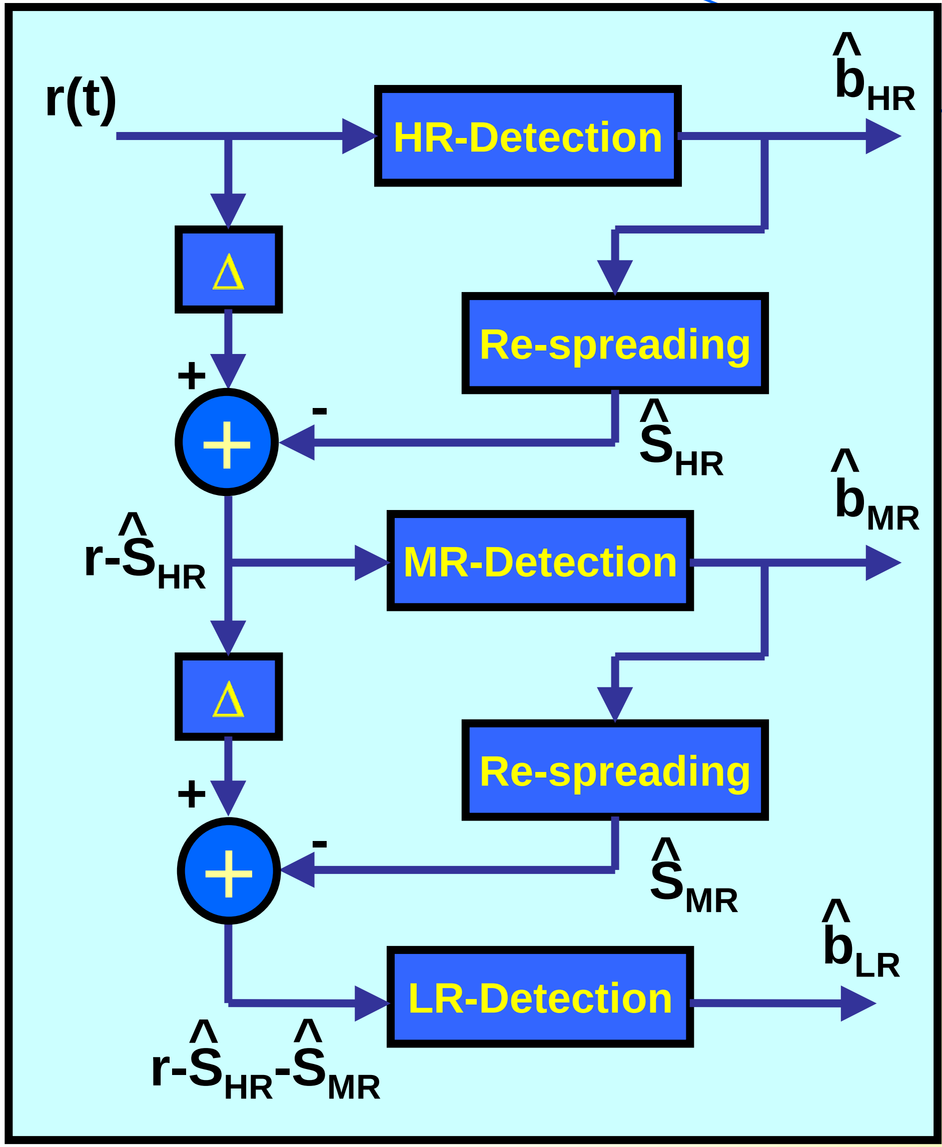
The next step is to develop receivers for these various multi-rate schemes. In general these receivers are of multi-stage type (with some kind of IC). This means that we have several successive stages for bit detection and each of them improve the current BER.

In the next figures we show some possible structures for MC and MPG type modulation.

First of all, this subsection could be another course ... so we will give some examples to illustrate this subject without any in depth pretension.

The two initial schemes represent a generic MPG-GSIC (Group Successive Interference Cancellation) solution.

All other figures show schemes that were exhaustively simulated in a MatLab environment. Interested readers can find these results (in Portuguese) and a huge list of additional references in [18].

