Chapter 7
Chaos-Based Communication Systems: Current Trends and Challenges

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7.1 Introduction

Much research has been devoted to synchronization and chaos-based communication in the last two decades or so. Since the early 1990s, many papers have addressed the subject, and much progress has been made. Considering that the research has been developed in many different – and, at certain extent, complementary – directions, it would be interesting to outline the advances and establish the bottom line to forthcoming research on the topic.

Chaos-based communication schemes, as originally conceived, use synchronization of chaotic oscillators as a backbone. It means that, under certain circumstances, the complex and highly sensitive nonlinear dynamics of coupled chaotic oscillators can synchronize, and such synchronous state can be exploited in several different manners to allow communication. Thus, to introduce this brief report, we attempt to tie synchronization and chaos-based communication together to make it clear how they are related. Ahead, the topic is briefly situated in the context of its evolution over time and the state of the art and challenges of the technique for future researches are outlined.

In the first place, concerning communication in general, what is the role of synchronization? In schemes based on coherent detection, synchronization enables *carrier recovery* and *timing recovery* at the receiver's end [43]. Carrier recovery refers to the reproduction or recovery, at the receiver's end, of the carrier signal produced in the transmitter. Once transmitter and receiver oscillators are matched,

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coherent demodulation of the modulated baseband signal is possible. On its turn, timing recovery refers to the need that both coherent and noncoherent receivers have to know the exact time and duration of each received symbol in a stream, in order to be able to assign decision times and reset the initial conditions of the correlator [43]. When mentioning *chaos synchronization* we mean a special type of carrier recovery.

In the second place, why use chaotic rather than sinusoidal carriers in communication systems? When a sinusoidal carrier is used to transmit information, the spectral power density concentrates in a narrow band of frequencies. Chaotic signals, on their turn, can occupy a large bandwidth, their autocorrelations can be impulsive and the cross-correlations between signals generated by different initial conditions present low values [15, 22, 25, 37, 38]. These characteristics have been behind the rationale for using chaotic signals as candidates for spreading information signals. When chaotic signals modulate independent narrowband sources increased bandwidths result with lower power spectral density levels in a fashion similar to what happens in Spread Spectrum (SS) systems [51]. Consequently, chaos-based and SS systems share several properties, namely (i) they are difficult to intercept by any unauthorized user; (ii) they are easily hidden from any unauthorized receiver and, in many cases, it is difficult even to detect their presence; (iii) they are resistant to jamming; and (iv) they provide a measure of immunity to distortion due to multipath propagation [51]. Figure 7.1 shows examples of periodic and chaotic carriers.

Apart from these reasons, chaos features some highly desirable characteristics from the point of view of communication, as it will be pointed in the sections ahead.

7.1.1 Context

The original idea of using chaos to transmit information was possible due to a couple of seminal publications on chaos control and synchronization in the year 1990.

Ott et al. [66] established that it is possible to control chaos to unstable periodic orbits or to fixed points by using small perturbations. They showed that, due to the property of transitivity of the chaotic motion in its attractive set, small perturbations are sufficient to render chaotic regime controlled. Subsequently, Hayes et al. showed both theoretically [33] and experimentally [34], how this idea could be used to make a chaotic signal bear information and so the idea of "communication with chaos" was introduced. Figure 7.2 shows an example of chaos synchronization using the Ott-Grebogi-Yorke (OGY) method for the Lorenz system [57].

Meanwhile, Pecora and Carroll [71] gave birth to the theoretical fundamentals of chaos synchronization by presenting a framework for determination of the stability of synchronization of a pair of identical coupled chaotic systems by means of the calculation of conditional Lyapunov exponents.

From these publications, a whole new perspective emerged in the study of nonlinear dynamics, especially chaos. The possibility of controlling and synchronizing chaos gave rise to the proposal of several practical applications, especially for communication.

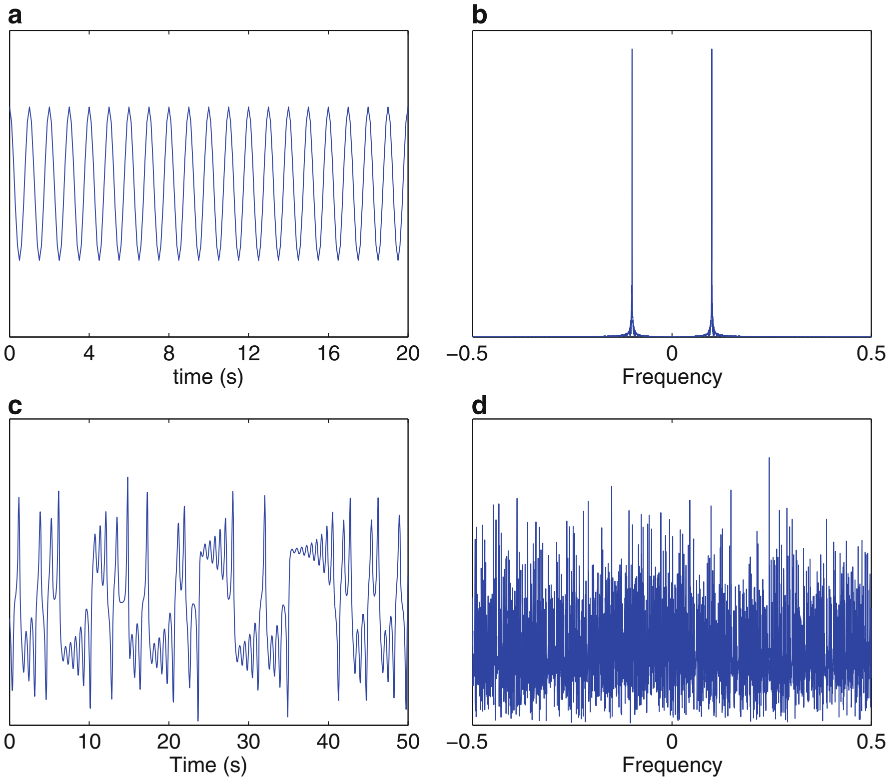

Fig. 7.1 The use of chaos as a carrier wave instead of a sinusoidal wave: the basic principle behind chaos-based communication; examples of a sinusoidal narrowband carrier in (**a**) time and (**b**) frequency domain and of a chaotic broadband carrier in (**c**) time and (**d**) frequency domain

Given that chaos became an important research topic, it did not take long until the first full chaos-based communication scheme was proposed and demonstrated experimentally by Cuomo and Oppenheim [13]. The idea was simple: to exploit the stable master-slave synchronous behavior between a pair of chaotic systems to encode, transmit and decode information in the chaotic evolution. The coupling signal consisted of a state variable of the master system, added with a low-power message. When adequately used as an input in the slave system, the chaotic waveform not only enabled systems to realize synchronization, but it also allowed the recovery of the information hidden in the chaotic dynamics. This system is known as *chaotic masking* and its block diagram is shown in Fig. 7.3.

