Proceedings of the third IFAC CHAOS Conference
Cancún (México) June 20-22, 2012

Using an Evolutionary Denoising Approach to Improve the Robustness of Chaotic Synchronization ⋆

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Abstract: Chaotic synchronization in master-slave networks has been extensively studied in the last years, with a relevant impact in application domains like communication systems and the modeling of neuronal and other biomedical signals and systems. Many recent papers have shown that chaotic synchronization is easily lost when there is additive noise in the link between master and slave. This lack of robustness can simply derail the use of chaos-based communication systems in non-ideal environments. In the present work we employ a bio-inspired optimization technique to increase the signal-to-noise-ratio of the chaotic signal that arrives in the slave node of a master-slave discrete-time network and we show that this technique can improve the robustness of the chaotic synchronization to noise.

Keywords: Synchronization, Optimization problems, Chaos, Noisy channels, Discrete-time systems

1. INTRODUCTION

Since Pecora and Carroll's seminal work (Pecora and Carroll, 1990), much has been written about the potential usefulness of chaotic synchronization in contexts like communication systems (Carroll and Pecora, 1991; Pecora and Carroll, 1991; Wu and Chua, 1993; Koh and Ushio, 1997; Rulkov and Tsimring, 1999; Baptista et al., 2000; Tôrres, 2007; Grzybowski et al., 2011) and modeling of biological and artificial neural networks (Faure and Korn, 2001; Tang et al., 2011; Qi et al., 2008)), as well as many other signal and systems (Strogatz, 2004).

In this paper, we focus on chaotic synchronization of a masterslave structure constituted by identical discrete-time multidimensional maps, where the slave system is driven by a signal derived from the master (Rulkov and Tsimring, 1999; Eisencraft et al., 2009). This set-up is a discrete-time version of the synchronization method proposed by Wu and Chua (1993). In this network, the synchronization error decays exponentially, being identical synchronization obtained after a few iterations (Eisencraft et al., 2009). However, when there is additive noise on the link between master and slave, the sensitive dependence on initial conditions that characterizes chaotic signals amplifies this error and synchronization is no longer obtained (Eisencraft and Batista, 2011; Eisencraft et al., 2011). This problem can have a significant practical impact: in chaos-based communication systems, for instance, it would lead to a decrease in que quality and/or in the rate of information exchange (Grzybowski et al., 2011).

Bearing this in mind, in the present work we numerically investigate an alternative to decrease the master-slave synchronization error when there is Additive White Gaussian Noise (AWGN) between master and slave: to use a denoising technique based on an evolutionary approach proposed by Soriano et al. (2011) to increase the signal-to-noise ratio (SNR) of the signal that arrives on the slave.

Evolutionary algorithms have been successfully employed in different contexts related to chaotic dynamics, including control, attractor reconstruction, synchronization, cryptography (Zelinka et al., 2010) and system identification(Gao et al., 2009). In this paper we take advantage of the flexibility of the global search potential of an algorithm of this class, an artificial immune system (de Castro and Timmis, 2002; de Castro, 2006), to find a trajectory that be as close as possible to the noiseless transmitted signal.

This paper is organized as follows: in Section 2, we present the considered master-slave models, and, in Section 3, we succinctly describe the employed denoising technique. In Section 4 some numerical results are iscussed and, finally, in Section 5, we expose the conclusions and perspectives for future works.

⋆ M.E. is partially supported by the Brazilian National Council for Scientific and Technological Development (CNPq).

2. DISCRETE-TIME CHAOTIC SYNCHRONIZATION

Wu and Chua (1993) address chaotic system synchronization differently from the seminal paper by Pecora and Carroll (1990). Instead of using conditional Lyapunov exponents to check the asymptotic stability of the slave system and hence the possibility of synchronism, Wu and Chua propose that the master and slave equations be written in such a way that the dynamics of the synchronization error is simple enough to permit the direct verification of its convergence to zero. The adaptation to discrete-time systems, proposed by Eisencraft et al. (2009), is succinctly described in the following.

Consider two discrete-time systems defined by

$$\mathbf{x}(n+1) = A\mathbf{x}(n) + \mathbf{b} + \mathbf{f}(\mathbf{x}(n)) \quad (1)$$

$$\mathbf{y}(n+1) = A\mathbf{y}(n) + \mathbf{b} + \mathbf{f}(\mathbf{x}(n)) \quad (2)$$

where $\{\mathbf{x}(n), \mathbf{y}(n)\} \subset \mathbb{R}^K$, $\mathbf{x}(n) = [x_1(n), \ldots, x_K(n)]^T$, $\mathbf{y}(n) = [y_1(n), \ldots, y_K(n)]^T$ and $n \in \mathbb{N}$. The real-valued matrix $A_{K\times K}$ and the vector $\mathbf{b}_{K\times 1}$ are constants. The function $\mathbf{f}(\cdot)$, $\mathbb{R}^K \to \mathbb{R}^K$ is nonlinear.