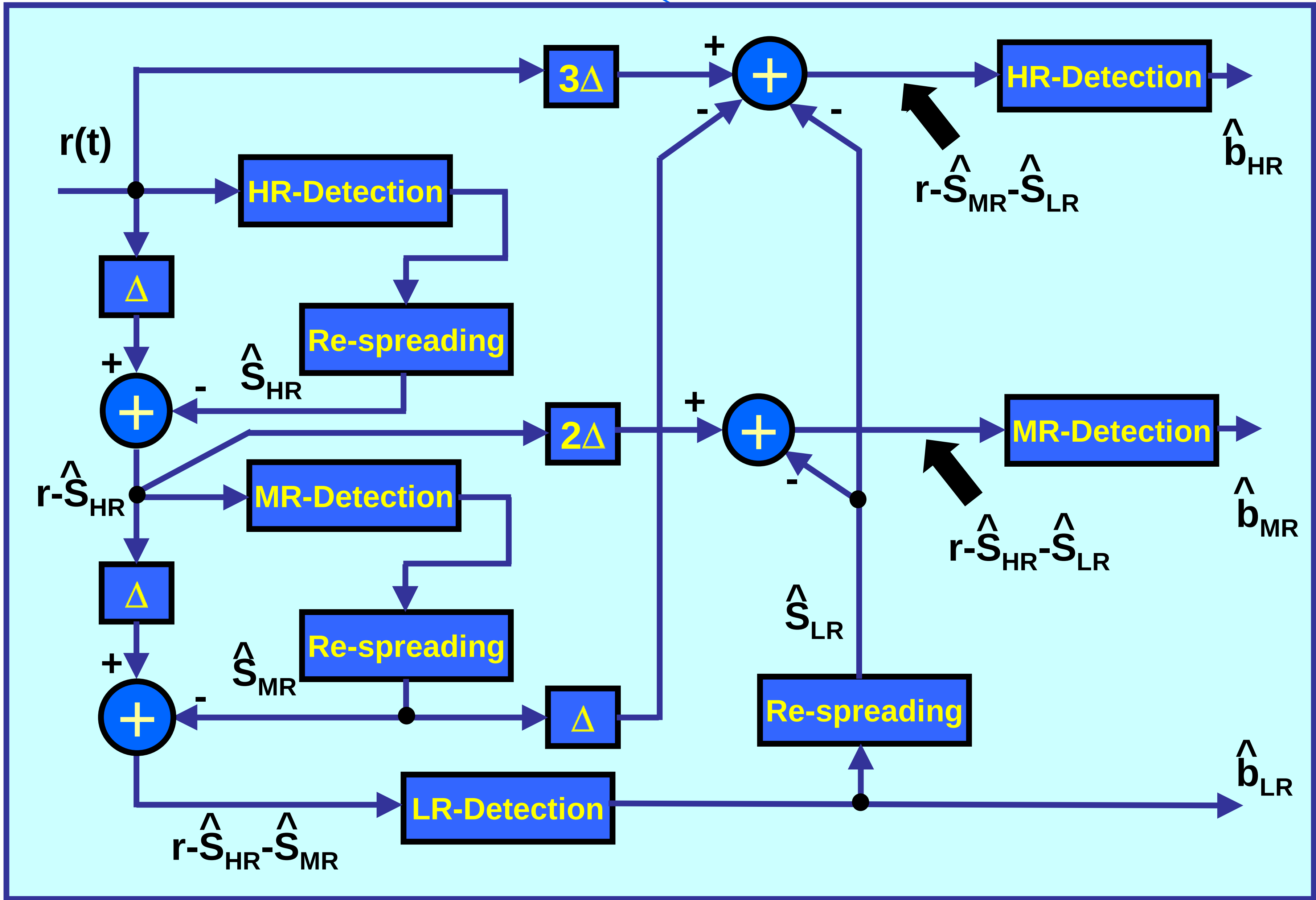



This figure presents a **MPG-GSIC** scheme. In this scheme we have a combination between serial and parallel IC, based on the natural amplitude's unbalance due the different rates (more details in references [26] and [27].

We have successive detection, reconstruction and cancelling for signals from the same group. Into the group the PIC detector was adopted because all signals in the group follow the same power and rate pattern.

Next, all detected signals from this group are reconstructed and subtracted from the delayed input for the next group detection.

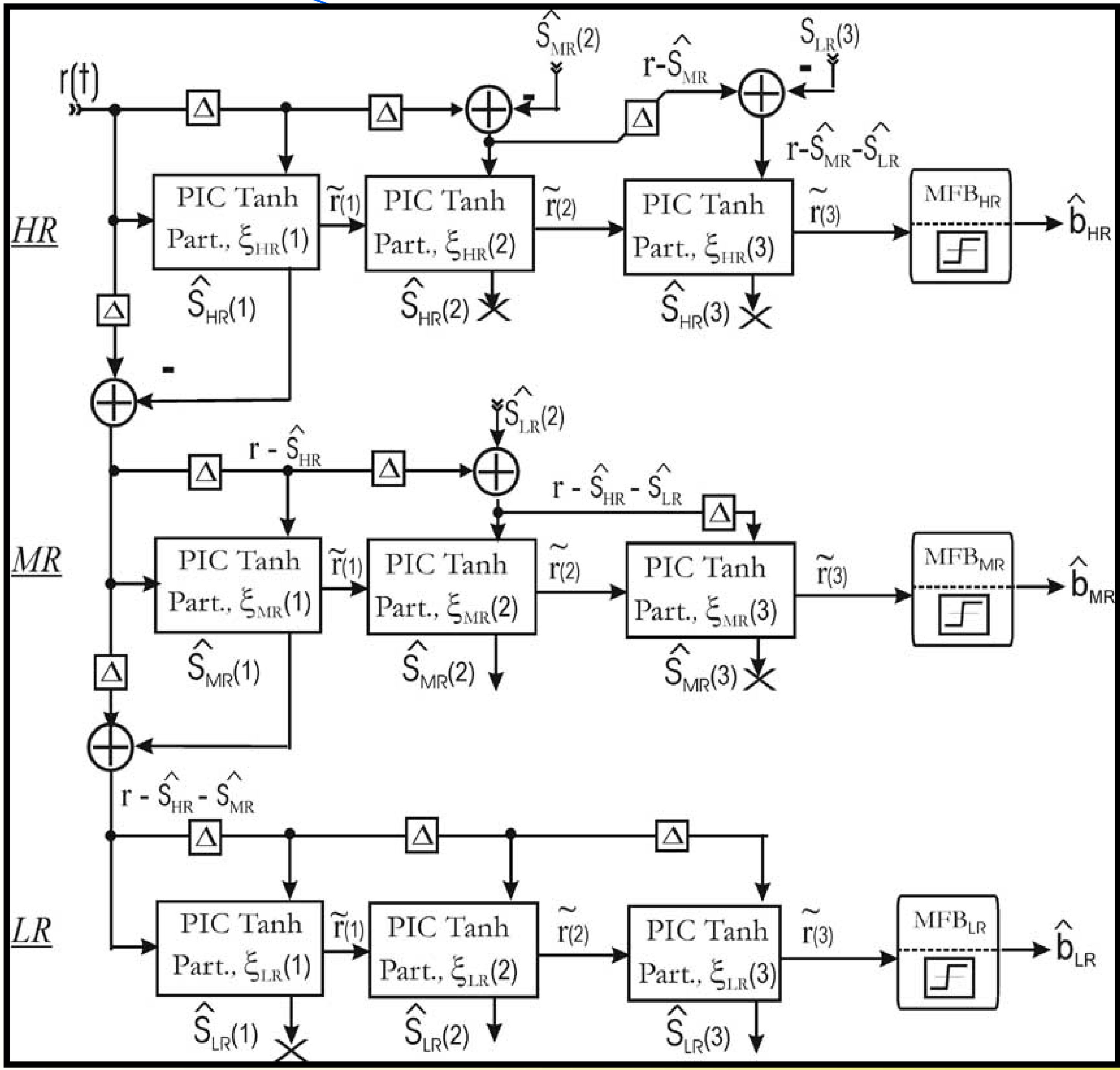
This figure shows us an **extended MPG-GSIC** scheme, which represents an improvement of former one and has a self explanatory operation.