Although the original idea itself was simple, it promoted insight into a remarkable underlying phenomenon: the slave system would synchronize only with the chaotic part of the input signal, regardless that some amount of noise and a low-power message were also present. Later on, this property became known as

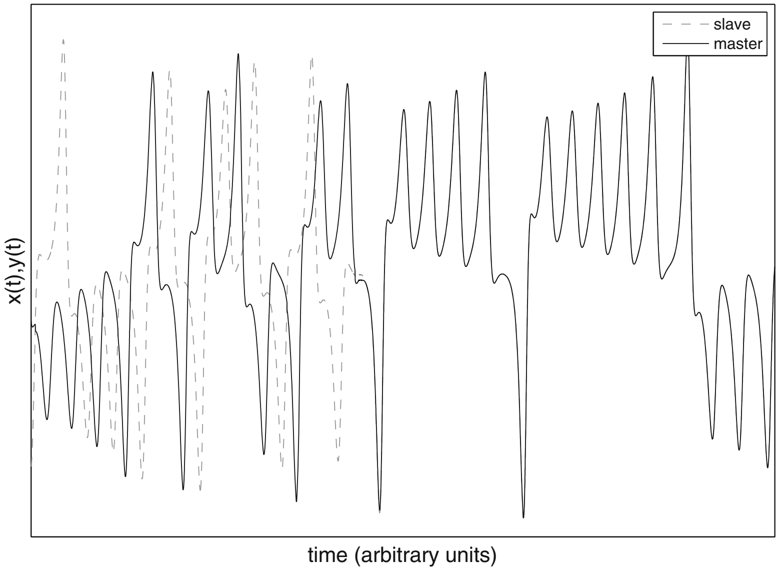

Fig. 7.2 Chaos under control: Lorenz systems get in synchrony by means of the OGY method [66]

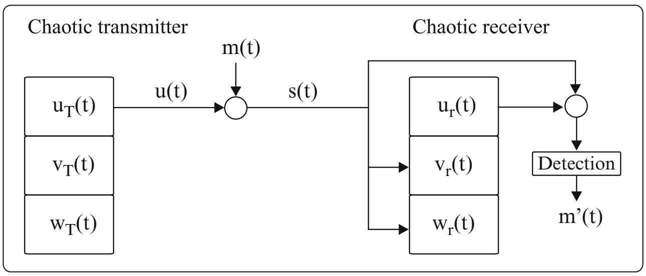

Fig. 7.3 Chaos communication design based on the Cuomo and Oppenheim [13] original scheme

chaos-pass filtering, that is, the property of synchronous systems to discard the non-chaotic part of the signal, which allows the message to be separated from the chaotic carrier signal [64, 70].

In the same year, Wu and Chua [87] proposed the *chaotic modulation*, which consists of the injection of the information signal in the equations of the chaotic system. The injection functions as a perturbation that alters the dynamics of the transmitter such that the chaotic signal itself contains information. At the receiver's end, by means of synchronization, the receiver is capable of regenerating the corresponding unperturbed chaotic signal and information retrieval follows by comparing the received perturbed signal and the generated unperturbed signal.

About ten years had passed since the first publications on chaos synchronization when another counterintuitive realization was made in delay-coupled chaotic systems subject to delayed self-feedback: a slave system can anticipate the trajectory of its master, what generates an emergent behavior that was named *anticipating synchronization*. According to Voss [83], such phenomena is nontrivial and of universal nature, since its underlying mechanism consists of the interplay between dissipation and memory. As such, anticipating synchronization can be exploited for communication purposes between remote oscillators or even for fast prediction, that is, state prediction without use of computation [83].

Many other possibilities of applications of chaos, ranging from digital and analog modulation to pseudorandom sequences generation and watermarking were proposed [37,38,77,81]. Chaos has also been shown in connection to devices used in signal processing such as nonlinear adaptive filters and phase-locked loop networks [23, 31, 63, 80].

As studies evolved and more elaborated scenarios were set to study chaos synchronization, time delays were introduced due to the fact that they are almost surely present in any real systems and, as a consequence, theoretical results with technological significance have to consider their influence. Besides, the realization that time delays in the feedback loop allow the emergence of chaos in laser emitters and also have potential to highly enhance chaos complexity has led to intensive study of delay differential equations, in order to precisely model and understand the emergence of chaos in such systems [24, 30].

As a result, the synchronization problem gained more elements, which allowed the emergence of unknown phenomena. A particularly interesting one resulted from the study of mutually delay-coupled identical chaotic oscillators, which could realize zero-lag synchronization despite of coupling delay. Such phenomenon is named *isochronal synchronization*. It was successfully tested for simultaneous chaos-based communication using identical mutually delay-coupled chaotic systems [40,82,84].

Since the first publications on chaos communication, much effort has been deployed towards the development of techniques and physical devices that can support and sustain efficient and cost-effective chaos-based communication systems. The efficiency aspect is mainly related to (i) the intensive exploitation of the chaotic dynamics to encode information, (ii) the low-power signal needed to control chaotic dynamics and (iii) the possibility of source coding, channel coding and encryption to be performed all at once in the same process [7]. The cost-effective aspect is related to (i) the simplicity of chaos-based communication setups [7], (ii) the low power-consuming devices needed to generate and control high-power chaotic signals and (iii) the use of all nonlinear operation excursion of electronic and optical components, which avoids the rather complicated and energy-consuming measures to avoid nonlinearities in the generation of the sinusoidal wave signals that serve as carriers in traditional communication schemes.

Nevertheless, desirable characteristics of chaos are not limited to the ones cited so far. Increased robustness against noise, parameter mismatch and multipath fading have been reported [7, 8, 56, 88], and consist of important points in favor of chaos-based communication schemes. Besides, recent results estimate that some chaos-based optical communication schemes can bear 15% more users in multi-user communication schemes for the same Bit Error Rate (BER) [55].

On the other hand, analytical upper limits in the BER performance for chaos-based communication schemes were found to be considerably lower than the performance of their conventional counterparts [37, 42, 46, 86]. This somewhat comes as a practical drawback since it diminishes the efficiency of information transmission.

As a result of such good and bad aspects, chaos-based communication is still a controversial subject. The main question is whether the overall performance of chaos-based communication schemes can surpass the performance of their conventional counterparts. This more concrete analysis would come as a result of the fair development of the various topics involving the practical implementation of chaos communication. So far, although it is not possible to answer that question in a broader sense, recent results indicate that there are concrete specific applications in which chaos-based communication can perform quite well, indeed [61, 69, 73].

As an imminently interdisciplinary matter, chaos-based communication has been investigated by physicists, engineers and mathematicians, among others. The comprehensive theory of chaos-based communication involves topics such as Nonlinear Dynamics and Chaos, Chaos Control and Synchronization, Lyapunov Stability of Motion Theory, Optics, Electronics, Telecommunication Engineering, among others. As a result of the contributions of different fields, the topic has evolved towards different and mutually complementary directions. Of course, in some of them the results have advanced more than in the others, and the consequence is that the state-of-the-art of chaos-based communication is hard to define if not in terms of its constituent individual matters. It is true that chaos-communication has been successfully tested in real environments, and a special reference should be made to a field experiment in Greece [4], probably the most relevant one up to date. But it is also true that not even in such experiment chaos-based communication was taken to the edge of the full possibilities it theoretically allows.

7.1.2 Advantages and Disadvantages of Chaos Communication

Is it worth investing so much time and energy in the research of chaos-based communication systems once conventional systems are managing perfectly? That seems to be the most fundamental and obvious question, as we watch the rise of faster and faster communication systems based on conventional techniques. The answer is not obvious, though. Chaos-based communication systems have unveiled a whole set of new possibilities, while at the same time they have been proven not to be able to reach the same BER performance as conventional systems [42, 86]. As such, the answer for this question may be obtained by pondering the advantages and disadvantages concerning the specific application that is desired.

