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Chaos-based communication systems in non-ideal channels

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ABSTRACT

Recently, many chaos-based communication systems have been proposed. They can present the many interesting properties of spread spectrum modulations. Besides, they can represent a low-cost increase in security. However, their major drawback is to have a Bit Error Rate (BER) general performance worse than their conventional counterparts. In this paper, we review some innovative techniques that can be used to make chaos-based communication systems attain lower levels of BER in non-ideal environments. In particular, we succinctly describe techniques to counter the effects of finite bandwidth, additive noise and delay in the communication channel. Although much research is necessary for chaos-based communication competing with conventional techniques, the presented results are auspicious.

1. Introduction

Chaotic signals are characterized by a deterministic, limited and aperiodic behavior, as well as sensitive dependence on initial conditions [1]. Their application has been considered in a variety of areas, see e.g. [2]. Signal processing and telecommunications are no exception specially after the seminal works by Pecora and Carroll [3] and Ott et al. [4]. Applications of chaos ranging from digital and analog modulation to cryptography, pseudorandom sequences generation and watermarking have been proposed [5–8]. Chaos has also been shown in connection to devices used in signal processing as nonlinear adaptive filters and phase-locked loop networks [9–11].

In this work, we classify a digital communication system as *chaos-based* when each symbol is transmitted using a fragment of a chaotic signal; their counterparts that do not use chaotic signals are referred to here as *conventional*.

Due to their properties, chaotic signals can occupy a wide bandwidth and have impulsive autocorrelation [12]. Furthermore, the cross-correlations between signals generated by different initial conditions present low values [5,7]. These characteristics have been behind the rationale for using chaotic signals as candidates in spectral spreading information signals.

By employing chaotic signals to modulate independent narrowband sources lead us to obtain systems that transmit signals with increased bandwidths and lower levels of power spectral density in a fashion similar to that of conventional spread spectrum systems [7]. This fact endows them with similar features, namely (i) they are difficult to intercept for an unautho-

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rized user; (ii) they are easily hidden, i.e. for an unauthorized receiver, it is difficult to even detect their presence in many cases; (iii) they are resistant to jamming; and (iv) they provide a measure of immunity to distortion due to multipath propagation [13]. Furthermore, chaos can supposedly provide a low-cost increase in security [5]. A practical 120 km fiber optic link using chaotic signals was recently reported [14].

Many of the proposed chaos-based communication systems are based on chaos synchronization, the property that coupled chaotic systems starting from different initial conditions can synchronize, under certain constrains, despite the sensibility to initial conditions [3,15].

Although the proposed schemes work well in almost ideal environments, the presence of the usual amount of additive noise, distortion or delay of almost any practical channel brings unsatisfactory results in terms of Bit Error Rate (BER) when compared to conventional communication systems [5,16].

In the last years, many researches have been conducted with the objective of approximating the performance of chaos-based communication systems to that of conventional ones in realistic environments. In this paper we review some of these new techniques that can allow chaotic signals to be used in practical applications in the near future. The results presented here were exposed in the two mini-symposia Communication with Chaos at Dynamics Days South America 2010 in São José dos Campos, Brazil.

In Section 2.1, we report a way of transmitting chaotic signals in bandlimited channels. The idea is simply to insert identical Finite Impulse Response (FIR) digital filters [17] in the transmitter and receiver feedback loops so that the synchronization process is unaffected but the chaotic signals spectra is fitted to the channel.

Next, in Section 2.2, we succinctly describe two approaches to minimize the effect of the additive noise in chaos synchronization. Firstly, we show numerical evidences that using lattices instead of single maps in discrete-time chaos synchronization problems increase the robustness to noise. After this, in SubSection 2.2.2 we address the use of blind signal separation techniques to separate deterministic chaos and stochastic noise and thus improving Signal-to-Noise Ratio (SNR).

Following, we briefly discuss bidirectional chaos-based systems that need to synchronize transmitter and receiver in scenarios that involve signal delays, as in satellite formation flying [18].

Finally, we draft our conclusions and perspectives in Section 3.