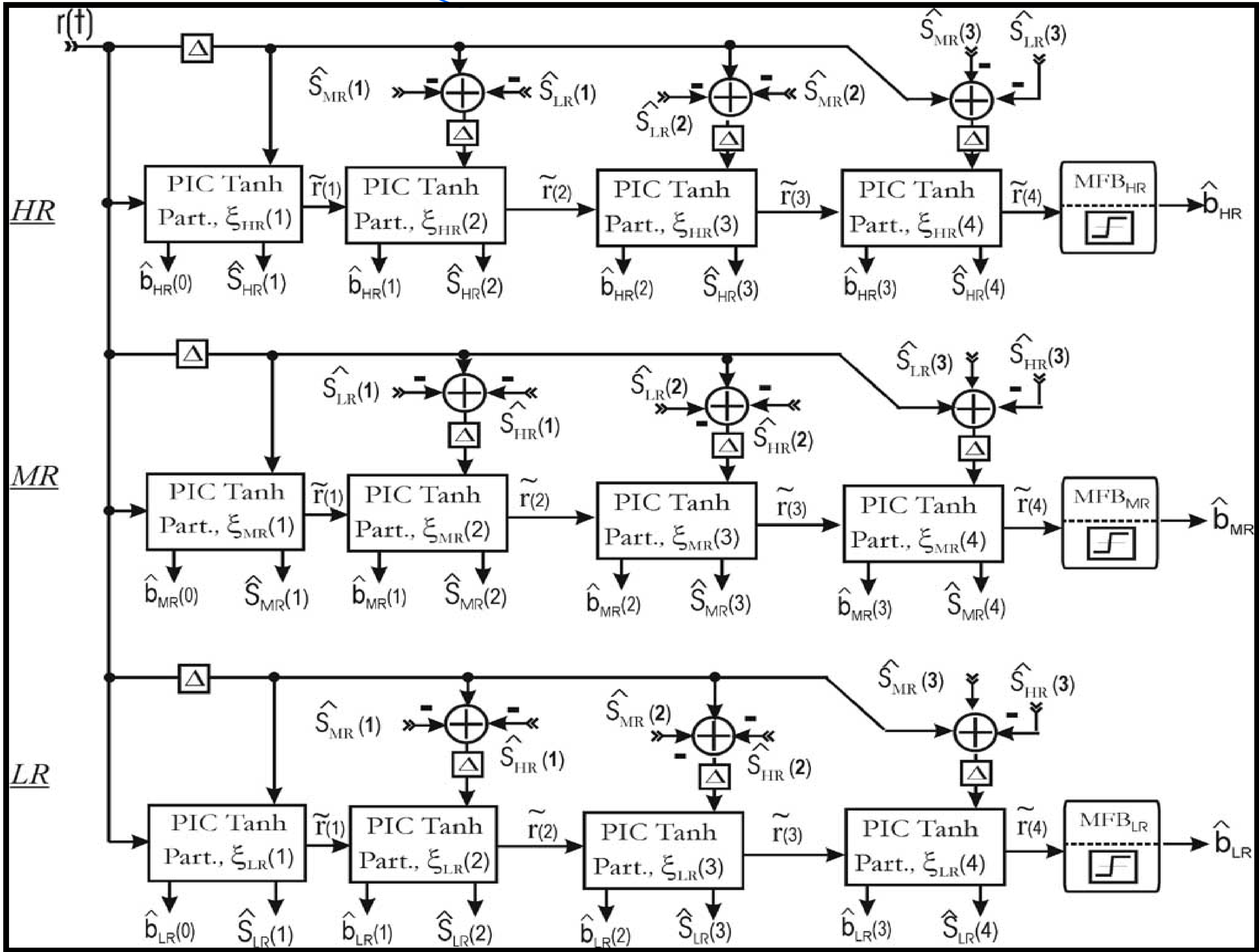


The next figures present a **MPG-GSIC three data rates and partial tanh decision** scheme in three versions.