The system described by Eq. (1) is autonomous and is called *master*. The one described by (2) depends on $\mathbf{x}(n)$ and is called *slave*.

The dynamics of the synchronization error between the two systems $\mathbf{e}(n) = \mathbf{y}(n) - \mathbf{x}(n)$, in this case, is given by

$$\mathbf{e}(n+1) = A\mathbf{e}(n). \quad (3)$$

They are said *completely synchronized* if $\mathbf{e}(n) \to \mathbf{0}$ as n grows. Consequently, master and slave synchronize completely if the eigenvalues λ_i of A satisfy (Agarwal, 1992)

$$|\lambda_i| < 1, \; 1 \leq i \leq K. \quad (4)$$

Hence, if a system can be written as in Eq. (1) with A satisfying the condition defined in Eq. (4), it is easy to set up a slave system that synchronizes with it.

In our numerical examples we employ, in the role of chaos generator the two-dimensional Hénon map (Hénon, 1976)

$$x_1(n+1) = 1 - \alpha x_1^2(n) + x_2(n) \quad (5)$$

$$x_2(n+1) = \beta x_1(n). \quad (6)$$

where α and β are constants. In this case, the master can be written in the form of Eq. (1) with A as

$$A = \begin{bmatrix} 0 & 1 \\ \beta & 0 \end{bmatrix}, \quad (7)$$

$\mathbf{b} = [1; 0]$ and $\mathbf{f}(\mathbf{x}(n)) = \mathbf{f}(x_1(n)) = \left[-\alpha x_1^2(n); 0\right]^T$. Thus, only the scalar signal $x_1(n)$ must be transmitted from master to slave.

The eigenvalues of A are $\lambda_1 = -\lambda_2 = \sqrt{\beta}$. So there is master-slave synchronization whenever $|\beta| < 1$.

Now we consider that the signal that arrives at the slave is not $x_1(n)$ but $r(n)$, given by

$$r(n) = x_1(n) + w(n) \quad (8)$$

where $w(n)$ is a zero-mean AWGN process with power σ_w^2. In this case, the receiver is described by

$$y_1(n+1) = 1 - \alpha r^2(n) + y_2(n) \quad (9)$$

$$y_2(n+1) = \beta y_1(n). \quad (10)$$

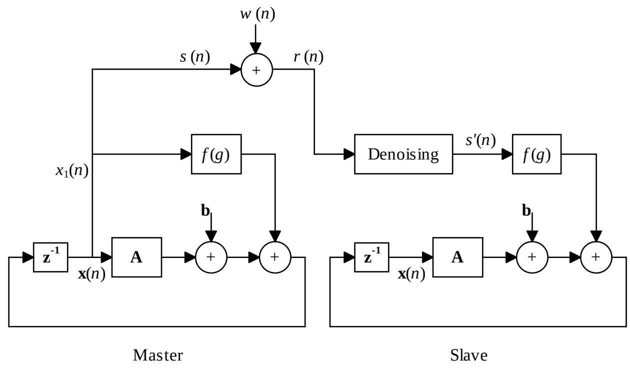

Fig. 1. Block diagram of the synchronization set.

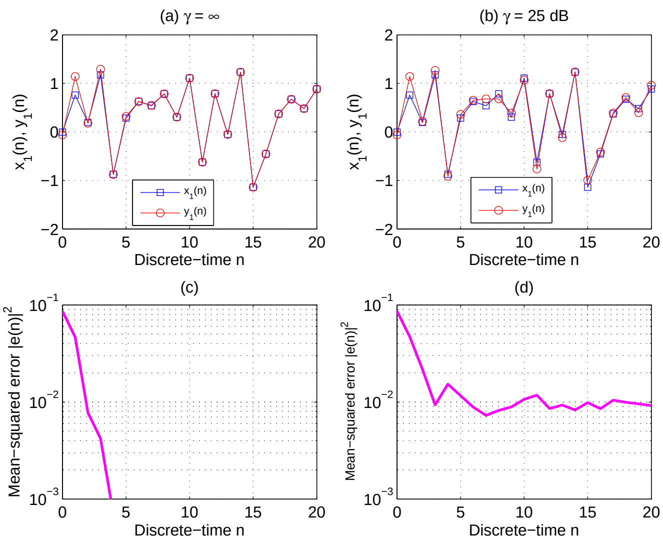

Fig. 2. Chaotic synchronization: Master and slave states $x_1(n)$, $y_1(n)$ and Mean-squared error $|\mathbf{e}(n)|^2$ for $\gamma = +\infty$ ((a) and (c)) and $\gamma = 15$ dB ((b) and (d))

This setup is represented in block diagram form on Figure 1.