2. Combating channel impairments

In this section, we provide an overview of some recent and innovative works whose objective is to provide a better chaos-based communication performance in realistic environments. In the sequence, we succinctly describe strategies that allow us to properly handle effects of band limiting properties of the communication channel (Section 2.1), additive noise (Section 2.2) and delay (Section 2.3).

2.1. Bandlimited channels

The works by Pecora and Carroll [3], Cuomo and Oppenheim [19], and Wu and Chua [20] have inspired numerical and theoretical studies on the feasibility of master–slave communication systems based on chaotic synchronization. However, these schemes do not usually present satisfactory performance when the bandwidth limitations imposed by the communication channel are taken into account [21,22]. This is a matter that in practice cannot be neglected. In fact, because of the nonlinear nature of the nodes composing the network, if any spectral component is amiss in the transmission, then all spectral components can be affected. Consequently, the message sent by the master can not be faithfully recovered in the slave. Rulkov and Tsimiring [23] and Eisencraft and Gerken [24] independently proposed a method for synchronizing master and slave, described by chaotic differential equations, under bandwidth limitations. The idea is to employ an identical filter on both nodes in order to confine the spectral content of the transmitted signal to the available bandwidth. Afterwards, these results were extended to difference equations [21] and they are succinctly reported here.

Let the states of master and slave systems be described by $\mathbf{x}(n) = [x_1(n), x_2(n)]^T$ and $\mathbf{y}(n) = [y_1(n), y_2(n)]^T$, respectively, where $n = 0, 1, \ldots$ is the discrete-time variable. In this synchronization scheme, the time evolution of the master–slave network is governed by

$$\mathbf{x}(n+1) = A\mathbf{x}(n) + \mathbf{b} + \mathbf{f}(s(n)), \tag{1}$$

$$\mathbf{y}(n+1) = A\mathbf{y}(n) + \mathbf{b} + \mathbf{f}(r(n)), \tag{2}$$

where the square matrix $A_{2\times2}$ and the column vector $\mathbf{b}_{2\times1}$ are constants. The function $\mathbf{f} : \mathbb{R}^2 \mapsto \mathbb{R}^2$ is nonlinear. Assume that $s(n)$ is the discrete-time signal generated by the master system before entering in the cannel and $r(n)$ is the signal arriving at the slave system after passing through the channel. For an ideal channel, then $s(n) = r(n)$. Therefore, the synchronization error $\mathbf{e}(n) \equiv \mathbf{y}(n) - \mathbf{x}(n)$ is given by $\mathbf{e}(n+1) = A\mathbf{e}(n)$. Of course, master and slave are completely synchronized if $\mathbf{e}(n) \to \mathbf{0}$ for $n \to \infty$. This occurs if $|\lambda_{1,2}| < 1$, where $\lambda_{1,2}$ are the eigenvalues of A.

For a bandlimited channel, in order to confine the spectrum of the transmitted chaotic signal to the channel band, a FIR filter of order N is placed in the master in the path of x_1. Thus, for the input $x_1(n)$, the filter output is $x_3(n) = \sum_{j=1}^{N+1} c_j x_1(n-j+1)$ where $c_1, c_2, \ldots, c_{N+1}$ are the filter coefficients. The message $m(n)$ is coded by using the component

$x_3(n)$ via $s(n) = c(m(n),x_3(n)) = m(n) + x_3(n)$, where $c(.)$ is the coding function. In the slave, the same filter is employed in the path of y_1. Analogously, $y_3(n) = \sum_{j=1}^{N+1} c_j y_1(n-j+1)$ and $m'(n) = d(y_3(n), r(n))$, where $m'(n)$ is the message decoded in the slave by using the decoding function $d(.)$, which performs the inverse operation of $c(.)$.

Figs. 1–3 show the results obtained from numerical simulations by using the Hénon map [25].