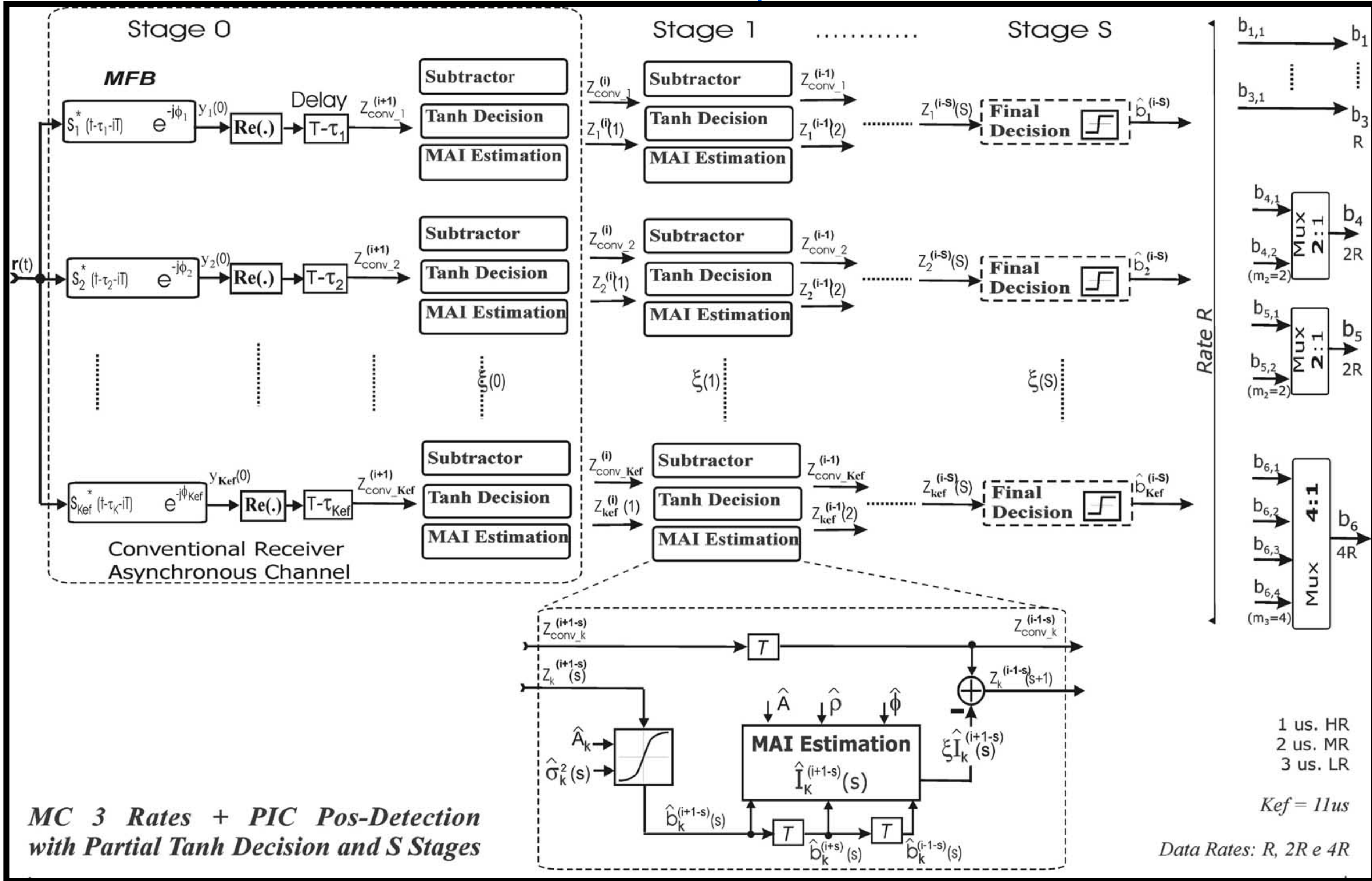
In these schemes we have a combination between serial and parallel IC based on the natural amplitude's unbalance due to the different rates. We have successive detection, reconstruction and cancelling for signals from the same group. In the group, the PIC detector was adopted because all signals in the group follow the same power and rate pattern. Next, all detected signals from this group are reconstructed and subtracted from the delayed input for the next group detection.

Variations of this scheme can consider additional detection for higher order groups, after cancellation of signals from lower order groups.


r(t)
HR
MR
LR
PIC Tanh
Part., $\xi_{HR}(1)$
PIC Tanh
Part., $\xi_{HR}(2)$
PIC Tanh
Part., $\xi_{HR}(3)$
PIC Tanh
Part., $\xi_{MR}(1)$
PIC Tanh
Part., $\xi_{MR}(2)$
PIC Tanh
Part., $\xi_{MR}(3)$
PIC Tanh
Part., $\xi_{LR}(1)$
PIC Tanh
Part., $\xi_{LR}(2)$
PIC Tanh
Part., $\xi_{LR}(3)$
$\tilde{r}(1)$
$\tilde{r}(2)$
$\tilde{r}(3)$
$\hat{S}_{HR}(1)$
$\hat{S}_{HR}(2)$
$\hat{S}_{HR}(3)$
$\hat{S}_{MR}(1)$
$\hat{S}_{MR}(2)$
$\hat{S}_{MR}(3)$
$\hat{S}_{LR}(1)$
$\hat{S}_{LR}(2)$
$\hat{S}_{LR}(3)$
$r-\hat{S}_{MR}$
$r-\hat{S}_{MR}-\hat{S}_{LR}$
$r - \hat{S}_{HR}$
$r - \hat{S}_{HR} - \hat{S}_{LR}$
$r - \hat{S}_{HR} - \hat{S}_{MR}$
$MFB_{HR}$
$MFB_{MR}$
$MFB_{LR}$
$\hat{b}_{HR}$
$\hat{b}_{MR}$
$\hat{b}_{LR}$
Δ
-


HR
MR
LR
r(t)
PIC Tanh Part., $\xi_{HR}(1)$
PIC Tanh Part., $\xi_{HR}(2)$
PIC Tanh Part., $\xi_{HR}(3)$
PIC Tanh Part., $\xi_{HR}(4)$
PIC Tanh Part., $\xi_{MR}(1)$
PIC Tanh Part., $\xi_{MR}(2)$
PIC Tanh Part., $\xi_{MR}(3)$
PIC Tanh Part., $\xi_{MR}(4)$
PIC Tanh Part., $\xi_{LR}(1)$
PIC Tanh Part., $\xi_{LR}(2)$
PIC Tanh Part., $\xi_{LR}(3)$
PIC Tanh Part., $\xi_{LR}(4)$
$MFB_{HR}$
$MFB_{MR}$
$MFB_{LR}$
$\hat{b}_{HR}$
$\hat{b}_{MR}$
$\hat{b}_{LR}$
$\tilde{r}(1)$
$\tilde{r}(2)$
$\tilde{r}(3)$
$\tilde{r}(4)$
$\hat{b}_{HR}(0)$ $\hat{S}_{HR}(1)$ $\hat{b}_{HR}(1)$ $\hat{S}_{HR}(2)$ $\hat{b}_{HR}(2)$ $\hat{S}_{HR}(3)$ $\hat{b}_{HR}(3)$ $\hat{S}_{HR}(4)$
$\hat{b}_{MR}(0)$ $\hat{S}_{MR}(1)$ $\hat{b}_{MR}(1)$ $\hat{S}_{MR}(2)$ $\hat{b}_{MR}(2)$ $\hat{S}_{MR}(3)$ $\hat{b}_{MR}(3)$ $\hat{S}_{MR}(4)$
$\hat{b}_{LR}(0)$ $\hat{S}_{LR}(1)$ $\hat{b}_{LR}(1)$ $\hat{S}_{LR}(2)$ $\hat{b}_{LR}(2)$ $\hat{S}_{LR}(3)$ $\hat{b}_{LR}(3)$ $\hat{S}_{LR}(4)$
$\hat{S}_{MR}(1)$ $\hat{S}_{LR}(1)$ $\hat{S}_{LR}(2)$ $\hat{S}_{MR}(2)$ $\hat{S}_{MR}(3)$ $\hat{S}_{LR}(3)$
$\hat{S}_{LR}(1)$ $\hat{S}_{HR}(1)$ $\hat{S}_{LR}(2)$ $\hat{S}_{HR}(2)$ $\hat{S}_{LR}(3)$ $\hat{S}_{HR}(3)$
$\hat{S}_{MR}(1)$ $\hat{S}_{HR}(1)$ $\hat{S}_{MR}(2)$ $\hat{S}_{HR}(2)$ $\hat{S}_{MR}(3)$ $\hat{S}_{HR}(3)$