Now, the synchronization error is not given by Eq. (3) anymore, and complete synchronization is no longer attainable. Figure 2 shows $x_1(n)$, $y_1(n)$ and $|\mathbf{e(n)}|^2$ for the noiseless case ($\sigma_w^2 = 0$) and for an SNR $\gamma = 15$dB. The SNR is defined here as

$$\gamma(dB) = 10 \log \left(\frac{\overline{s^2(n)}}{\sigma_w^2} \right) \quad (11)$$

where $\overline{s^2(n)}$ is the mean-squared value of $s(n)$, the transmitted signal.

As Figure 2 shows, for a noiseless channel ($\gamma = \infty$), complete synchronization is quickly attained. However, in this example, for a relatively high SNR of 15dB, which means that the noise power is approximately 3% of the power of $s(n)$, $|\mathbf{e}(n)|^2 \approx 10^{-1}$, which is approximately 17% of the mean value of $|\mathbf{y}(n)|^2$. Thus, a small additive noise in the transmitted signal can result in a significant higher synchronization error due to the feedback and intrinsic nonlinearity of the involved systems.

This nonlinear effect of channel noise in the synchronization error is certainly one of the factors responsible for the relatively poor performance of digital communication systems based on chaos coherent compared to their conventional counterparts in terms of bit error rate in AWGN channels (Williams, 2001).

In the next sections we apply the denoising approach proposed by (Soriano et al., 2011), succinctly described in Section 3, to increase the channel SNR and, consequently, reduce the synchronization error.

3. DENOISING OF CHAOTIC TIME SERIES USING AN EVOLUTIONARY APPROACH

Under the premise that the system model $F(\mathbf{x}, \mathbf{p})$ underlying the observed signal is known, but not its free parameter vector $\mathbf{p}$ and initial condition $\mathbf{x}_0$, it is possible to obtain a denoising method based on a trajectory identification procedure (Soriano et al., 2011).

In order to reduce the noise in the observed signal one could try to estimate the true parameters and initial conditions of the system, which would allow a perfect reconstruction of the deterministic orbit, thus eliminating the noise. However, it is well known that any attempt to estimate the initial condition would lead to a deterministic orbit that inevitably exponentially diverges from the observed one, given the finite precision of the estimate of the initial state and the chaotic nature of the system.

Therefore, in (Soriano et al., 2011), the authors proposed a piecewise estimation, i.e., for each segment of N_s samples of the observed signal, a set of parameters and initial conditions is obtained and an orbit close to the true one is reconstructed. Mathematically, the task consists in the following minimization problem:

$$\arg\min_{\mathbf{x}_0,\mathbf{p}}\{f_{\text{score}}(\mathbf{x}_0,\mathbf{p}) = [\frac{1}{N_s}\sum_{n=1}^{N_s}(s'(n) - r(n))^2]\} \quad (12)$$

where $\mathbf{x}_0$ and $\mathbf{p}$ are, for the Hénon map, $\mathbf{x}_0 = [x_1(0)\ x_2(0)]^T$ and $\mathbf{p} = [\alpha\ \beta]^T$). In simple terms, the idea is to minimize the score function $f_{\text{score}}(\cdot)$ given by the mean-squared error between a candidate chaotic time series solution $s'(n)$ (dependent on the optimization parameters) and the observed vector $r(n)$. The candidate solution $s'(n)$ is obtained by solving (iterating or integrating) $\mathbf{F}(\mathbf{x}, \mathbf{p})$ given the parameter vector $\mathbf{p}$ and initial condition $\mathbf{x}_0$.

In order to optmize the cost function presented in (12), the authors employ a search heuristic that uses the immune system as an inspiration, an optimization approach that has a significant global search potential (de Castro and Timmis, 2002; de Castro, 2006), explained in the following.

3.1 Optimization by Artificial Immune System

Artificial immune systems (AIS) correspond to a bio-inspired computational strategy that uses concepts derived from the study of immune systems of superior organisms (de Castro and Timmis, 2002; de Castro, 2006). It has several applications to engineering problems, and is particularly useful for optimization, presenting a good performance in terms of global convergence rate even in problems with significant multimodality.

