Abstract—This work analyses a genetic algorithm (GA) applied to multiuser detection (MuD) direct sequence code division multiple access (DS/CDMA) communication systems in multipath fading channels with errors in the channel parameters estimates, received amplitudes, and inter-users and paths delays. Monte Carlo simulation (MCS) results show that the detection based on GA is a viable option when compared with the optimum MuD (OMuD) and the classic Decorrelator (DD). The algorithm complexity is determined compared with the optimum MuD (OMuD) and the classic Decorrelator (DD). The algorithm complexity is determined based on the required number of computational operations.

Index Terms—Multiuser channels, code division multiple access, genetic algorithm, maximum likelihood detection, complexity theory, fading channels, Monte Carlo methods.

I. INTRODUCTION

For most of the practical cases of engineering interest, MuD based on heuristic techniques result in almost optimum performance, i.e., very close to the performance reached by the OMuD [1], however with the advantage of smaller computational cost and a smaller detection time, an attractive tradeoff between converge speed and complexity. In spite of existence of several works using approximative procedures for the sub-optimum MuD, most of the investigations are restricted to the AWGN synchronous channels [2]. Very few works analyze the detection problem in frequency selective channels [3]. Differently of [3], this work uses one-shot GA-MuD over all bits from all users in the same frame, considering multipath exponential power profile channels with errors in parameters estimates. Additionally, the maximal rate combining (MRC) was used in order to find the initial candidate bits.

II. SYSTEM MODEL

In a DS/CDMA system with binary phase-shift keying modulation (BPSK) shared by K asynchronous users the k-th user transmitted signal is given by:

\[ x_k(t) = \sqrt{2P_k} \sum_{i} h_k(t-iT_b) \cos(\omega, t) \]  

(1)

where \( P_k = A_k^2/2 \) represents the k-th user’s transmitted power; \( h_k(t) \) is the i-th BPSK symbol with period \( T_b \); \( \omega \) is the carrier frequency; \( s_k(t) = \sum_{n=1}^{N-1} E(t-nT_c)s_{k,n} \) corresponds to the spreading sequence defined in the interval \( [0,T_b] \) and zero outside, where \( s_{k,n} \in \{-1,1\} \) is the n-th chip of the sequence with length \( N \) used by the k-th user; \( T_c \) is the chip period and the spread spectrum processing gain, \( \frac{T_b}{T_c} \), is equal to \( N \); the pulse shaping \( p(t) \) is assumed rectangular with unitary amplitude in the interval \( [0,T_c] \) and zero outside. Assuming a frame with \( I \) bits for each user, propagating over \( L \) independent slow Rayleigh fading paths, the baseband received signal in the base station is given by:

\[ r(t) = \sum_{i=0}^{I-1} \sum_{k=1}^{K} \sum_{l=1}^{L} A_k h_k^{(i)} s_k(t - \tau_{k,l}) + n(t) \]

(2)

where \( K \) is the number of active users, \( t \in [0,T_b] \), the amplitude \( A_k \) is assumed as constant for all I transmitted bits, \( b_k \in \{-1,1\} \) is the transmitted information bit, \( s_k \) is the signature sequence assigned to the k-th user, with \( \tau_{k,l} \) representing the random delay associated to the k-th user; this random delay takes into account the asynchronous nature of the transmission, \( d_k \), as well as the propagation delay, \( \Delta_k \), for k-th user, \( \ell \)-th path, resulting in \( \tau_{k,l} = \Delta_k + d_k \); \( n(t) \) represents the AWGN with bilateral power density equal to \( N_0/2 \) and the complex low-pass impulse response of the channel for the k-th user over the i-th bit interval is

\[ h_k^{(i)}(t) = \sum_{\ell=1}^{L} c_{k,\ell} \delta(t - \Delta_k) \]