In this case, $A_{2\times 2} = [0,1;\beta,0]$, $\mathbf{b}_{2\times 1} = [1;0]$, $\mathbf{f}(z)_{2\times 1} = [-\alpha z^2;0]$, where α and β are constants. In these simulations, $\alpha = 1.4$ and $\beta = 0.3$. Fig. 1 exhibit the performance of this scheme for an ideal channel. Notice that after a transient, the original message sent by the master is fully recovered in the slave. In Figs. 2 and 3, the channel effect is considered by introducing a FIR filter of order $N = 50$ and cut-off frequency $\omega_c = 0.8\pi$ between the master and slave systems. In Fig. 2, no filter is used in the master and in the slave, in order to confine the spectrum of the transmitted message; hence, it can not be recovered. In Fig. 3, FIR filters with $N = 30$ and $\omega_s = 0.4\pi$ are employed in both nodes and the message is fully recovered after a transient.

Therefore, chaos-based communication systems using chaotic synchronization can satisfactorily work even when bandwidth limitations imposed by the communication channel are considered. The performance of this scheme is analytically being investigated.

2.2. Additive noise

One of the main problems of chaos-based communication schemes is their poor performance under Additive White Gaussian Noise (AWGN) [16]. Certain intrinsic features of chaotic signals – e.g. aperiodicity, sensitivity to initial conditions and broadband spectrum – pose significant difficulties to the performance of essential signal processing steps, such as denoising. A straightforward way to illustrate this point is to notice that the spectral similarities between chaotic signals and noise renders unsuitable the classical linear filtering approach based on direct frequency response shaping [17].

Many papers on recovering chaotic time-series from noisy environments have been recently published. Among them, we can mention several different approaches: estimation theory, e.g. [26–31]; local polynomial approximation [32]; singular value decomposition and local geometric projection [33–35] and, more recently, blind signal separation [36,37].

In what follows, we comment on two of these recent approaches: using lattices instead of single maps as master and slave systems and using blind signal separation techniques.

2.2.1. Using lattices to increase robustness to noise

Here we present numerical results of a comparison between synchronization error due to AWGN when the transmitter and receiver are implemented by single or coupled maps [38].

Firstly, consider a model with two coupled 3D-Hénon maps [39] in a master–slave configuration with additive noise in the link between them. This way, the coupling is given by

$$x_1(n+1) = -a(x_1(n))^2 + x_3(n) + 1, \tag{3}$$

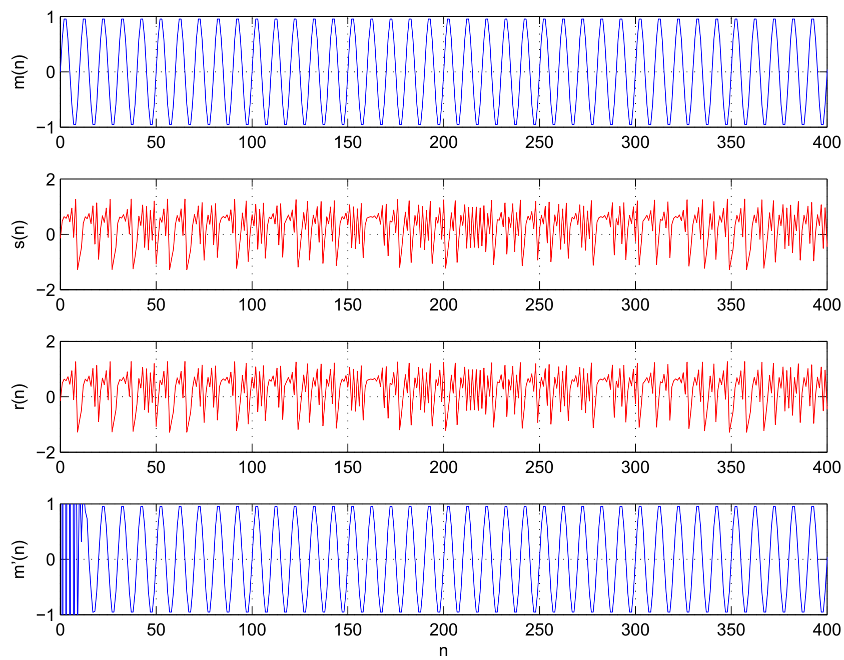

Fig. 1. Message $m(n)$, transmitted $s(n)$ and received $r(n)$ signals and recovered message $m'(n)$ in the ideal channel case.