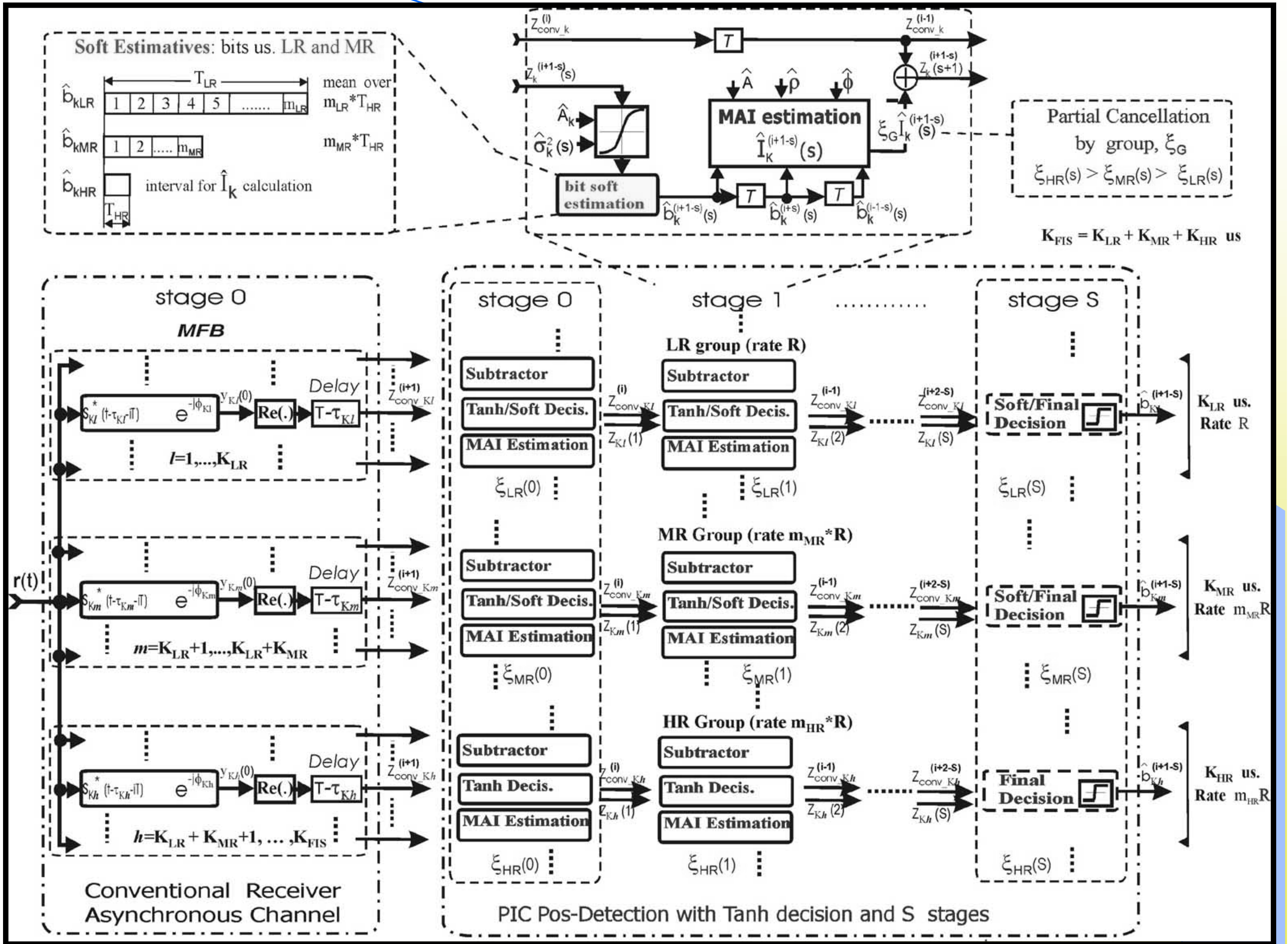
The **MC-PIC three data rates, multi-stage and partial tangh** scheme represented below allows users with rates R, 2R and 4R. The reconstructed signal is multiplied by factors that grow as we increase the stage's order. The last stage has hard decision circuit.