(2)

where \( c_{k,\ell} \) is the complex channel coefficient for the k-th user, \( \ell \)-th path; it is assumed that the \( c_{k,\ell} \) phase has a uniform distribution over \( \phi_{k,\ell} \in [0,2\pi] \) and the channel coefficient’s amplitude \( \beta_{k,\ell} \) represents the small scale-fading envelope following a Rayleigh distribution with probability density function \( f(\beta) = \frac{2\beta}{\sigma^2} e^{-\beta^2/\sigma^2} \), where \( \beta \) is the coefficient’s module and \( \varsigma \) the multipath’s component average power \( \varsigma = E[\beta^2] \). Additionally, it is assumed that the channel gain is normalized for all users. Using vectorial notation the equation (2) can be stated as:

\[ r(t) = \sum_{i=0}^{I-1} s_t(t-iT_b) a c^{(i)} b^{(i)} + n(t) \]

(3)

where: \( s(t) = [s_1(t - \tau_{1,1}), s_1(t - \tau_{1,2}), \ldots, s_1(t - \tau_{1,L}), \ldots, s_K(t - \tau_{K,1}), \ldots, s_K(t - \tau_{K,L})]^T \) is the users signature sequence vector, \( a = \)
diag \[ \sqrt{P_1^T I}, \sqrt{P_2^T I}, \ldots, \sqrt{P_K^T I} \] is a diagonal matrix for the average received amplitudes of the users including the path losses and shadowing effects, where \( I_{L\times L} \) is the identity matrix with dimension \( L \); 
\[ c^{(i)} = \text{diag} \left( c^{(i)_1}, \ldots, c^{(i)_{L}}, c^{(i)_{2L}}, \ldots, c^{(i)_{K\times L}} \right) \]
is the diagonal channel gain matrix, and the data vector is given by 
\[ b^{(i)} \triangleq \left[ b^{(i)}_1, b^{(i)}_2, \ldots, b^{(i)}_K \right]^T \]
with \( b^{(i)}_k \) representing the \( 1 \times L \) \( k \)-th user bit vector. 

For simplicity and without loss of generality, it was assumed an ordering of the random delays, such that 
\( 0 \leq \tau_{1,1} \leq \tau_{1,2} \leq \cdots \leq \tau_{1,L} \leq \tau_{2,1} \leq \cdots \leq \tau_{K,L} \leq T_b \). 

For multipath fading channels, the conventional receiver (Rake) consists of a bank of \( KL \) filters matched to the users signature sequence. The matched filter outputs with coherent reception for the \( k \)-th user corresponding to the \( \ell \)-th multipath component (finger) sampled at the end of the \( i \)-th bit interval is 
\[ y_{k,\ell}^{(i)} = \sqrt{P_k^T T_{k,\ell}} b_{k,\ell}^{(i)} + S_{k,\ell}^{(i)} + n_{k,\ell}^{(i)} \]
where the first term corresponds to the desired signal, the second to the auto-interference, and the third to the multiple access interference (MAI) over the \( \ell \)-th multipath component of \( k \)-th user and the last to the filtered AWGN. 

The output of the matched filter bank (MFB) at the \( i \)-th symbol interval can be written using vectorial notation as: 
\[ y^{(i)} = \begin{bmatrix} y^{(i)}_1, y^{(i)}_2, \ldots, y^{(i)}_{L}, y^{(i)}_{2L}, \ldots, y^{(i)}_{KL} \end{bmatrix}^T \]
\[ = R^T [1] a c^{(i+1)} b^{(i+1)} + R [0] a c^{(i)} b^{(i)} + + R [1] a c^{(i-1)} b^{(i-1)} + n^{(i)} \]
where the matrices \( R [0] \) and \( R [1] \) with dimension \( L K \times L K \) are defined by the elements: 
\[ R_{jk} [0] = \begin{cases} 1, & \text{if } j = k \ 
\mathcal{R}_{jk}(\tau_{jk},0), & \text{if } j < k \text{ and } \ j > k \ \mathcal{R}_{kj}(\tau_{jk},0), & \text{if } j > k \end{cases} \]
\[ R_{jk} [1] = \begin{cases} 0, & \text{if } j \geq k \ 
\mathcal{R}_{kj}(\tau_{jk},0), & \text{if } j < k \end{cases} \]