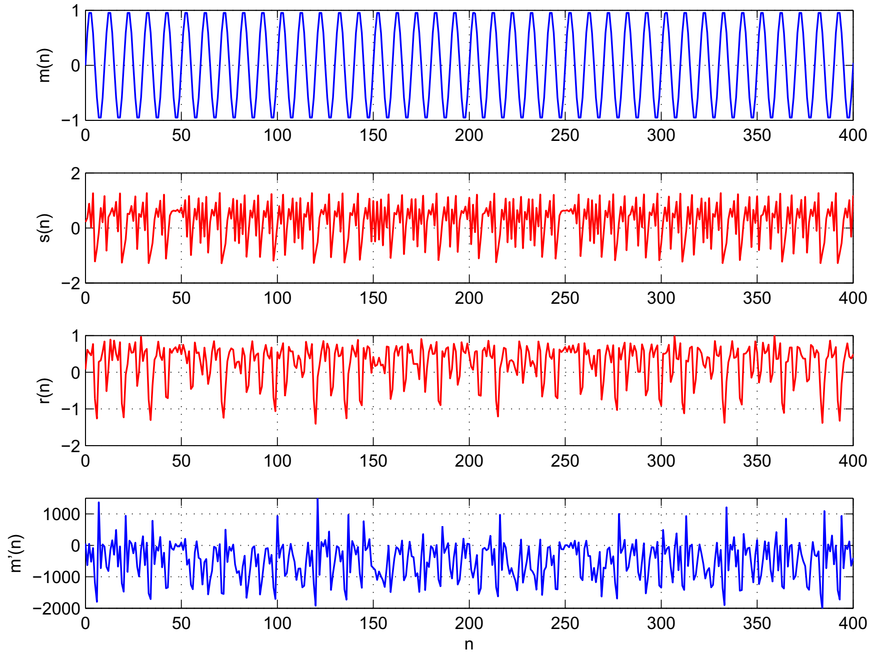

Fig. 2. Message $m(n)$, transmitted $s(n)$ and received $r(n)$ signals and recovered message $m'(n)$. The channel is a low-pass filter. The message could not be recovered.

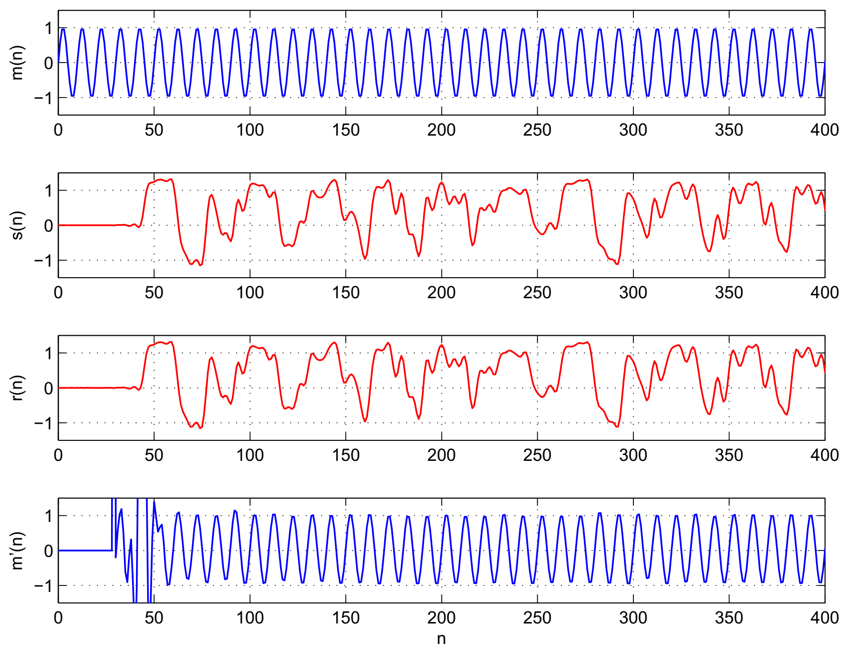

Fig. 3. Message $m(n)$, transmitted $s(n)$ and received $r(n)$ signals and recovered message $m'(n)$ for the same channel as in Fig. 2 but now low-pass filters are included in master and slave systems. The message is fully rebuilt after a transient.