At this point of evolution of hardware devices, such as lasers and electronic components, there are cases when the use of conventional sinusoidal waveforms to carry information can itself be seen as drawback. While optical devices are known to feature chaotic regime for a wide range of its parameters, the exclusive use of sinusoidal waves may consist of a waste of energy, due to the need for rather complicated measures to maintain linearity in inherently nonlinear systems. Besides, as lasers and optical fiber technologies are fairly developed, the use of their capacity to the fullest implies the efficient use of the wide range of signals that can be generated through their use. Indeed, the fact that both laser and electronic devices have much larger parameter range operation in chaotic regime than in periodic regime per se magnifies the capacity of generating broadband carrier signals.

While in conventional SS communication systems the broadband signals are generated using pseudorandom sequences to spread signals in frequency, in chaos-based schemes the bare fact that a chaos-generating device is running is enough to generate broadband signals [53, 79]. Typically, such broadband signal features a dense set of unstable periodic orbits within a range of frequencies with flat power spectral density. In practice, it means that chaotic systems make use of the input energy to generate broadband signals.

It is well known that arbitrarily nearby trajectories of chaotic systems exponentially diverge in the time evolution, due to the hallmark property of exponential sensitivity to initial conditions [2]. As a consequence, given two chaotic signals started in arbitrarily close initial conditions, such signals become statistically uncorrelated over time, which enables the use of multiplexing techniques. In the case of frequency division multiplexing, it is known that even the band-pass filtered chaotic signal will remain chaotic due to the existence of a dense set of unstable periodic orbits in the chaotic attractive set [36].

Concerning the overall implementation, conventional communication systems are known to require the intensive use of modulators, source encoders, channel encoders and filters. On the other hand, it is claimed that chaos-based communication schemes can be implemented by using only one subsystem at each end of the communication link yet providing all the basic processes needed for digital information transmission [7].

Furthermore, chaos-based communications are intended to make use of nonlinearities and complexity instead of suppressing them. In fact, by using such complexity adequately, as a result of a good understanding of the nonlinear dynamics, efficiency of chaos-based communication schemes can possibly supplant patterns of communication efficiency based on conventional schemes, yet saving energy and using the nonlinear characteristics of components to their fullest.

Note also that, however source coding and compaction techniques are available, they have rather limited resources when compared to eminent chaos-based coding techniques. This is due to the fact that in chaos-based coding, the redundancy needs not be sent over the channel; rather, they are built in the deterministic nature of the chaotic dynamics [7].

Concerning applications of communication such as mobile or indoor radio, an effect arises out of the interaction of signals traveling different paths and arising at the receiver. Such effect, known as *multipath fading*, consists of the destructive interference among signals and can result in severe signal degradation [51]. Theoretically, such effect can be reduced in chaos-based communication schemes, due to the inherent lower cross-correlation existing between any two time-shifted segments of the chaotic waveform, comparatively to that of periodic signals [53].

Apart from the theoretical advantages of chaos-based communication schemes, hardware equipments are somewhat converging to the use of nonlinear features of optical, optoelectronic and electronic components at low noise rates. Besides, according to [3], some of them could be easily included in the existing optical network infrastructure, such as the built-on-chip chaotic emitters.

Digital communication systems using chaotic carriers have been demonstrated to possess inferior upper limits for BER performance in Additive White Gaussian Noise (AWGN) channel than their corresponding versions based on conventional digital techniques [86]. This means that, under the same conditions of signal-to-noise ratio, conventional techniques have lower BER than their chaotic counterparts. Although it is true that the BER performance can be enhanced through the use of optimized coding and decoding or estimation techniques [32, 58], it is also true that conventional systems often present better BER performance even in such case. The alternative might be to direct chaos-based communication schemes to applications in which such disadvantage is not critical, such as in ultra-wideband radio, low data rate personal networks and optical laser systems [54].

From the last paragraphs, it is possible to enjoy some of the potentials of chaos for communications. In Sect. 7.2 we attempt to briefly review some state-of-the-art research areas. Afterwards, in Sect. 7.3 we comment on some challenges chaotic systems must surpass to become practical. Finally, on Sect. 7.4 we draft some conclusions.

7.2 State-of-the-Art of Chaos-Based Communication

In this section, a glimpse of some recent research topics on chaos-based communications is provided. We clearly do not intend to cover all the field. Rather, we succinctly describe areas that are closely related to practical real-world applications, for historical or technical reasons.

7.2.1 Source Coding Based on the Chaotic Dynamics

Chaos-based communication schemes present great potential for source coding and information transmission due to the inherent determinism of chaotic dynamics. Consider a chaotic map $x_{n+1} = f(x_n)$ that is able to generate a sequence $X_0 = \{x_0^0, x_1^0, \ldots, x_n^0\}$ of length $n+1$ out of a given initial condition x_0^0. Such chaotic sequence can be made to correspond to a given data stream $M_0 = \{m_0, m_1, \ldots m_n\}$ by creating a topological correspondence between the units of the alphabet and the disconnected regions of the invariant chaotic set Thus, a given data stream M_i corresponds to a unique chaotic sequence $X_i = \left\{x_0^i, x_1^i, x_2^i, \ldots\right\}$, as shown in [7].

Once the quantized initial condition x_0 is transmitted over a communication channel, the sequence X_0 can be generated at the receiver's end. As a consequence, the data stream M_0 can be retrieved. It is worth noting that only the initial condition x_0^0 of the chaotic sequence X_0 travels through the communication channel. As such, this principle allows the transmission of large amount of data as a much lower amount actually travels through the communication channel [7]. In other words, for each set of n+1 symbols produced by the information source, only one value must travel through the communication channel, namely, the initial condition of the chaotic sequence.

Note that the chaotic signal as it is used in this scheme is not only the carrier of information; rather, it also contains information in itself as it allows the receiver to recreate a whole data stream $M_0 M_1 \dots M_l$ from pieces of information $x_0^0 x_0^1 \dots x_0^l$. As such, the chaotic dynamics can be exploited for source coding.

7.2.2 Controlling Chaos and Modulating Information with Small Perturbations: The OGY Method

What does chaos control has to do with chaos communication? Chaos control strategies are responsible for generating a desired trajectory within the chaotic attractive set, such that information is encoded in the chaotic evolution.

In this context, the OGY method has been successfully used for more than two decades in the problems of controlling and synchronizing chaos. According to Grebogi and Lai [27], the key ingredient for chaos control is the observation that, within the chaotic set, there are infinite unstable periodic orbits which are recurrently visited by the trajectory as the chaotic process evolves in time. Some of these periodic orbits may correspond to a desired trajectory on which we may want the system to settle. Also according to [27], another key ingredient is that chaotic systems are subject to sensitive dependence on initial conditions, which also means that we can easily alter the trajectory by using small perturbations. The OGY method consists of using small perturbations to 'capture' and stabilize a chaotic trajectory on a given final state, i.e., a periodic orbit or fixed point, whenever such trajectory passes sufficiently close to this desired final state [66].

The use of small perturbations to control chaos is intimately related to the efficiency of chaos-generating devices, since the process of modulating a digital message into the chaotic evolution can be managed in practice with a small amount of energy. Although it was the first well-documented effective strategy for chaos control and synchronization, the OGY method remains widely used due to its energy-saving characteristic yet preserving effectiveness of the overall process.