where the crosscorrelation elements is \( \mathcal{R}_{jk} = \int_0^{T_0} s_j(t) s_k(t+\tau) dt \) and the filtered noise vector \( n^{(i)} \) has autocorrelation matrix \( E \left[ n^{(i)}(n^{(i)})^T \right] = 0.5 N_0 R [i] \), with \( i = -1, 0, +1 \) and \( R [-1] = R^T [1] \). 

The conventional detector for frequency selective channels consists in combining the available MFB outputs of each user (fingers) in a coherent way and weighting it by each channel gain [4]. The MRC combines the \( D \) correlators’ output signals, followed by an abrupt decision circuit: 
\[ z_k^{(i)} = \sum_{\ell = 1}^{D} \text{Re} \left( y_{k,\ell}^{(i)}(s) b_{k,\ell}^{(i)} \right) \]
\[ \hat{b}_k^{(i)} = \text{sgn} \left( z_k^{(i)} \right) \]

where \( D \leq L \) represents the number of correlators in the receiver for each user, which in a real system needs the estimation of the following parameters for all users: channel coefficients, \( \beta \), power, \( P' \), delay, \( \tau \), and phase, \( \phi \). 

For the joint decision of all bits from all users this work adopts the one-shot approach in asynchronous channels [1]. In this context the \( K \)-user, \( L \)-paths, \( I \)-frame and asynchronous channel scenario can be viewed as a \( KLI \)-user synchronous channel scenario, and then the \( KLI \)-user receiver \( B \) can be written as: 
\[ B = [b^{(0)}_1, b^{(1)}_2, b^{(2)}_2, \ldots, b^{(I-1)}_T]^T \]

The objective is to find the matrix \( B \) with dimension \( KLI \) that maximizes: 
\[ \exp \left[ -\int_0^{T_T} [y(t) - S(B)]^2 dt \right] \]

where \( S(B) = \sum_{k=1}^K \sum_{l=1}^L \sqrt{P_k^T} b_{k,l}^{(i)} s_k(t - \tau_{k,l}) \). Based on the MFB observations, vector \( y^{(i)} \) in (5), the maximization is equivalent to select the vector \( B \) that maximizes the so-called log-likelihood function (LLF): 
\[ \Omega(B) = 2 \text{Re} \left\{ B^T C A Y \right\} - B^T C A R A C H B \]

where the coefficients and amplitudes diagonal matrices, with dimension \( KLI \), are defined by \( C = \text{diag} \left( c^{(0)}, c^{(1)}, c^{(2)}, \ldots, c^{(I-1)} \right) \) and \( A = \text{diag} \left[ a[a, a, \ldots, a] \right] \) respectively, \( Y = \left[ y^{(0)}_T, y^{(1)}_T, y^{(2)}_T, \ldots, y^{(I-1)}_T \right]^T \), where the transposed hermitian operator is defined by \( \cdot^H = [\cdot]^T \) and the cross correlation matrix \( R \) is a block-tridiagonal, block-Toeplitz, with the same dimension, composed by the sub-matrices \( R[0] \) and \( R[1] \) [1]. 