$$x_2(n+1) = -bx_1(n), \tag{4}$$

$$x_3(n+1) = bx_1(n) + x_2(n), \tag{5}$$

$$s(n) = f(x_1(n), x_3(n)) + r(n), \tag{6}$$

$$y_1(n+1) = (1-\varepsilon_l)\left(-a(y_1(n))^2 + y_3(n) + 1\right) + \varepsilon_l s(n), \tag{7}$$

$$y_2(n+1) = -by_1(n), \tag{8}$$

$$y_3(n+1) = by_1(n) + y_2(n), \tag{9}$$

where $\mathbf{x}(n) = [x_1(n), x_2(n), x_3(n)]^T$ and $\mathbf{y}(n) = [y_1(n), y_2(n), y_3(n)]^T$ are the master and slave states at time n, $f(x_1, x_3) = -ax_1^2 + x_3 + 1$, $0 \leqslant \varepsilon_l \leqslant 1$ is the coupling strength and $r(n)$ is zero-mean AWGN with variance σ^2. Considering $a = 1.07$ and $b = 0.3$, this three-dimensional generalization of the Hénon map presents chaotic orbits for almost all initial conditions in the unity sphere [39].

The synchronization error is defined by

$$\delta_n = |x_1(n) - y_1(n)| \tag{10}$$

and master and slave are considered synchronized at instant n if $\delta_n < 10^{-3}$.

For the chosen parameters, it may be shown that $\delta_n \to 0$ as $n \to \infty$ if $\sigma = 0$ [21].

In Fig. 4(a) one can observe that the system synchronizes after a transient time when there is no noise. On the other hand, decreasing the SNR, defined as the mean power of the transmitted signal $s(n)$ divided by σ^2, the maps pass from synchronized to the non-synchronized behavior of Fig. 4b.

In order to increase robustness of synchronization with respect to channel noise, Batista and Eisencraft [38] consider coupled maps lattices instead of single maps in the master–slave configuration. Coupled maps lattices are spatially extended dynamical systems, that present discrete space and time, while the state variable is continuous [40]. They analyze two lattices where each one presents a global coupling prescription, and the coupling between the lattices is unidirectional. The coupled maps lattices are given by

$$x_1^j(n+1) = (1-\varepsilon_1)\left(-a\left(x_1^j(n)\right)^2 + x_3^j(n) + 1\right) + \frac{\varepsilon_1}{J}\sum_{j=1}^{J} f\left(x_1^j(n), x_3^j(n)\right), \tag{11}$$

$$x_2^j(n+1) = -bx_1^j(n), \tag{12}$$

$$x_3^j(n+1) = bx_1^j(n) + x_2^j(n), \tag{13}$$

$$s(n) = \sum_{j=1}^{J}\left[f\left(x_1^j(n), x_3^j(n)\right) - f\left(y_1^j(n), y_3^j(n)\right)\right] + r(n), \tag{14}$$

$$y_1^j(n+1) = (1-\varepsilon_2)\left(-a\left(y_1^j(n)\right)^2 + y_3^j(n) + 1\right) + \frac{\varepsilon_2}{J}\sum_{j=1}^{J} f\left(y_1^j(n), y_3^j(n)\right) + \frac{\varepsilon_l}{J} s(n), \tag{15}$$