The point here is that knowledge about the dynamics allows the use of little energy to control the chaotic dynamics and to encode information in its evolution. This fact has been widely explored in the development of chaos-based communication schemes through the use of the OGY method [9, 33]. For instance, Fig. 7.4 shows Poincaré sections to which the symbols 0 and 1 can be assigned, such that the chaotic evolution generates a binary bit stream, as shown in Fig. 7.5.

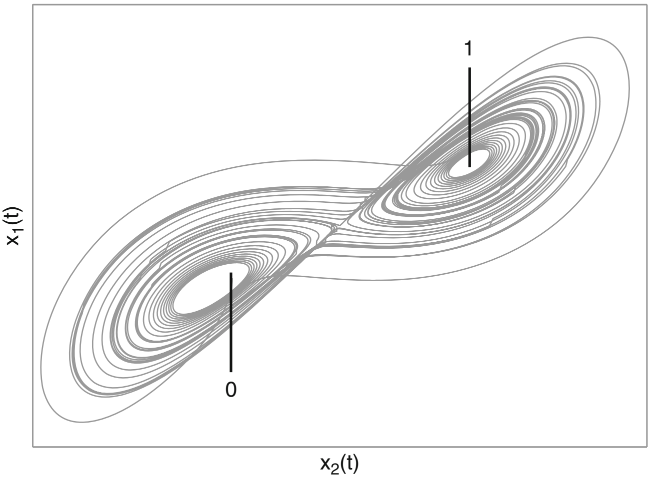

Fig. 7.4 Bits 0 and 1 are modulated in the chaotic evolution as the chaotic trajectory crosses a predetermined threshold [33]

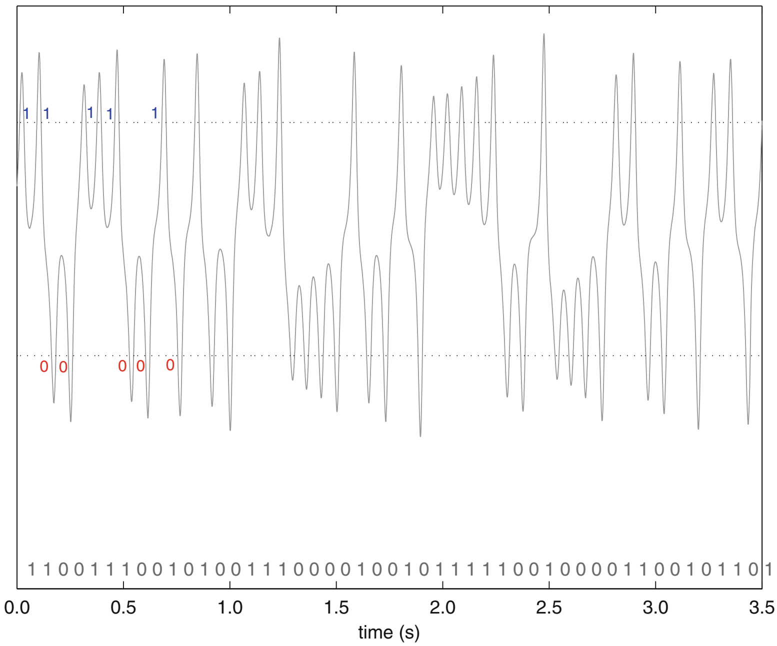

Fig. 7.5 Modulation process using the chaotic evolution [33]

7.2.3 Chaos-Based Communication in Delay Channels

Chaos communication can be effective in scenarios involving channel delays. Simultaneous bidirectional communication between delay-coupled oscillators has been presented in numerical simulations using lasers [82] and in experimental setups using chaotic electronic circuits [84]. In both cases, it was achieved in a scenario involving two coupled chaotic systems linked together by a delay channel. As one considers real situations, where time delays are almost surely present, chaos-based communication may be exploited as a means to overcome the inconveniences caused by channel delay yet maintaining a conceptually simple framework. Nevertheless, as the concept of simultaneous bidirectional chaos-based communication relies on synchronization, one needs to determine what synchronization means in the context of delay-coupled oscillators. The aim of this section is to briefly address the phenomenon of isochronal synchronization, which is the backbone of the simultaneous bidirectional chaos-based communication framework, and to outline how such synchronous state can be explored in simultaneous bidirectional communication schemes.

Recent results have shown that mutually coupled chaotic systems are capable of realizing zero-lag synchronization even in an environment with channel delay. Such form of synchronization is named isochronal synchronization and it has appeared in numerical simulations [41, 50, 85, 89], in experimental setups [6, 41, 84] and, more recently, in analytical results based on the Lyapunov-Krasovskii stability theory [29].

As the problem of communication in real environments is considered, time delays in the information transmission must be taken into account. The effect of such transmission time can be experienced in different forms, according to the unidirectional or bidirectional nature of the communication scheme. In unidirectional communication schemes in the form transmitter-receiver, such effect is limited to time delay in the information reception. As an example, suppose that a waveform $x(t)$ representing a symbol is sent by the master or transmitter system over a channel at the instant t and it is subject to delay τ due to the inherent characteristics of such channel. In this case, the transmitted waveform $x(t)$ is received at the receiver end at the instant $t+\tau$, and the principle behind decoding the symbol is the achronal synchronization between transmitter and receiver systems, that is, $x(t) = y(t+\tau)$, in which the receiver follows the transmitter with a time lag of τ [84] and recovers the transmitted symbol due to the chaos-pass filtering property of the synchronized oscillators.

On its turn, considering bidirectionally delay-coupled oscillators, isochronal synchronization allows the conception of simultaneous bidirectional communication schemes due to the fact that $x(t) = y(t)$ despite of the delay time introduced by the channel. Isochronal synchronization allows not only the realization of simultaneous bidirectional communication, but also the encryption and decryption of information and negotiation of secret keys [40, 84]. Although mutual driving of chaotic systems subject to coupling delay are sensitive to the magnitude of time delay, parameter

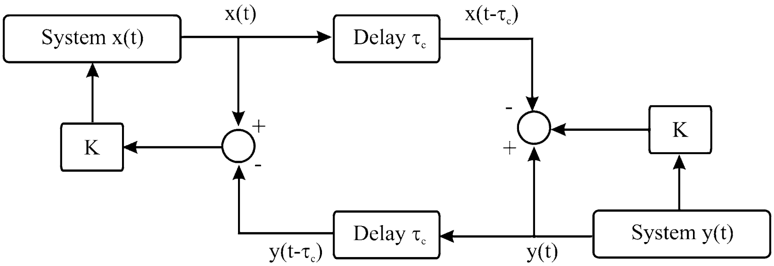

Fig. 7.6 Scheme of mutually driven systems [84]

mismatches and noise, successful realizations of chaos-based communication in simple experimental setups reinforce the possibility of fully exploiting chaotic dynamics to transmit information in such real environments presenting delay.