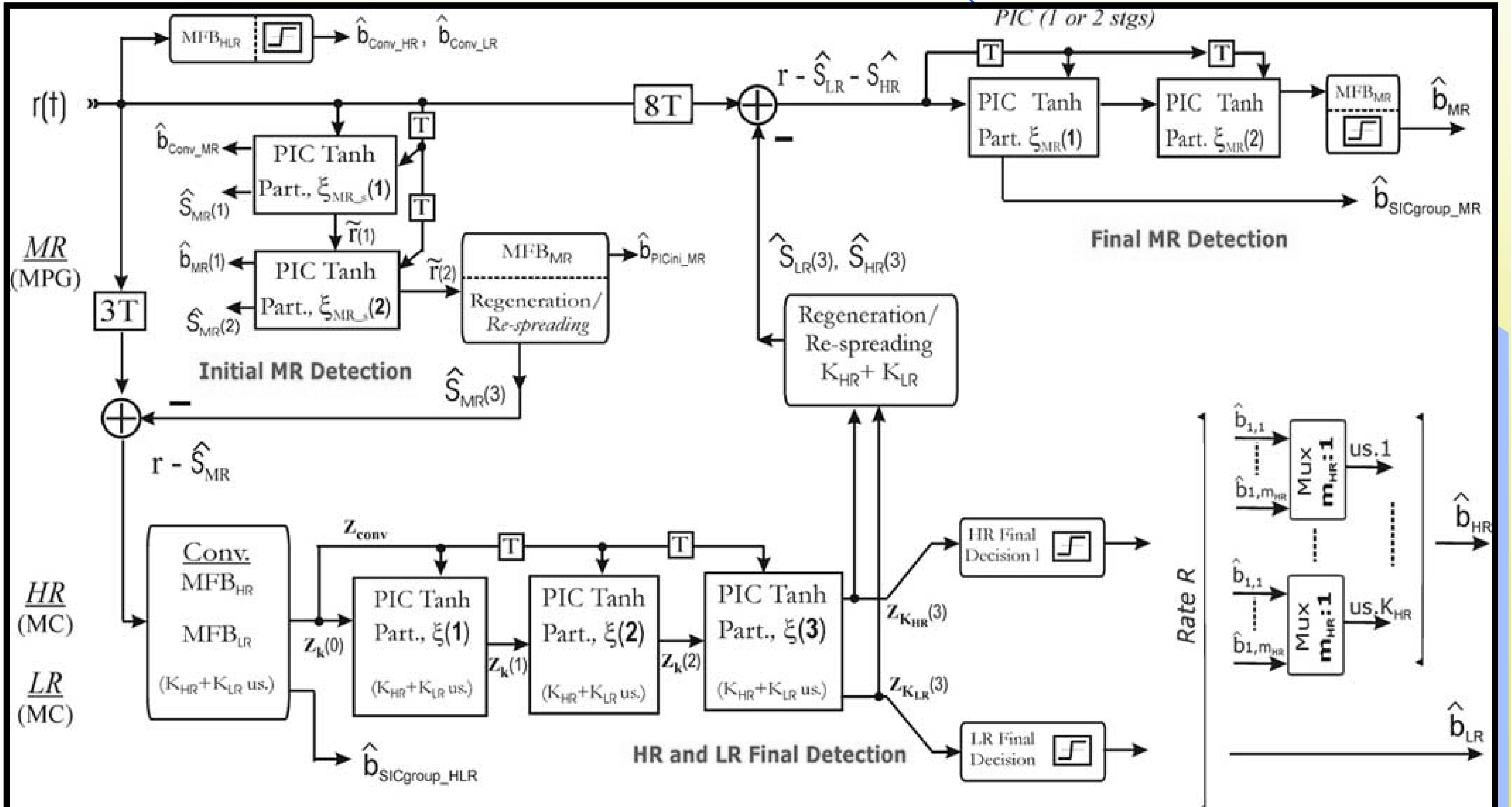

*MC 3 Rates + PIC Pos-Detection with Partial Tanh Decision and S Stages*

The **MPG-PIC three data rate multi-stage and partial tanh decision** scheme allows users with rates LR, $MR=m_{MR}LR$ and $HR=m_{HR}LR$. The reconstructed signal is multiplied by factors that grow as we increase the stage's order. The last stage has hard decision circuit. More details can be seen in the next figure.

Note that, we need to have partial MAI estimation (named here as soft estimation) for each $T_{HR}$ interval, that is the smallest time interval considered for data variations. Now, the amplitude of transmitted signal is related with its rate in order to maintain constant bit energy (for instance, rate change from R to 4R implies on amplitude change from 1 to 2). In this scheme the cancellation factor was fixed in concordance with the rate: higher rates imply higher amplitudes, which imply more confidence in decisions. So we should impose factors as $1>\xi_{HR}>\xi_{MR}>\xi_{LR}>0$.


Soft Estimatives: bits us. LR and MR
mean over $m_{LR}*T_{HR}$
$m_{MR}*T_{HR}$
interval for $\hat{I}_k$ calculation
MAI estimation
bit soft estimation
Partial Cancellation by group, $\xi_G$
$\xi_{HR}(s) > \xi_{MR}(s) > \xi_{LR}(s)$
$K_{FIS} = K_{LR} + K_{MR} + K_{HR}$ us
stage 0
MFB
Delay
$l=1,\ldots,K_{LR}$
$m=K_{LR}+1,\ldots,K_{LR}+K_{MR}$
$h=K_{LR}+K_{MR}+1,\ldots,K_{FIS}$
Conventional Receiver
Asynchronous Channel
stage 1
stage S
LR group (rate R)
MR Group (rate $m_{MR}$*R)
HR Group (rate $m_{HR}$*R)
Subtractor
Tanh/Soft Decis.
Tanh Decis.
MAI Estimation
Soft/Final Decision
Final Decision
$K_{LR}$ us. Rate R
$K_{MR}$ us. Rate $m_{MR}$R
$K_{HR}$ us. Rate $m_{HR}$R
PIC Pos-Detection with Tanh decision and S stages

The **Hybrid MC/MPG extended GSIC** scheme is a compromise that tries to avoid the near-far effect caused by MPG users with high rates (that needs to transmit its signal with higher power) and the low availability of codes for MC type users with high rates. Users with high or low rate transmit their data by MC and users with medium rate adopt MPG (the scheme is well adapted for a scenario in which we have great difference in users' rates).