Among the various AIS employed for solving optimization tasks, we decided to employ, in this work, a version of the CLONALG algorithm (de Castro and Zuben, 2002) adapted to operate with real coding. This choice was motivated by two features of this tool: 1) a relatively simple modus operandi and s) an interesting balance between local and global search mechanisms.

The modus operandi of the CLONALG is based on two conceptual pillars: clonal selection and affinity maturation (de Castro and Timmis, 2002). The clonal selection principle establishes that, when an organism is invaded by antigens (e.g. virus or bacteria), specific cells of the immune system recognize the exogenous element and are selected to proliferate, which gives rise to a cloning process with rates proportional to the affinity - defined by some measure of recognition - of these cells to the antigens. In the affinity maturation process, the individuals produced in the current generation can exhibit mutations with rates inversely proportional to their affinity with the antigens, and the mutated generation can eventually present individuals with higher affinity (de Castro and Timmis, 2002; de Castro and Zuben, 2002).

To effectively convert these ideas into an efficient optimization algorithm, it is necessary to make some considerations. First, each candidate solution to the optimization problem corresponds to an individual, that is, a vector of real values that represents, in simple terms, the structure of an immune cell. The quality of an individual (called here fitness measure) is defined by the cost function (with the caveat that it is necessary to convert minimization into maximization), providing means of quantifying the antibody-antigen affinity (de Castro and Timmis, 2002; de Castro and Zuben, 2002).

Finally, there is also a periodic insertion of new randomly-generated individuals to replace individuals with poor fitness in order to perform a better exploration of the search space. These steps can be summarized in the CLONALG (de Castro and Zuben, 2002) pseudo-code shown in Algorithm 1.

In our simulations, after a number of preliminary tests, we decided to use $N = 50$ individuals and $N_c = 20$ clones. An adaptive value of η - which represents the mutation rate - was used. η was initially defined as 1 and, after half of the total number of generations (which was set in 4000), it linearly increased until a final value of 1000 was reached. It is also valuable to remark that the algorithm parameters (as N, N_c, η, number of generations) define its performance and some numerical experimentation is recommended before setting these parameters (for more details, see (Soriano et al., 2011)). In this case, some "thumb-rules" or heuristic settings can be applied to perform a reasonable search, which consists in one of the major issues in bio-inspired optimization (de Castro and Timmis, 2002).

It is important to note that the CLONALG algorithm performs the maximization of a fitness measure (defined as J_{FIT}) and not the direct minimization of the cost function presented in Eq. (12). Hence, we have used the following relation between the cost function and the fitness measure to be maximized, Eq. (14):

$$J_{FIT} = \frac{1}{f_{\text{score}}(\mathbf{x}_0, \mathbf{p})} \quad (14)$$

4. SIMULATION RESULTS

In order to assess the possible gain of employing a denoising technique in the synchronization of two dynamical systems, we considered the scenario depicted in Fig. 1. For an observation vector with 200 samples obtained from the Hénon map, we have applied the denoising methodology described in Section 3, considering an estimation window of $N_s = 10$ samples. Once a denoised version $s'(n)$ of the received signal is obtained, it is used as the input of the slave system, as indicated in Fig. 1. The synchronization error is then evaluated, and is defined by

$$\text{SyncError} = \frac{1}{180} \sum_{n=20}^{200} ||\mathbf{x}(n) - \mathbf{y}(n)||^2 \quad (15)$$

where $\mathbf{x}(n)$ and $\mathbf{y}(n)$ denote, respectively, the state vectors of the master system and slave system. The summation starts with $n = 20$ in order to discard the transient in the synchronization process.

As mentioned earlier, the denoising procedure employs a piecewise estimation of the received signal, thus obtaining a set of parameters and initial conditions for each estimation window. Even though it would not be possible to perfectly estimate the parameters and the initial conditions due to the chaotic nature of the system and the noise, the obtained parameters fluctuate around the true ones used

[CLONALG]

Algorithm 1. Pseudo-code for CLONALG algorithm
(1) Randomly initialize the population (N);
(2) Determine the fitness of each individual: J_{FIT};
(3) While the maximum number of generation is not attained, do
  (a) Create N_c clones for each individual;
  (b) Keep the original individual, and apply a mutation process for each clone as described in Eq. (13):

$$c' = c + \epsilon Y(0,1) \quad (13)$$
$$\epsilon = \frac{1}{\eta} exp(-J_{FIT})$$

  where c' and c represent the clones modified by mutation and the original one, respectively. $Y(0,1)$ represents Gaussian random variable with zero mean and unitary variance and η represents a control parameter to establish the applied mutation;
  (c) Evaluate the fitness of each individual of the population and keep in the population only the best solution of each group given by the individual and its derived mutated clones;
  (d) At each t generations, eliminate the m elements with lowest fitness and substitute them by randomly generated individuals;
(4) Return to the step 3.

