FEEDBACK CONTROL SYSTEM WITH AN ADDITIVE DISTURBANCE IN THE CONTEXT OF BIOLOGICAL SCIENCES

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Resumo— Nos cursos introdutórios de Engenharia, a análise de sistemas de controle em malha fechada com perturbação aditiva geralmente envolve dispositivos eletromecânicos. Tais sistemas de controle merecem ser estudados pela sua própria importância, mas o significado dessa análise pode-se enfatizada, por meio de alguns exemplos fora dos domíniosda Engenharia pura. Nesse artigo, propoe-se um modelo simplificado do sistema termorregulador de aves e mamíferos que é responsável pelo controle de temperatura, de modo a mostrar como os conceitos-chave de sistemas em malha fechada podem emergir em um contexto diferente. As nossas expectativas são melhorar e dar caminhos alternativos para o estudo deste tipo de sistemas de controle.

Abstract— In introductory engineering courses, the analysis of closed-loop control systems with additive disturbance usually involves electromechanical devices. Such control systems deserve to be studied by their own merits, but the significance of this analysis might be highlighted by means of some examples outside the pure engineer domains. Here we propose a simplified model of the avian/mammalian thermoregulatory system responsible for temperature control in order to show how the key concepts of closed-loop systems could emerge in a different context. Our expectations are to improve and to give alternative paths for the study of this type of control systems.

Keywords— Dynamical Systems Applications, Thermoregulatory Model, Disturbance Analysis, Closed-loop Control Systems

1 Introduction

In introductory courses on classical control theory, students learn that closed-loop systems are less sensitive to perturbation than the corresponding open-loop ones. Such a conclusion is usually drawn by analyzing, for instance, a feedback control system with an additive disturbance, as shown in Fig. 1 (e.g. (Dutton et al., 1997; Franklin et al., 2005; de Klerk and Craig, 2004; Kuperman and Rabinovici, 2005)). In such a scheme, the controlled variable $C(s)$ of the process is measured by the sensor $H(s)$ and compared with the reference $R(s)$. The difference $R(s) - H(s)C(s)$, called system error $E(s)$, feeds the compensator $G(s)$, and the compensator output is the signal actuating on the process. The steady-state error is affected by $R(s)$ and by the external disturbance $D(s)$ added to the process output.

In undergraduate engineering classes, the block diagram shown in Figure 1 usually involves electromechanical devices, like satellite attitude control systems (e.g. (Franklin et al., 2005)) or electric motors (e.g. (Kuperman and Rabinovici, 2005)). In order to enrich the presentation of possible systems of this type in a different context, here we derive it for a simplified model of the thermoregulatory system responsible for temperature control in birds and mammals. Even though we focus in some aspects of a biological temperature control system, there is a vast area of different purposes related to this matter. For instance, the design of temperature control systems is obviously important in industrial applications (e.g. (de Miranda Montenegro et al., 2006; Salau et al., 2005)).

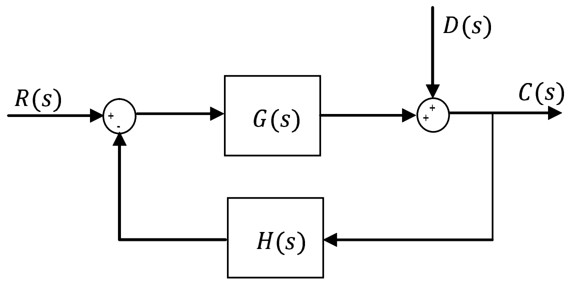

Figure 1: The block diagram represents a generalized feedback control system with additive disturbance $D(s)$. The controlled variable $C(s)$ of the process is required to track the reference input $R(s)$. The tracking error $E(s) = R(s) - H(s)C(s)$ is the input to the plant.