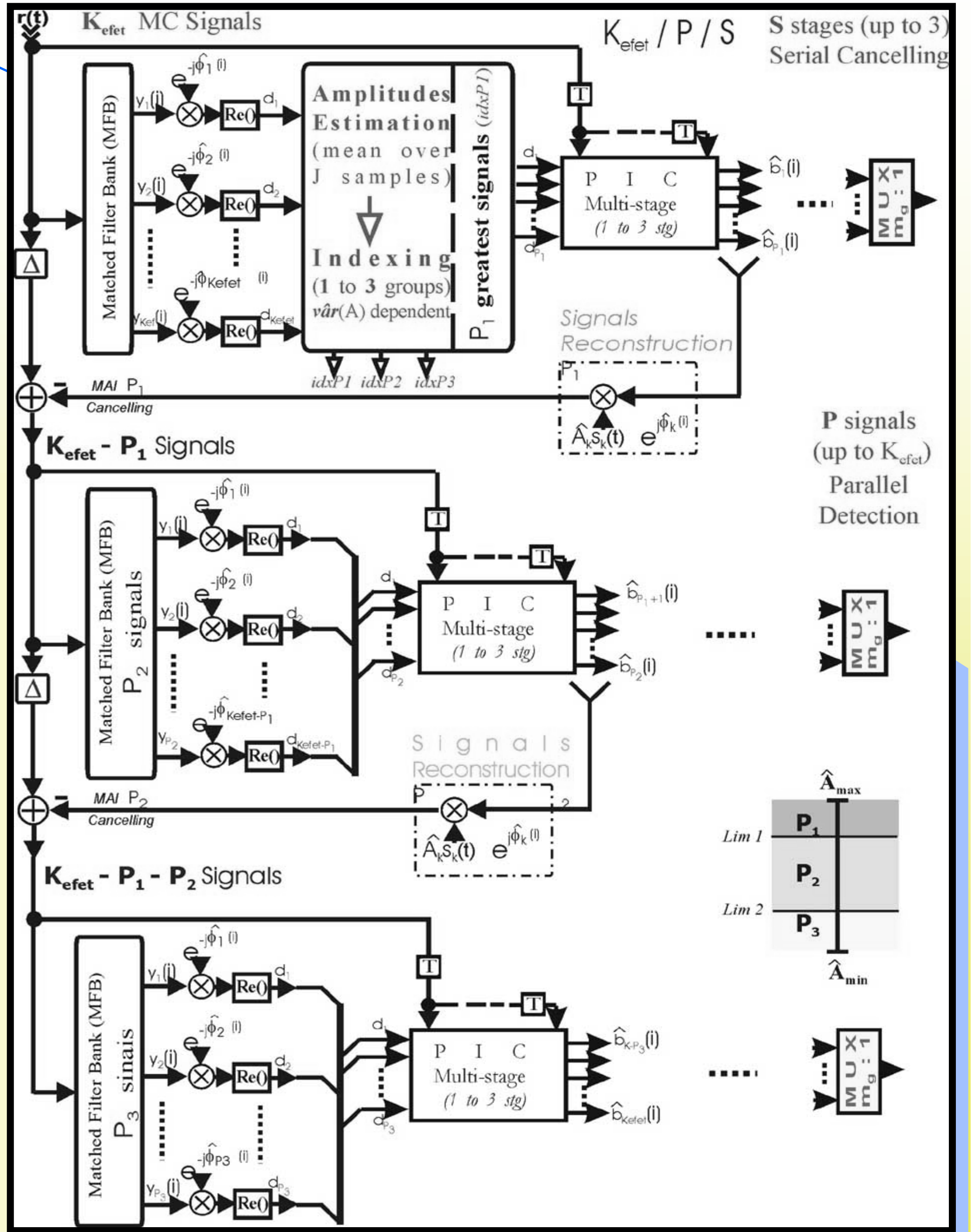



Hybrid Multi-rate MC/MPG associated with extended G-SIC Group Detector

In the **Hybrid K/P/S MC-GSIC three data rates and partial tanh decision** scheme first we detect, with a PIC multi-stage, the signals with greater amplitudes (more confident detection) that are in the predefined interval $P_1$.

Next these reconstructed signals are subtracted from the delayed input, the remainders are reordered according to their amplitudes and detected on the next stage ($K_{efet}-P_1$).

We repeat the process again so in the next stage we will have ($K_{efet}-P_1-P_2$) signals for final detection.



# References and Bibliography

[1] P. J. E. Jeszensky, **Notas de Aula do Curso de Comunicação por Espalhamento Espectral: Uma motivação para o estudo de seqüências de códigos**, Escola Politécnica da Universidade de São Paulo, Departamento de Engenharia Eletrônica, 1992, 19 p.

[2] J. R. Fitzgerald Jr., **Tópicos sobre a Caracterização de Desempenho de Sistemas DS-CDMA** , M. Sc. Dissertation, Escola Politécnica da Universidade de São Paulo, Departamento de Engenharia Eletrônica, São Paulo, 1996, 134 p.

[3] M. B. Pursley, **Performance Evaluation for Phase-Coded Spread Spectrum Multiple-Access Communication - Part I: System Analysis**, IEEE Transactions on Communications, Vol. 25, Nº 8, August 1977, pp. 795-799.

[4] M. B. Pursley and D. V. Sarwate, **Performance Evaluation for Phase-Coded Spread Spectrum Multiple-Access Communication - Part II: Code Sequence Analysis**, IEEE Transactions on Communications, Vol. 25, Nº 8, August 1977, pp. 800-803.

[5] K. Yao, **Error Probability of Asynchronous Spread Spectrum Multiple Access Communication Systems**, IEEE Transactions on Communications, Vol. 25, Nº 8, August 1977, pp. 803-809.

[6] C. L. Weber, K. H. Gaylord and B. H. Batson, **Performance Considerations of Code Division Multiple-Access Systems**, IEEE Transactions on Vehicular Technology, Vol. 30, Nº 1, February 1981, pp. 3-10.

[7] D. V. Sarwate and M. B. Pursley, **Crosscorrelation Properties of Pseudorandom and Related Sequences**, Proceedings of the IEEE, Vol. 68, Nº 5, May 1980, pp. 593-619.

[8] N. Nazari and R. E. Ziemer, **Computationally Efficient Bounds for the Performance of Direct-Sequence Spread Spectrum Multiple-Access Communications Systems in Jamming Environments**, IEEE Transactions on Communications, Vol. 36, Nº 5, May 1988, pp. 577-587.