The idea of simultaneous bidirectional transmission and reception is based on the fact that the systems can synchronize with zero lag in the presence of channel delay and thus information can be injected and retrieved properly at both ends of the communication link, as if no delay were present. As simultaneous transmission occurs, there are two possibilities of symbol encoding at a given moment: (i) either both systems are encoding the same symbol or (ii) each system is encoding a different symbol. That being considered, two different circumstances concerning the maintenance of the synchronous state arise. In the first case, as mutually coupled systems are coding the same symbol at a given instant, the synchronization is maintained due to the fact that both systems are subject to the same perturbation caused by the information injection. In this case, an eavesdropper would have no clue of which symbol is being encoded [40, 84]. In the second case, as the systems encode different symbols at a given instant the message is treated as additive noise at the receiving circuit and it is filtered due to chaos-pass filtering of the synchronized systems. It is claimed that as the systems encode the same symbol at the same time, an eavesdropper could not possibly infer the symbol being encoded, and thus simultaneous bidirectional communication can be used in practice to negotiate secret keys, as both sides of the link can agree on a secret key consisting on the first N symbols that coincide, in such way that a key with dimension as large as desired can be negotiated in a public channel [84]. This framework is advantageous since it eliminates the need for a private channel to exchange key information and thus greatly simplifies the communication process. The block diagram of this system is shown in Fig. 7.6. Figure 7.7 illustrates the outcome of simultaneous data transmission: $m_x(t)$ and $m_y(t)$ are bit streams transmitted by systems $x(t)$ and $y(t)$, respectively. The dynamics of the synchronization error allows the identification of spikes that correspond to each system encoding a different bit. It follows that the received bit streams can be recovered after a simple XOR operation, as it is shown in the figure.

It is worth noting that the modulation of different symbols in a given instant causes bursts in the synchronous state. As resynchronization occurs, the next symbol

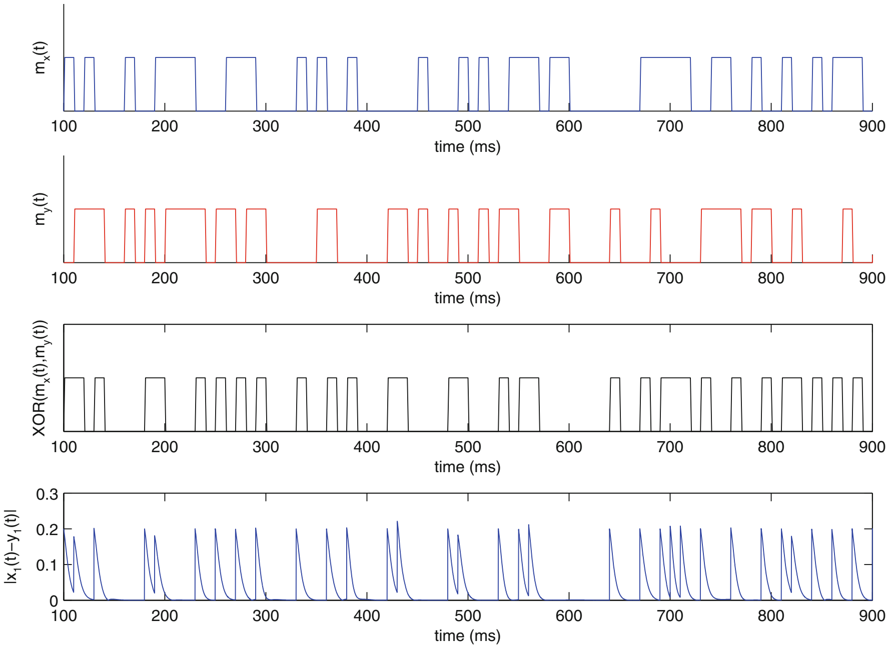

Fig. 7.7 Simultaneous bidirectional chaos-based communication scheme [84]

in the symbol sequence can be encoded at both ends and, as a consequence, the encoding rate is given by the inverse of the resynchronization time. For instance, in the case of semiconductor lasers, resynchronization takes around 0.3 ns, what allows a maximum encoding rate of 3Gbps per system [82].

Considering that source coding efficiency and data compression rates using chaotic dynamics can overpass conventional source coding and data compressing techniques [7], chaos-based simultaneous bidirectional communication based on isochronal synchronization tend to be of increasing research interest due to its great information bearing capacity yet maintaining simple framework and cost-effective implementation.

7.2.4 Chaos-Based Communication in Bandlimited and Noisy Channels

Although chaos-based communication systems based on chaos synchronization work well in ideal environments, the presence of noise and distortion in the channel, which are almost surely present, for instance, in wireless channels, brings unsatisfactory results in terms of BER when compared to conventional communication systems [37, 53, 58, 86].

In the last years, there have been many researches whose objective is to approximate the performance of chaos communication systems to that of conventional ones, considering realistic environments. Here we briefly review some of these current techniques that may allow chaotic signals to be used in practical applications in the near future.

7.2.4.1 Bandlimited Channels

The papers by Pecora and Carroll [71], Cuomo and Oppenheim [13] and Wu and Chua [87] have inspired numerical and theoretical studies on the feasibility of master-slave communication systems based on chaotic synchronization. However, these schemes do not usually present satisfactory performance when the bandwidth limitations imposed by the communication channel are taken into account [21, 60]. This is a matter that in practice can not be neglected. In fact, because of the nonlinear nature of the nodes composing the network, if any spectral component is amiss in the transmission, then all spectral components can be affected. Consequently, the message sent by the master can not be faithfully recovered at the slave.

To overcome this problem, Rulkov and Tsimring [74] and Eisencraft and Gerken [18] independently proposed a method for synchronizing master and slave, described by chaotic differential equations, under bandwidth limitations. The idea is to employ an identical filter on both nodes in order to confine the spectral content of the transmitted signal to the available bandwidth. Afterwards, these results were extended to difference equations [21].

Figures 7.8–7.10 show the results obtained from numerical simulations by using the two-dimensional Hénon map [35]. Figure 7.8 exhibit the performance of the scheme described in [21] for an ideal channel. Notice that after a transient, the original message sent by the master is fully recovered in the slave. In Figs. 7.9 and 7.10, the channel is considered a Finite Impulse Response (FIR) filter [65] of order 50 and cut-off frequency $\omega_c = 0.8\pi$. In Fig. 7.9, no filter is used in the master and in the slave, in order to confine the spectrum of the transmitted message; hence, such a message can not be recovered. In Fig. 7.10, FIR filters of order 30 and $\omega_s = 0.4\pi$ are employed in both nodes and the message is fully rebuilt after a transient.

Therefore, within this framework, master-slave systems based on chaotic synchronization can satisfactorily work even when bandwidth limitations imposed by the communication channel are considered.