Our biological model follows from Newton's law of cooling, which states that the rate of energy exchange due to heat transfer of a body is proportional to the difference between body temperature and surrounding (ambient) temperature. It incorporates the physiological feature of those animals that, in their majority, rely on metabolic energy conversion for the maintenance (regulation) of a somewhat limited range of body temperature throughout the day. These two features, i.e., metabolic energy conversion for thermoregulation and a limited range of body temperature, are named endothermy and homeothermy, respectively.

The proposed thermoregulatory model is described in Section 2 and its behavior is analytically determined in Section 3. In Section 4, the main conclusions are presented.

2 A Thermoregulatory Model

A very basic fact about body temperature is that it represents, at any given time t, a balance between the amount of energy received or produced by an animal and the energy lost to the surroundings, either actively or passively.

In the case of endothermic animals, body temperature $T_b(t)$ is actively regulated by the control of both energy production – by shivering and non-shivering thermogenesis – and energy loss – by mechanisms of thermal exchange, as changes in posture, peripheral blood perfusion and evaporative water loss. Notice that we will not discuss T_b long-term regulation related to changes in fur or feathers, dermal fat thickness etc. Also, body temperature control in mammals resides, mainly, in hypothalamic centers, while in birds there are other peripheral controllers; therefore, we will refer simply to a "temperature controller" in a general sense in the text.

In this way, for an endotherm to maintain a constant body temperature (homeothermy), energy production must equal energy loss by heat. Without loss of generality in the present context, we will assume that $T_b(t)$ is homogeneous throughout the organism. Then, a simple model of body temperature variation would be given as the difference between $T_b(t)$ and environmental temperature $T_a(t)$ generating an energy loss to the surroundings proportional to the thermal conductance[1] χ (J^o C s^{-1}); the difference between such a loss and the energy input represented by the metabolic rate M (J s^{-1}) results in a $T_b(t)$ (^{o}C) variation inversely proportional to the product of body mass B (g) and the mean specific heat of tissues C (J g^{-1o}C^{-1}) (Cooper, 2002; Chaui-Berlinck et al., 2005). Mathematically we could write:

$$\frac{dT_b(t)}{dt} = \frac{1}{BC}\left(hM(t) - \chi\left(T_b(t) - T_a(t)\right)\right). \quad (1)$$

The dimensionless coefficient h is the inefficiency in metabolic energy conversion, thus representing the amount of energy available to increase body temperature, $0 < h \leq 1$. Observe that $1-h$ is the useful work done by the system, i.e., the organism, in the surroundings.

In order to regulate body temperature, metabolic rate M increases as the difference between a set-point and the core temperature increases, so the time variation of metabolic rate can be expressed, in our minimalist linear model, as:

$$\frac{dM(t)}{dt} = K\left(T_s(t) - T_b(t)\right) \quad (2)$$

where K is the gain in the closed control loop and T_s is the set-point temperature of the organism.

3 Analytical Results

3.1 Stability analysis

Let $T_a(t)$ and $T_s(t)$ be constants. The eigenvalues λ of linear system (1)-(2) are obtained from (Monteiro, 2011):

$$\det\begin{pmatrix} -\frac{\chi}{BC} - \lambda & \frac{h}{BC} \\ -K & 0 - \lambda \end{pmatrix} = 0. \quad (3)$$

Thus, linear system (1)-(2) has the characteristic polynomial:

$$\lambda^2 + \frac{\chi}{BC}\lambda + \frac{Kh}{BC} = 0 \quad (4)$$

which has the roots:

$$\lambda_{1,2} = \frac{-\chi \pm \sqrt{\chi^2 - 4KhBC}}{2BC}. \quad (5)$$

Given that the parameters χ, B, C and h in Eq. (1) are always positive numbers, then both eigenvalues in (5) have negative real parts (that is, $\mathrm{Re}(\lambda_1) < 0$ and $\mathrm{Re}(\lambda_2) < 0$). Therefore, the system (1)-(2) is always asymptotically stable and the way $T_b(t)$ tends to T_s depends only on the signal of $\chi^2 - 4KhBC$: if $\chi^2 - 4KhBC < 0$, then the system is underdamped (thus, $T_b(t)$ tends to T_s by oscillating around T_s); if $\chi^2 - 4KhBC > 0$, then it is overdamped ($T_b(t)$ exponentially tends to T_s); if $\chi^2 - 4KhBC = 0$, it is critically damped (this is the fastest way that $T_b(t)$ can converge to T_s without overshoot, which is typical of underdamped systems).