[9] A. Papoulis, **Probability, Random Variables, and Stochastic Processes**, 3rd Edition , McGraw-Hill, 1991, 666 p.

[10] R. Cameron and B. Woerner, **Performance Analysis of CDMA with Imperfect Power Control**, IEEE Transactions on Communications, Vol. 44, Nº 7, July 1996, pp. 777-781.

[11] K. I. Kim, **CDMA Cellular Engineering Issues**, IEEE Transactions on Vehicular Technology, Vol. 42, Nº 3, August 1993, pp. 345-350.

[12] L. B. Milstein and T. Rappaport, **Performance Evaluation for Cellular CDMA**, IEEE Journal on Selected Areas in Communications, Vol. 10, Nº 4, May 1992, pp. 680-689.

[13] S. Moshavi, **Multiuser Detection for DS-CDMA Communications**, IEEE Communications Magazine, Vol. 34, Nº 10, October 1996, pp. 124-136.

[14] F. D. Priscoli and F. Sestini, **Effects of Imperfect Power Control and User Mobility on a CDMA Cellular Network**, IEEE Journal on Selected Areas in Communications, Vol. 14, Nº 9, December 1996, pp. 1809-1816.

[15] R. K. Morrow Jr. and J. S. Lehnert, **Bit-to-Bit Dependence in Slotted DS/SSMA Packet Systems with Random Signature Sequences**, IEEE Transactions on Communications, Vol. 37, Nº 10, October 1989, pp. 1052-1061.

[16] C. S. Hemsi, **Análise de Sistemas DS/CDMA de Múltipla Taxa**, M. Sc. Dissertation, Escola Politécnica da Universidade de São Paulo, Departamento de Engenharia de Telecomunicações e Controle, 2000, 247 p.

[17] M. W. D. Rolim, **Tópicos sobre a Determinação da Capacidade de Sistemas DS/CDMA**, M. Sc. Dissertation, Escola Politécnica da Universidade de São Paulo, Departamento de Engenharia de Telecomunicações e Controle, 2000,173 p.

[18] T. Abrão, **Canceladores de Interferência Multiusuário Aplicados a Sistemas DS/CDMA de Múltipla Taxa**, Ph. D. Thesis, Escola Politécnica da Universidade de São Paulo, Departamento de Engenharia de Telecomunicações e Controle, 2001, 378 p.

[19] A. J. Viterbi, **CDMA: Principles of Spread Spectrum Communication**, Addison Wesley Publishing Co., 1995, 245 p.

[20] W. C. Y. Lee, **Mobile Communications Engineering**, McGraw-Hill Book Co., 2nd Edition, 1998, 689 p.

[21] P. J. E. Jeszensky and J. R. Fitzgerald Jr., **Sequences Selection for Quasi-Synchronous CDMA Systems**, ISSSTA'98-Fifth IEEE International Symposium on Spread Spectrum Techniques and Applications, Sun City, South Africa, 1998, pp. 706-708.

[22] J. K. Holmes, **Coherent Spread Spectrum Systems**, John Wiley & Sons Inc., 1982, 624 p.

[23] S. Verdú, **Minimum Probability of Error for Asynchronous Gaussian Multiple Access Channels**, IEEE Transactions on Information Theory, Vol. 32, Nº 1, January 1986, pp. 85-96.

[24] S. Verdú, **Multiuser Detection**, Cambridge University Press, 1998, 451 p.

[25] J. G. Proakis, **Digital Communications**, McGraw-Hill Book Co., 2nd Edition, 1989, 905 p.

[26] C. S. Wijting, T. Ojanpera, M. J. Juntti, K. Kansanen and R. Prasad, **Groupwise Serial Multiuser Detectors for Multirate DS-CDMA**, Proceedings of IEEE Vehicular Technology Conference (VTC'99), Houston, USA, May 16-20, 1999.

[27] M. J. Juntti, **Performance of Multiuser Detection in Multirate CDMA Systems**, Wireless Personal Communications, Kluver Academic Publishers, 1999.

[28] C. A. Balanis, **Antenna theory: analysis and design**, John Wiley & Sons, 2nd Edition, 1996, 960 p.

[29] D. Divsalar and M. Simon, **Improved CDMA Performance Using Parallel Interference Cancellation**, JPL (Jet Propulsion Laboratory) Publication 95-21, October 1995.

[30] G. H. Golub and C. F. V. Loan, **Matrix Computations**, The John Hopkins University Press, 3rd Edition, 1996, 694 p.

[31] J. C. Liberti and T. S. Rappaport, **Smart Antennas for Wireless Communications : IS-95 and Third Generation CDMA Applications**, Prentice Hall, 1999, 528 p.

[32] L. K. Rasmussen, T. J. Lim and A. L. Johansson, **A Matrix-Algebraic Approach to Successive Interference Cancellation in CDMA**, IEEE Transactions on Communications, Vol. 48, Nº 1, January 2000, pp. 145-151.

[33] D. Guo, L. K. Rasmussen, S. Sun and T. J. Lim, **A Matrix-Algebraic Approach to Linear Parallel Interference Cancellation in CDMA**, IEEE Transactions on Communications, Vol. 48, Nº 1, January 2000, pp. 152-161.

[34] A. L. C. Hui and K. B. Letaief, **Successive Interference Cancellation for Multiuser Asynchronous DS/CDMA Detectors in Multipath Fading Links**, IEEE Transactions on Communications, Vol. 46, Nº 3, March 1998, pp. 384-391.

[35] T. Abrão and P. J. E. Jeszensky, **Detectores Multiusuários para DS/CDMA - Lineares Fixos**, Brazilian Telecommunication Society Magazine, vol. 16, n. 02, 2001, pp. 122-137.

[36] T. Abrão and P. J. E. Jeszensky, **Detectores Multiusários para DS/CDMA - Canceladores de Interferência**, Brazilian Telecommunication Society Magazine, vol. 16, n. 02, 2001, pp. 138-156.