7.2.4.2 Additive Channel Noise

As it has been stated before, one of the main problems of chaos-based communication schemes is their poor performance under AWGN. If, on the one hand, the use of signals engendered by chaotic phenomena is a promising alternative towards

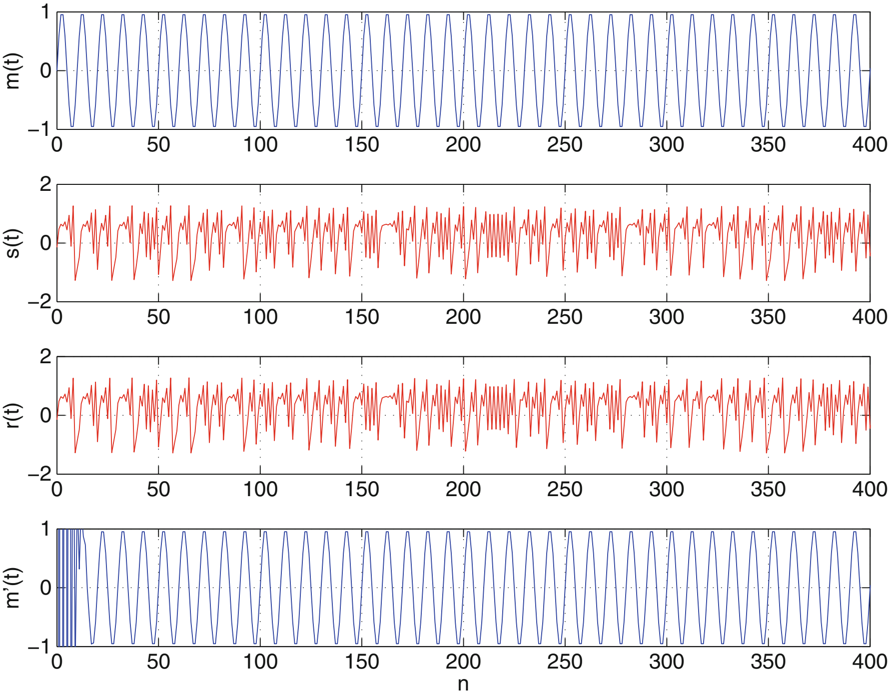

Fig. 7.8 Message $m(t)$, transmitted $s(t)$ and received $r(t)$ signals and recovered message $m'(t)$ in the ideal channel case [21]

efficient, secure and low power-consuming communication systems, on the other hand, certain intrinsic features of these signals – e.g. aperiodicity, sensitivity to initial conditions and broadband spectrum – pose significant difficulties to the performance of essential signal processing steps, such as denoising. A straightforward way to illustrate this point is to notice that the spectral similarities between chaotic signals and noise renders unsuitable the classical linear filtering approach based on direct frequency response shaping.

However, there have been many papers on recovering chaotic time-series from noisy environments. Among them, we can mention several different approaches: estimation theory, e.g. [14, 16, 17, 20, 59, 67, 68]; local polynomial approximation [47,48]; singular value decomposition and local geometric projection [11,26,49,75] and, more recently, blind signal separation [12, 76].

Based on these methods, many chaos-based communication schemes that use denoising techniques before or during demodulation have been proposed lately [20, 39]. Although their performance in terms of BER in AWGN are better than the chaos-based systems that do not use them, there is still a long path to them become comparable with conventional system in this aspect.

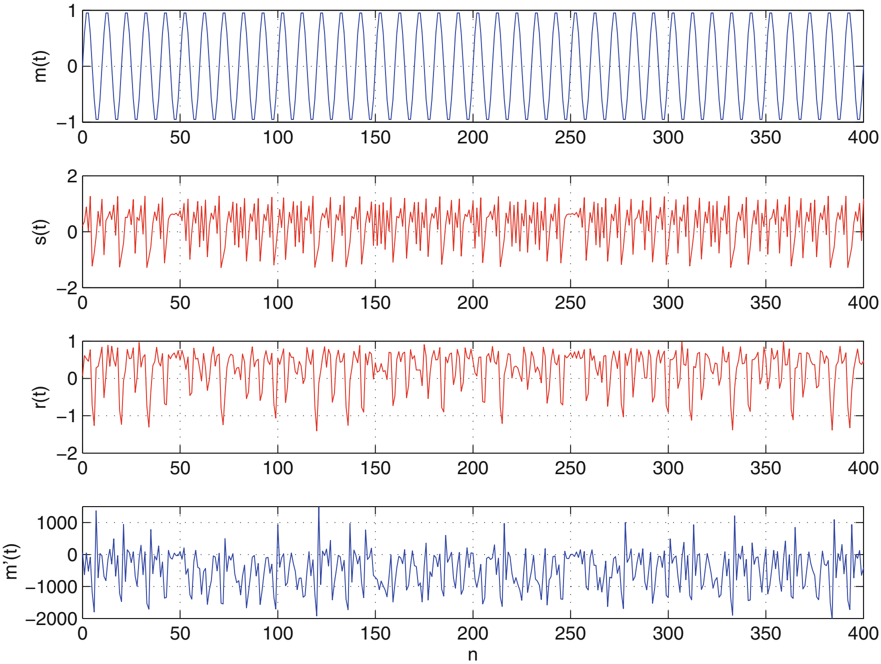

Fig. 7.9 Message $m(t)$, transmitted $s(t)$ and received $r(t)$ signals and recovered message $m'(t)$. The channel is a low-pass filter with normalized cut-off frequency $\omega_c = 0.8\pi$ [21]. The message could not be recovered

7.2.4.3 Limits of BER Performance

To qualitatively explain why digital chaos-based communications systems have BER performance issues, we focus now in one of the simplest ones, the Chaos Shift Keying (CSK) and its variants based on noncoherent or differential demodulation [37,43–45,53,77]. In one of the most relevant experiments on chaos communication up to date, the Optical Communications Laboratory of the Athens University in Greece, implemented an 120 km optical fiber link in metropolitan Athens and managed to transmit at gigabit rates using CSK [4, 78].

CSK is a digital modulation where each symbol to be transmitted is encoded as coefficients of a linear combination of signals generated by different chaotic attractors [43].

CSK can be basically implemented in two ways: the *coherent* and the *non-coherent* CSK. In the first case, the chaotic signals are regenerated at the receiver via chaos synchronization and the transmitted symbol is decoded using correlators. In the second case, each symbol is transmitted by chaotic signals with different mean energy and the receiver can decode the transmitted symbol using a threshold decision [37].

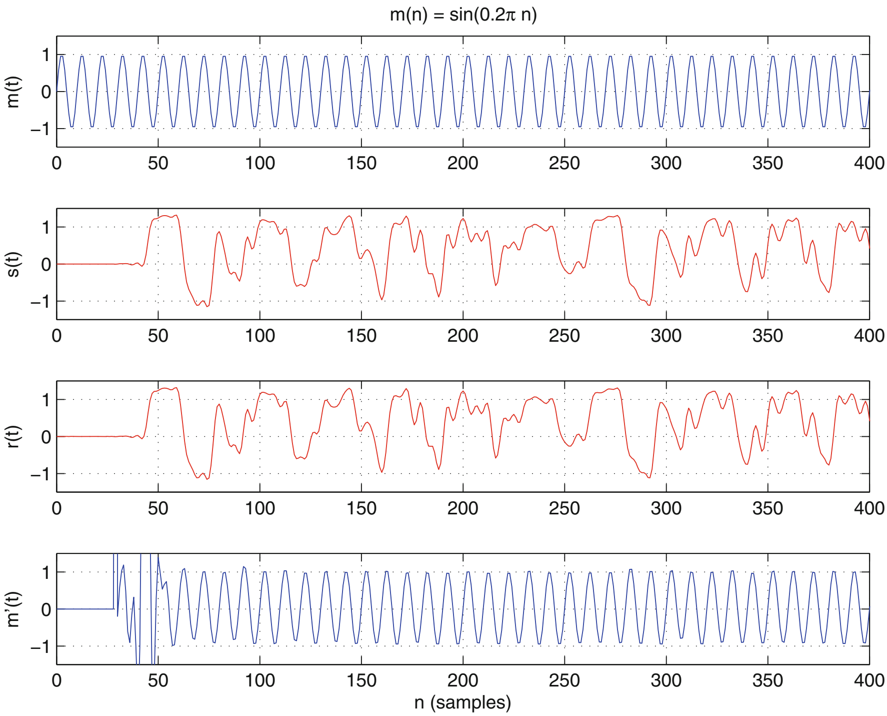

Fig. 7.10 Message $m(t)$, transmitted $s(t)$ and received $r(t)$ signals and recovered message $m'(t)$. for the same channel as in Fig. 7.9 but now low-pass filters are included in master and slave systems [21]. The message is fully rebuilt after a transient

As in conventional digital communication schemes, coherent demodulation is optimal if the basis function can be perfectly recovered at the receiver [51]. However, as chaos synchronization is sensitive to noise it turns out that non-coherent CSK has better performance in terms of BER in AWGN channel [86]. Nevertheless, an important drawback of non-coherent CSK is that the decision threshold depends on the power of channel noise that has to be estimated before demodulation.