$$y_2^j(n+1) = -by_1^j(n), \tag{16}$$

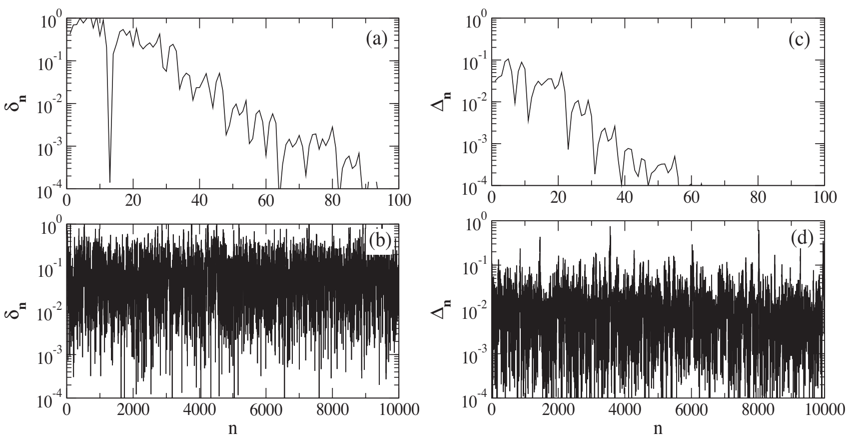

Fig. 4. Synchronization error for two coupled 3D-Hénon maps with $\varepsilon_l = 0.3$ and (a) SNR = ∞, (b) SNR = 8. Synchronization error for two coupled map lattices for $J = 10$, $\varepsilon_1 = \varepsilon_2 = 0.7$, $\varepsilon_l = 0.3$, and (c) SNR = ∞, (d) SNR = 8.

$$y_3^j(n+1) = by_1^j(n) + y_2^j(n), \tag{17}$$

where the superscript index $j = 1, \ldots, J$ identifies a particular map in the J-maps lattice. In this case, the synchronization error is defined as

$$\Delta_n = \frac{1}{J}\sum_{j=1}^{J}\left|x_1^j(n) - y_1^j(n)\right| \tag{18}$$

and master and slave are considered synchronized at instant n if $\Delta_n < 10^{-3}$. The time behaviors of Δ_n for different SNR values and $J = 10$ maps are shown in Fig. 4(c) and (d). Comparing two coupled maps Fig. 4b with coupled lattices Fig. 4d it is possible to verify that the coupled lattices are more robust to noise than two unidirectional coupled maps, at least in this numerical experiment.

Fig. 5 represents a portion of the coupling parameter plane (ε_I versus SNR) exhibiting regions of synchronized and non-synchronized chaotic states. In (a) it is considered the case of a unique map and in (b) the case of master and slave consisting of 100 3D-Hénom maps lattice each. The synchronization is considered to occur when at least at 95% of the time (a) $\delta_n < 10^{-3}$ or (b) $\Delta_n < 10^{-3}$ in the time interval $5000 < n < 50000$. The bound between the two regions is the *critical SNR*, SNR_C, the minimum SNR required for synchronization, given the coupling strength and lattice size.

Simulations reveal that SNR_C decreases according to a power-law with slope ≈ -1 with the lattice size [38]. Notwithstanding, for $\varepsilon_I \geqslant 0.5$ the coupling strength almost does not affect SNR_c.

These results enable to affirm that by using coupled lattices instead of single maps may be a way to improve robustness to noise of chaos synchronization. The next step is to test the use of lattices as transmitter and receiver of coherent chaos-based communication systems, a work in progress.

2.2.2. Denoising chaotic time series using blind signal separation

To increase robustness of chaos-based communication against additive noise, an interesting alternative is to employ denoising techniques before demodulation, e.g. [26,32].

The most studied denoising problem in the context of chaotic systems is probably that of extraction of a pure deterministic chaotic orbit from an observation vector corrupted by additive noise, which can be described as

$$x(n) = y(n) + r(n), \tag{19}$$

where $x(n)$ is the observed signal, $y(n)$ is the original chaotic time series and $r(n)$ is the random noise term. In this case, if knowledge about the dynamics is assumed, but there is a certain degree of uncertainty with respect to the initial condition, the problem turns to be intimately associated with the problem of shadowing (that of finding the purely deterministic orbit $x(n)$ that stays maximally close to $y(n)$). The works by Hammel [41] and by Farmer and Sidorowich [42] bring important contributions to this formulation. Generally, their approach assumes that nonlinear dynamics are locally linear and that a local polynomial map can be adjusted to constrain the noise-free trajectory to fall within the smooth sub-manifold. This procedure can also be applied to experimental data by reconstructing the attractor in the phase space using Takens' theorem and then by considering a local polynomial approximation to the noise-free chaotic trajectory [32]. These methods require the calculation or the estimation of the Jacobian matrices underlying the nonlinear dynamics for every time-step, which may lead to the same numerical drawbacks present in the calculus of Lyapunov exponents, that is, a numerical instability due to the presence of positive eigenvalues in the composition of the Jacobian matrices.