III. GENETIC ALGORITHM 
A pseudocode for the GA-MuD is described below.

**Input:** \( p, B_1, M, G \)  
**Output:** \( B_1 \) 
begin 
1. Initialize first population \( B; \ g = 0; \) 
2. Evaluate the fitness(\( B \)); 
3. While \( g < G \) then; 
4. \( B_{\text{selected}} = \text{Selection}(B, T); \) 
5. \( B_{\text{cross}} = \text{Crossover}(B_{\text{selected}}); \) 
6. \( B_{\text{new}} = \text{Mutation}(B_{\text{cross}}); \) 
7. Evaluate the fitness(\( B_{\text{new}} \)); 
8. \( B = \text{Replacement}(B \cup B_{\text{new}}); \) 
9. end 
end 

The GA-MuD total search universe will be characterized by all possible combination of received data bits that present the same bit for all \( D \) processing branches.
This work uses an adapted equation in order to find the population size for the MuD problem based on [5]:

\[ p = 10 \cdot \left[ 0.3454 \left( \sqrt{\pi (K \cdot I - 1) + 2} \right) \right] \]  

(11)

where the operator \( \lfloor x \rfloor \) returns the smallest integer contained in \( x \). This equation is calculated in the GA initialization stage and maintained constant in all generations.

The estimates from the Rake receiver outputs is adopted as an initial individual. The other members of the first population \( B_1 \) are obtained from the initial individual with convenient perturbations [3], [6]. In the MuD context the aptitude is measured through the LLF function (10) and it is directly responsible for the death or life of individuals.

The mating pool size (M) should be selected in order to guarantee the convergence velocity and the final solution quality. For the MuD problem \( M = 0.1p \) was adopted.

In this work, the selection process chooses the best \( M \) individuals from the population \( p \) as the parent for the next generation. Consequently, the \( p - M \) individuals with low fitness scores are removed for the reproduction stage.

For the GA-MuD genetic operators, we adopted the uniform crossover [7] and the mutation based on noise:

\[ \text{newindividual} = \text{sign} \left( \text{individual} + \mathcal{N}(0, \sigma^2) \right) \]  

(12)

where \( \mathcal{N}(0, \sigma^2) \) represents a Gaussian distribution with standard deviation \( \sigma \) and expectation zero. The standard deviation is strongly related with the mean rate mutation [6].

This work uses replacement strategy called global elitism, where only the best \( p \) individuals from the joint population of parents and offsprings are maintained for the next generation. Finally, the optimization process for the GA-MuD problem is finished after a fixed number of generations \( G \).

IV. NUMERICAL RESULTS

In all MCS we have adopted the following parameters: the spread sequences are selected as pseudo-noise (PN) with processing gain \( N = 31 \); the number of active asynchronous users is \( K = 10 \); two-paths slow Rayleigh channels with the second ray delayed 1.5\( T_c \) from the first and exponential power profile with \( E[\beta^2_1] = 0.832 \) and \( E[\beta^2_2] = 0.168 \). All \( K \) users were considered with an uniformly distributed velocity in the interval \([0; v_{\text{max}}]\), resulting in a maximum Doppler frequency of \( f_m = \frac{v_{\text{max}}}{4} = 222.2 \) Hz for a carrier frequency of \( f_c = \frac{1}{\lambda} = 2 \) GHz; the adopted Rake diversity is \( D = L = 2 \) and all users are transmitting with the same data rate \( R_b = 384 \) Kb/s.

In order to simulate the effect of the errors in the estimates of the delays the number of samples for each chip was fixed as \( N_s = 10 \).

For the channel coefficients generation a modified Gans model was adopted [8], with coefficients generated in the frequency domain. The average \( E_b/N_0 \) at the receiver input is given by \( \hat{\gamma} = \sum_{\ell=1}^{L} \gamma_{\ell} \), where \( \gamma_{\ell} = \frac{1}{2} E[|\beta_{\ell}|^2] \).

For comparison purpose the performances of the Rake, DD and the single user bound (SuB) were also included. The analytical SuB for BER is given by [4].