An alternative to eliminate the problem of dependence of the decision threshold on the noise power in the channel is to use differential schemes, as the Differential CSK (DCSK) [44]. DCSK is a variant of CSK with two maps whose basis sequences consist of repeated segments of chaotic waveforms. To transmit a "1" two identical segments of length $N/2$ integer are sent. To transmit a "0" the second segment is multiplied by -1. The decision on the transmitted bit is based on the correlation between these two segments and the decision threshold is zero, independently of the channel noise.

A typical binary DCSK signal $x(n)$ corresponding to the symbol sequence $\{1,1,0,1,0,0,1,0\}$ using the tent map [22] as the chaotic generator is shown in Fig. 7.11.

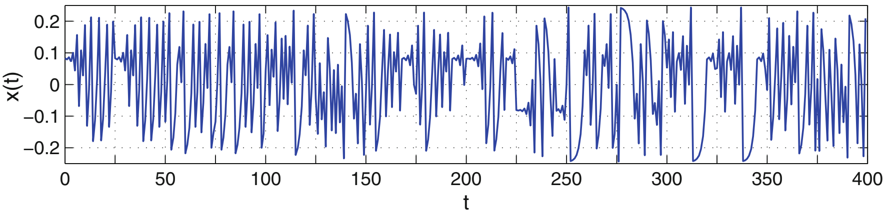

Fig. 7.11 DCSK signal transmitted for the data sequence $\{1, 1, 0, 1, 0, 0, 1, 0\}$ with $N = 50$ samples per symbol [44]

For a conventional modulation scheme using only one periodic basis function $s(n)$ composed of N samples per symbol as an integer multiple of its period, the energy per symbol E_s is constant for each distinct symbol. In contrast, chaotic signals are by definition aperiodic. Thus, when using a chaotic basis, $s(n)$ is different at each interval and the energy can be different for each transmitted symbol.

Compared to conventional systems, the fact that the energy per symbol is not constant is a major disadvantage of noncoherent CSK and DCSK. The practical consequence of such drawback is that errors in reception can occur even in ideal noiseless channels, which is undesirable in practice.

An alternative solution is to modify the modulation scheme so that the transmitted energy for each symbol is kept constant. That is the aim of Frequency Modulated DCSK (FM-DCSK) [45].

The FM-DCSK transmitter generates a DCSK signal with constant energy per symbol. The idea is to take advantage of the fact that the power of a frequency modulated signal is independent of the signal, as long as it is slowly-varying compared to the carrier [51]. Thus, the chaotic signal is fed into a frequency modulator. If the output of this modulator is used in implementing DCSK, then the output of the correlator at the receiver will be a constant in the absence of noise and the problem of energy variability disappears.

Despite the fact that FM-DCSK eliminates the threshold dependence, the energy variability and the synchronization problems, it still suffers from the fact that it does not use any information on the chaos generating system in the demodulation process. In conventional systems, as the carrier is perfectly known by the receiver, optimal receivers as correlators or matched filters can be used with superior performance.

Table 7.1 summarizes the problems encountered in the digital modulations described. The column **Threshold** concerns the problem of dependence of the decision threshold on the noise power in the channel. The column **Energy** represents the problem of variability of energy per symbol. The column **Sync.** means the need for recovery of basis chaotic functions at the receiver and the last column, **Map Info** when signalized means that the system does not use properties of the chaotic attractor in the estimation of the transmitted symbol. Among the modulations cited, FM-DCSK has the best results because it does not depend on chaotic synchronization, its decision level threshold is independent of noise and the mean energy per symbol is constant.

Table 7.1 Problems of chaotic modulations studied in this section

System	Threshold	Energy	Sync.	Map Info
Coherent CSK		X	X	
Noncoherent CSK	X	X		X
DCSK		X		X
FM-DCSK				X

The analyzed non-coherent and differential receivers have a common feature: they do not use any characteristic of the dynamics of the systems that generate the chaotic signals to process the demodulation. These techniques are limited to estimating characteristics of the received signal and to comparing them with an adequate decision threshold.

So, to obtain better BER performance it is necessary to use the transmitter dynamics information on the receiver in a way robust to noise. Many works on this research field have been published lately as [14,16,17,20,59,67,68]. This research is fundamental for chaos-based communication to have a chance on noisy non-optical communications channels.

7.2.5 Experimental Realization of Chaos-Based Communication: Photonic Integrated Devices

Optical networks are regarded as a highly suitable environment for the proper functioning of chaos-based communication, due to (i) the great potential of devices that generate chaotic light (semiconductor lasers, laser diodes) and (ii) the low level of distortion and noise to which chaotic light is submitted when traveling through optical fiber links. As a consequence, optical links appear as a subject of intensive and fruitful study in the topic nowadays.

Fruitful, indeed, as it has been reported in [3], where compact, fully controllable and stably operating monolithic photonic integrated circuits (PICs) were successfully used for peer-to-peer optical link communication over 100km of optical fiber links, with bit rates of 2.5Gb/s and BER below 10^{-12}. Such devices are said to generate high-complexity broadband chaos, which are desirable characteristics from the standpoint of chaos-based communication. The major argument in favor of chaos in [3] is that chaotic carriers offer an extra layer of encryption that help secure data from unauthorized users, such that eventual eavesdroppers would be unable to recover the information embedded in the chaotic signal due to the technical difficulties of tuning their PICs accordingly. The main element responsible for the proper functioning of this setup seems to be the precise thermoelectric cooling of the devices, which provides stability and controllability of important synchronization properties, such as wavelength, optical power, spectral distribution and phase matching conditions [3].

More importantly, according to Argyris et al. [3], PICs are available to be directly and efficiently incorporated to the existing optical network.

7.3 Challenges of Chaos-Based Communication

In this section, we point out some challenges regarding the future applications of chaos in telecommunications.

7.3.1 *Chaos-Based Versus Conventional Communication*

The fact that conventional communication is managing perfectly and still improving incredibly fast, as optical techniques, such as *Dense Wavelength Multiplexing* (DWM), providing potential for efficient and reliable transmission at rates of the order of terabits per second [1], makes out of conventional techniques and devices something of a more and more well-established communication standard. Concerning chaos-based communication, there are still many points that must be improved and tested before they can be effectively used beyond experimental setups with scientific purpose.