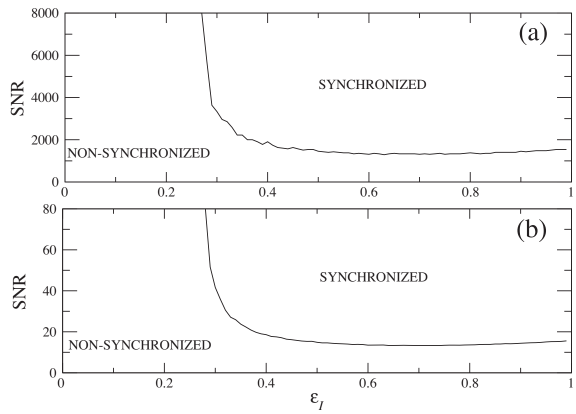

Fig. 5. Synchronized and non-synchronized regimes in the parameter plane of coupling strength $\varepsilon_I \times$ SNR for (a) 3D-Hénon maps and (b) lattices with 100 3D-Hénon in master–slave configuration.

Recently, some interesting alternative methods have been developed for blind signal separation, something that may have important applications concerning chaos-based communication systems, and, to the best of our knowledge, have not been properly addressed in this context, excepting for Chen et al. [36] and Soriano et al. [37]. In particular, from a theoretical standpoint, denoising chaotic time series can be seen either as a Blind Source Extraction (BSE) or a Blind Source Separation (BSS) problem [43,44]. The BSE and BSS formulations arise in a very natural way if there is no knowledge about the motion equations, but more than one mixture between the chaotic signal and the noise parcel are available.

In more formal terms, let us consider that two sources - one being a chaotic signal $s_c(n)$ and the other being a stochastic signal $s_s(n)$ – are linearly mixed, giving rise to $\mathbf{x}(n) = A\mathbf{s}(n)$, where $\mathbf{x}(n) = [x_1(n), x_2(n)]^T$ is the mixture vector, $A_{2\times 2}$ is the mixing matrix (which is assumed to have full rank) and $\mathbf{s}(n) = [s_1(n), s_2(n)]^T$ is the source vector. The aim of BSE is to extract a specific source from the mixtures without the need for a reference signal or for knowledge about the coefficients of the mixing matrix, while BSS aims to recover all the original signals that give rise to the mixture vector. BSE can be performed in practice by multiplying the mixture vector by an adequately chosen separating vector $\mathbf{w}$, so that the output vector $y(n) = \mathbf{w}^T\mathbf{x}(n)$ yield $Gs_c(n)$, where G is a scaling factor. The extension to BSS is direct and involves the use of multiple vectors – which form a separating matrix – to simultaneously recover all sources. Fig. 6 illustrates both problems.

A well-established and convenient criterion to adapt the separating system can be provided by Independent Component Analysis (ICA), which allows the recovery of the original sources up to scale and permutation ambiguities [43,44]. In the following discussion of this approach, it is assumed, without loss of generality, that the mixing matrix is orthogonal and, as a consequence, $\mathbf{w}$ can be parameterized in terms of a single variable θ, i.e., $\mathbf{w} = [\cos\theta, \sin\theta]^T$ [44]. In this context, ICA looks for solutions in θ that ensure, for instance, maximal nongaussianity, which can be evaluated with the aid of the kurtosis of the output components, or, alternatively, maximization of independence between the elements of the output vector $\mathbf{y}(n) = [y_1(n), y_2(n)]^T = W\mathbf{x}(n)$ (e.g. by minimizing a mutual information measure), being W the separating matrix given by:

$$W = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}. \tag{20}$$

As an example, let us consider $s_c(n)$ to be the first state variable of the emblematic Lorenz system pre-processed to have zero mean and unit mean power and $s_c(n)$ a white gaussian process 5 dB below the chaotic source in power. These signals are mixed by the orthogonal mixing matrix