3.2 Disturbance Analysis

By applying Laplace transform to Eqs. (1) and (2), we obtain the following system in the s-

[1] Representing a general term of thermal conductance that may occur by conduction, convection, radiation and water evaporation.

domain:

$$T_b(s) = \left(\frac{h}{BCs+\chi}\right) M(s) + \left(\frac{\chi}{BCs+\chi}\right) T_a(s) \quad (6)$$

$$M(s) = \frac{K}{s}\left(T_s(s) - T_b(s)\right). \quad (7)$$

It is represented by the block diagram in Fig. 2.

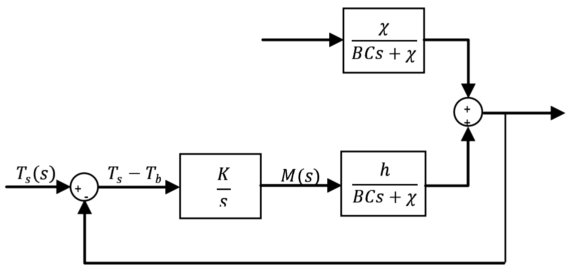

Figure 2: Block diagram of the body temperature control system of Eqs.(6) and (7). The model represents a feedback closed-loop system of body temperature influenced by environmental temperature as an input error.

The system (6)-(7) is clearly a closed-loop system, and it is easy to check, by the block diagram (Fig. 2), that variations in environmental temperature are compensated by changes in metabolic rate in order to maintain T_b near T_s.

By carrying out some algebraic manipulation in (6) and (7), we obtain:

$$T_b(s) = \frac{Kh}{BCs^2+\chi s+Kh} T_s(s) + \frac{\chi s}{BCs^2+\chi s+Kh} T_a(s). \quad (8)$$

In a closed-loop system, negative feedback triggers a correcting response that opposes deviations from the set-point. So, negative feedback is the main attribute to a closed-loop system to act as a regulatory system.

The metabolic control is given by the difference between body temperature and body temperature set-point, representing the closure in the negative feedback system. Therefore, by Eq (1), if the organism is in energy equilibrium (energy input equal to energy output), at a given time t, then the following identity applies:

$$M(t) = \frac{K}{h}\left(T_b(t) - T_a(t)\right). \quad (9)$$

If we call the effects of ambient temperature as disturbances in the model, we can, generically, analyze the thermoregulation model (Eq (1) and (2)) and see how these disturbances affect $T_b(t)$.

The generic closed loop model showed in Fig. 1 can be described by:

$$C(s) = \frac{G(s)}{1+H(s)G(s)} R(s) + \frac{1}{1+H(s)G(s)} D(s). \quad (10)$$

The thermoregulatory model in equation (8) has, exactly, this form.

In face of this, next we show that, besides stabilization of body temperature at the set-point, the thermoregulatory system is also capable of reducing disturbances induced by $T_a(s)$.