The adopted parameter values for the GA-MuD were obtained in two steps: preliminary simulations with typical values found in the literature; additional simulations in order to optimize these parameters, not in an exhaustive form, however assuring a superior performance than those found in the preliminary step. The population was chosen from equation (11) resulting in \( p = 110 \), the mating pool size was adopted as \( M = 0.1p = 11 \), the mutation percentage was adopted as \( p_m = \frac{1000}{20T} = 1.43\% \), i.e., one mutation (one bit) by individual, in average [6]. In all simulations the GA-MuD processes and optimizes one frame with \( K \cdot I \cdot D \) bits each time, where \( I = 7 \) bits/user and \( G = 40 \).

The GA-MuD performance degradation was analyzed considering errors in the channel estimates, delays and amplitudes. Errors were introduced separately and jointly being modeled through uniform distributions:

\[ \hat{\beta}_{k,\ell} = U[1 \pm \epsilon_\beta] \times \beta_{k,\ell} \; ; \; \hat{\phi}_{k,\ell} = U[1 \pm \epsilon_\phi] \times \phi_{k,\ell} \; ; \; \hat{\gamma}_{k,\ell} = U[1 \pm \epsilon_\gamma] \times \gamma_{k,\ell} \; ; \; \hat{A}_{k,\ell} = U[1 \pm \epsilon_A] \times A_{k,\ell} \]

with \( \epsilon_\beta \) and \( \epsilon_\phi \) \( \in \) [0.05;0.10;0.25;0.30]; \( \epsilon_\gamma \) \( \in \) [0;0.1;0.2;0.3] and \( \epsilon_A \) \( \in \) [0.1;0.5].

Fig. 1 to Fig. 3 show the performance as a function of signal to noise ratio (\( \gamma \)). For comparison purpose the DD performance in frequency selective fading channel was also included [9]. In this case, the decorrelating matrix (\( \mathbf{R}^{-1} \)) has dimension \( K \cdot I \cdot D \). When the number of users or the processed frame length or the multipath diversity increases the inverse matrix calculation becomes impracticable. Further, we can verify from these figures that the GA-MuD performs better than the DD.

Fig. 1 shows that even with great module or phase estimates errors, about up to 30\%, the GA-MuD reaches better performance than those obtained with the Rake detector in the absence of errors, evidencing its tolerance to the errors in the estimates. The GA-MuD has better performance than the DD with perfect estimates for \( |\epsilon_\beta| \) and \( |\epsilon_\phi| < 0.1 \). Also, Fig. 1 shows that the GA-MuD is more sensitive to phase errors than to module errors.

For delays errors estimates, Fig. 2, shows that the GA-MuD reaches better performance than those obtained with the Rake detector. For \( |\epsilon_\gamma| < 0.1 \), the GA-MuD showed better performance than DD with perfect estimates.

The GA-MuD is relatively robust against the amplitude estimates errors. Simulation results showed that even with amplitudes estimates errors of the order of 50\%, the GA-

\(^1\)The DD is insensitive to variations in the amplitude.
MuD performance suffers a tolerable degradation in the
range of medium $\gamma$.

![Graph showing performance degradation with errors in the estimates of channel coefficients.](image1)

**Fig. 1.** GA-MuD performance degradation with errors in the estimates of channel coefficients.

![Graph showing performance degradation with delays.](image2)

**Fig. 2.** GA-MuD performance degradation with errors in the estimates of delays.

![Graph showing performance degradation with jointly errors in estimates.](image3)

**Fig. 3.** GA-MuD performance degradation with jointly errors in the estimates of channel coefficients, delays and amplitudes.

V. SUMMARY

The GA-MuD receiver in Rayleigh fading channels approaches the SuB limit for perfect parameters estimates condition. With errors in the estimates of parameters the GA-MuD showed better performance than the DD, with the advantage of not being necessary accomplished the $\mathbf{R}^{-1}$ operation. Additionally, it results in a huge complexity reduction in comparison with the OMuD, making its implementation feasible in the base stations of cellular systems. The GA-MuD has a relative immunity against errors in the estimates of channel coefficients and delays and great robustness against errors in the estimates of amplitudes.

REFERENCES