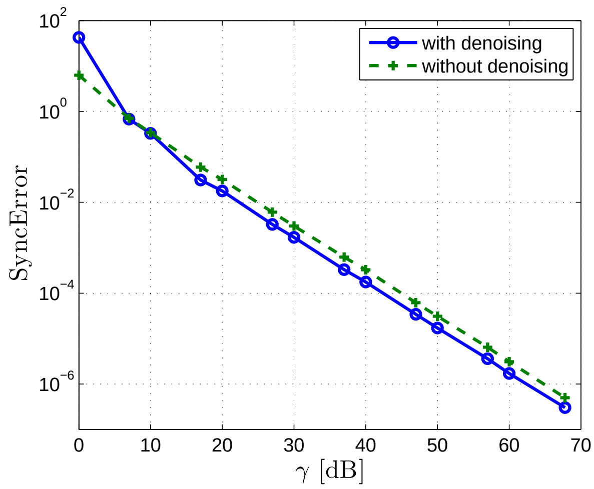

Fig. 3. Synchronization MSE for different values of γ.

to generate the transmitted signal – $\alpha = 1.4$ and $\beta = 0.3$, as indicated in Fig. 4, which depicts the evolution of the estimated parameters along the various segments of the signal. As a matter of fact, this observation was used to increase the convergence speed of the algorithm. Instead of randomly initialize all parameters for each estimation window, the estimated parameter values for one data window were used as the initial values for the parameters in the subsequent estimation window.

Fig. 3 shows the synchronization error for different values of γ, and it becomes clear that the denoising procedure is effective for a wide range of noise levels ($\gamma >$ 10dB). In other words, it means that the CLONALG algorithm was able to obtain a set of parameters that allowed a very reasonable reconstruction of the original trajectory, thus enhancing the synchronization performance of the slave system. For this range of SNR, it can be noted that the denoising method exhibit a $3dB$ gain with respect to the non-processed data, i.e., it is possible to achieve the same synchronization error with an SNR $3dB$ lower, which represents a significant improvement.

For low γ values, it was not possible to obtain the same performance level achieved for higher SNR. This limitation is partially explained by the fact that the CLONALG parameters were kept the same for all SNR values - in a practical situation, if we know in advance that the SNR is not so high, these parameters could be adjusted and a better synchronization error would be obtained.

5. CONCLUSIONS

In this work we employ a bio-inspired optimization technique to increase the signal-to-noise-ratio of the chaotic signal that arrives in the slave node of a master-slave discrete-time network. The simulation results show that, for a wide range of SNR values, the proposed methodology is able to increase the channel SNR, thus reducing the synchronization error of the slave system. Moreover, despite of the computational cost of the employed optimization approach, we hope that the advent of greater computer resources and the development of more efficient optimization techniques can provide a promise real time signal pro-

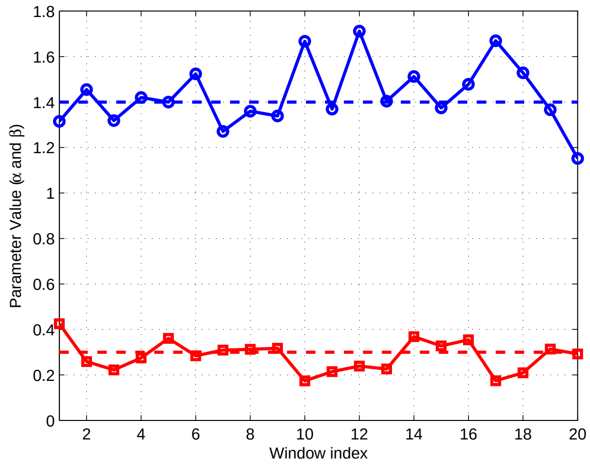

Fig. 4. Evolution of the system parameters – α and β – among the estimation windows.

cessing environment for evolutionary chaotic time series denoising.

This encouraging results indicate many possibilities to be explored in future works, which include the evaluation of the methodology for different dynamical systems, and its application in chaos-based communication systems with the aim of understanding how the denoising procedure would affect the bit error rate (BER) of the system.

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