It seems, however, that rather than mutually exclude each other, both techniques can coexist. This is due to the fact that the interesting aspects of each of them find their way in different applications, depending on the demands of the scenarios involved. For example, while conventional mobile radio communication experiences the undesirable effects of multipath fading and narrow band interference, chaos-based experimental setups have shown that chaos synchronization manages poorly in noisy channels. On the other hand, while conventional communication is quite suitable for ultra-high-speed optical communication using DWM, for example, it is not likely that data encryption at the software level will possibly be able to keep up at such high transmission rate. On its turn, chaos-based communication can provide an extra layer of high-speed real-time encryption at the physical level, which may allow the reduction of software encryption strength while maintaining the original level of secrecy, thus allowing the secure transmission of data at very high rates.

Finally, it seems that at this point, the challenge consists of directing the adequate technique to the applications it is best suited for, taking into account the desired aspects and demands of the final application and at the same time ensure that conventional and chaos-based appliances can interact properly.

7.3.2 *Chaos-Based Communication in the Existing Infrastructure*

It is not likely that the operating communication systems will be simply substituted, no matter how developed any other technique might be. Some relevant subjects related to the development of new techniques and technologies are (i) at what extent it can use and (ii) how easily it can be incorporated in the existing infrastructure and (iii) how compatible it is with the standard technology.

Bearing this in mind, some recent papers have dealt with this subject. Lau et al. [52] and Chaps. 7 and 8 of [53] have studied the coexistence of CSK and DCSK with conventional narrowband digital communication systems and with conventional spread spectrum systems. They have obtained theoretical and numerical BER results which allow the evaluation of the performance of the resulting hybrid system for different (i) power ratios and spreading factors, (ii) chaotic signal powers, (iii) conventional spread-spectrum signal powers, and (iv) noise power spectral densities. Although the results themselves are not much conclusive, they are an important step towards deeper understanding of the advantages and drawbacks of the coexistence of the communication techniques.

It is also worth citing the results presented in [69] as examples of the use of on-the-shelf pieces of equipment to create practical chaos-based applications. This paper presents the design and validation by means of suitably improved randomness tests of two different implementations of high-performance true-random number generators which use a discrete-time chaotic circuit as their entropy source. The proposed system has been developed from a standard pipeline analog-to-digital converter design, easily available, modified to operate as a set of piecewise-linear chaotic maps.

7.3.3 Further Development of Efficient Multi-User Schemes

For any practical communication system, the multiple-access capability is essential. Early studies on chaos-based communication systems were focused on a single-user case. In the past few years, more effort has been employed in the investigation of systems with multiple-access capability, which is a key feature of spread-spectrum communication systems [79]. However, the study of the multi-user capacities of chaos-based communications in real environments is still in its early developments.

A method based on multiplexing chaotic signals has been proposed by Carroll and Pecora [10] and some chaos-based approaches for generating spreading codes have been applied to conventional code division multiple access systems [62, 72]. A patent on the subject was granted to Yang and Chua (US Patent 6331974).

Multiple access using DCSK and FM-DCSK has also been proposed, see [79] and references therein. To minimize the co-channel interference [51], Kolumban et al. [46] proposed a multi-user FM-DCSK scheme, in which a chaotic signal is combined with two Walsh functions to form the basis functions representing the symbols $+1$ and -1 for each user.

As another possibility, Eisencraft and Kato [19] have numerically demonstrated the possibility of generating band-limited chaotic signal and to employ conventional frequency-division multiplexing for multi-user capability of chaos-based communication system.

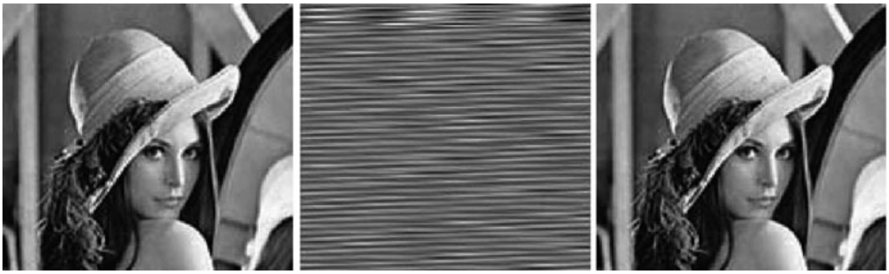

Fig. 7.12 Lenna picture encrypted and decrypted using chaos [28]

7.3.4 Encryption at the Physical Level and Legal Issues

One of the remarkable characteristics of chaos-based communication is the possibility of implementing cryptography at the physical level, since chaos has many desirable characteristics from the point of view of cryptography, namely, complexity, ergodicity, transitivity, determinism and sensitivity to initial conditions and perturbations [40]. Although the cryptographic predicates of chaos are rather controversial, especially concerning security issues, the widespread use of chaos might touch restrictions in several countries that have strict policies concerning the private use of encryption.

The massive use of real-time high-speed encryption at the physical level would be barely a by-product of chaos-based communication and, if on the one hand security of chaotic encryption is questionable, on the other hand it does make it harder to control and access information. Such fact can surely become a barrier to widespread use of chaos-based communication schemes, since it can touch long-standing legal issues concerning information secrecy in many countries.

Despite this questions, chaos-based cryptographic systems and their analysis have been areas with large number of contributing papers lately, mainly on image cryptography and watermarking, where conventional solutions are not established yet, see e.g. [5] and references therein. For example, Fig. 7.12 shows the Lenna picture encrypted and decrypted using chaos [28].

7.4 Final Remarks

This chapter is aimed to outline the main aspects involved in chaos-based communication in terms of the development of the technique and physical devices up to date. Topics are not intended to be treated in depth. Rather, newcomers to the topic are provided a suitable starting point to their investigation. At the same time, expert researchers may profit from a global overview that may allow the identification of their own work while acquiring a general idea of what has been done in different directions of research.

Based on the topics presented in this communication and in the cited references, final remarks are given regarding current applications of chaos-based communications:

- Optical Communications: coherent CSK was successfully implemented with performance similar to conventional systems. As optical channels approach a noiseless one, sensitivity of chaos synchronization to noise is not a relevant issue and chaos can provide an extra security level;
- Wireless Communications: real wireless channels present noise, multipath and delay, so that many of the proposed chaos-based communication systems simply do not work. However, many researches are in course to investigate solutions to the problems that arise due to these non-ideal and inherent characteristics of the channel;
- Satellite Formation Flying: in this application, many smaller satellites cooperate to perform tasks that would not be possible with a single monolithic satellite, such as distributed measurement for synthetic aperture radar. Due to the distributed nature of the mission, synchronization among the clocks of the satellites is necessary for coordination, for instance, when data acquired by different satellites must be matched up. Due to the distances, time-delays are generated when information is transmitted between satellites, and isochronal synchronization has potential to be exploited in this scenario;
- Cryptography: this is perhaps the topic receiving most part of the publications within chaos applications. The characteristics of chaos have been exploited in hundreds of methods proposed in the literature. Nowadays, the applications are concentrated in image encryption and watermarking, where conventional solutions are not established.

Since conventional communication techniques are managing quite well, the deployment of so much time and energy in the development of chaos-based techniques gives rise to the question of whether it is worth the try. However, given the whole new theoretical possibilities that chaos brings about and the increasingly demanding scenarios where reliable, high-speed and effective communication is demanded, it is a general consensus that researching chaos communication is a far-sighted measure. Either in the case the outcome will be successful or not in terms of widespread use of such technique, chaos-related subjects have provided many insights into different important areas of scientific interest, such as Medicine, Social Sciences, Physics and Engineering. This, per se, can be regarded as a substantial scientific contribution.

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