By applying Laplace transform to the set-point $T_s(t) = S$ and to a fixed ambient temperature $T_a(t) = A$ (where S and A are constants) leads to:

$$T_s(s) = \frac{S}{s} \quad (11)$$

and:

$$T_a(s) = \frac{A}{s} \quad (12)$$

Thus, our thermoregulatory model could now be written as:

$$T_b(s) = \left(\frac{Kh}{BCs^2+\chi s+Kh}\right)\frac{S}{s} + \left(\frac{\chi s}{BCs^2+\chi s+Kh}\right)\frac{A}{s}. \quad (13)$$

The error $E(s)$ between set-point (11) and body temperature (13) is given by:

$$E(s) = T_s(s) - T_b(s). \quad (14)$$

Therefore:

$$E(s) = \left(\frac{BCs^2+\chi s}{BCs^2+\chi s+Kh}\right)\frac{S}{s} - \left(\frac{\chi s}{BCs^2+\chi s+Kh}\right)\frac{A}{s}. \quad (15)$$

According to the Final Value Theorem for Laplace transforms (e.g. (Nise, 2006)) as long as $E(s)$ does not have more than one pole at the origin and any poles in the right half of the complex plane, then the steady-state error e_{ss} has as limiting value:

$$e_{ss} = \lim_{s\to 0}[sE(s)]. \quad (16)$$

In the present case, by using the error expressed by (15), then:

$$e_{ss} = \lim_{s\to 0}\left[\left(\frac{BCs^2+\chi s}{BCs^2+\chi s+Kh}\right) S - \left(\frac{\chi s}{BCs^2+\chi s+Kh}\right) A\right]. \quad (17)$$

This way:

$$e_{ss} = 0. \quad (18)$$

That is, body temperature tends to the set-point value of the controller.

Now, let us analyze the case when ambient temperature is not fixed. Our model, given by Eq. (8), can be written as:

$$BCs^2 T_b(s) + \chi s T_b(s) + KhT_b(s) = KhT_s(s) + \chi s T_a(s). \quad (19)$$

By considering null initial conditions and applying the anti-Laplace transform, this expression becomes:

$$BC\ddot{T}_b(t) + \chi\dot{T}_b(t) + KhT_b(t) = KhT_s + \chi\dot{T}_a(t) \quad (20)$$

where the dot means derivative with respect to the time.

In steady-state of T_b, i.e., when $\dot{T}_b(t) = 0$ and $\ddot{T}_b(t) = 0$, we have:

$$T_b(t) = T_s + \frac{\chi}{Kh}\dot{T}_a(t). \quad (21)$$

Therefore, in the cases where disturbances in ambient temperature are present, expression (21) shows us that such fluctuations are attenuated in systems that have a reduced thermal conductance χ or an increased product Kh.

Numerical simulations of the thermoregulatory model, presented in Fig. 3, show how the reduction in χ affects the temporal behavior of body temperature in the presence of ambient temperature oscillations.

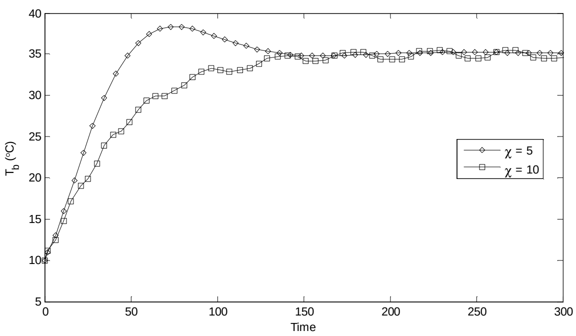

Figure 3: Body temperature under influence of oscillating environmental temperature. As expected, a reduction on thermal conductance causes a reduction on the effects of external disturbances. Values used in the simulations are $C = 1$, $B = 50$, $K = 0.5$, $h = 0.4$ and set-point temperature is 35 ^{o}C. Environmental temperature is given by $T_a(t) = 10 + 5\sin(t)$.

4 Conclusion

In the present work, we developed a model not usually employed in undergraduate courses to explore some properties of feedback control systems. We showed how a simple thermoregulatory model can be used as a good example of such control systems with an additive disturbance. This example provides an alternative context to closed-loop systems in general and, also, more particularly, of feedback control systems where many key concepts, as set-point and steady state error, emerge almost naturally due to the biological processes themselves.

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