$$A = \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}, \tag{21}$$

with $\theta = \pi/6$. Let us also consider the classical kurtosis ($\mathrm{fit_k}$) and mutual information ($\mathrm{fit_m}$) estimators as defined in [43,44]:

$$\begin{aligned} \mathrm{fit_k} &= (E\{y_1(n)\})^4 - 3\left(E\left\{(y_1(n))^2\right\}\right)^2 \\ \mathrm{fit_m} &= H\{y_1(n)\} - H\{y_1(n)|y_2(n)\}, \end{aligned} \tag{22}$$

where the operator $E\{\alpha\}$ is the statistical expectation of the random variable α and $H\{\alpha\}$ is the Shannon differential entropy [45] of α.

In this scenario, it is possible to optimally recover the chaotic time series if two mixtures of these signals are available. As a matter of fact, perfect recovery of the original signals is, in a general case, subject to the existence of a number of mixtures at least equal to the number of sources. To illustrate this issue, Fig. 7(a) shows that it is possible to find the solution in θ for the separating system that perfectly inverts the mixing matrix by looking for the peak of the kurtosis of an element of the output vector or for the minimum mutual information associated with the output vector as a whole, which occurs at $\theta = \pi/6$. The mutual information estimator also presents a second minimum, which is associated to the solution in θ that recovers the stochastic source, in accordance with the broader aim of BSS.

To illustrate the fact that independence recovery leads, in this case, to effective separation, Fig. 7(b) shows the original Lorenz chaotic time series and its noisy version (one of the mixtures), while Fig. 7(c) shows the same corrupted observation

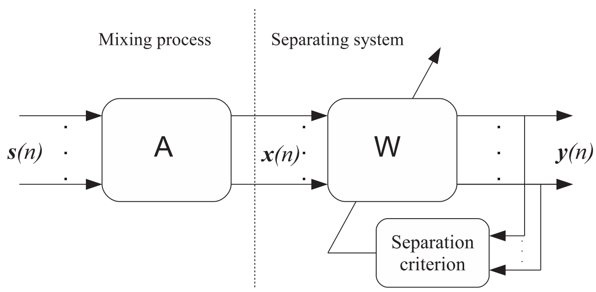

Fig. 6. Scheme for blind extraction or separation problem, where $\mathbf{s}(n)$ is the source vector, A is the mixing matrix, $\mathbf{x}(n)$ is the observed mixtures, W is the separating matrix (or separating vector in the BSE case) and $\mathbf{y}(n)$ is the recovered vector up to scaling factor.

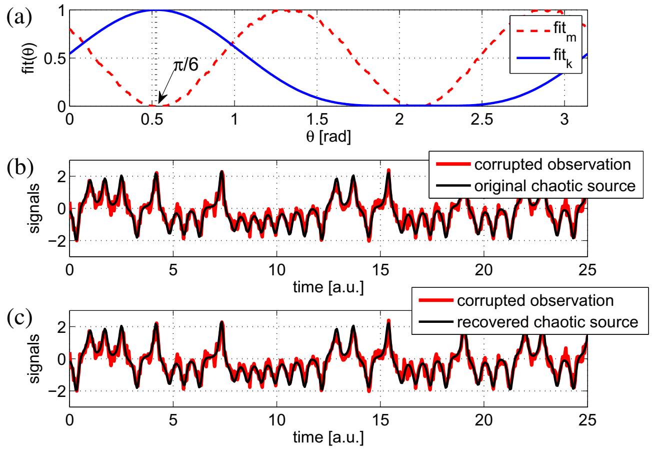

Fig. 7. (a) kurtosis (fit_k) of $y_1(n)$ and mutual information (fit_m) between $y_1(n)$ and $y_2(n)$ for different θ values; (b) one of the observed mixtures and the original Lorenz chaotic signal (where time is given in arbitrary units – a.u.); (c) same corrupted observation and the recovered chaotic source when the separating system is adapted to provide maximal independence in the output vector.

and the recovered noise-free chaotic signal after adjusting the separating system. It can be noted that middle and lower panel are similar, which is consequence of the good achieved performance.

In addition to the denoising task, ICA can also be used to separate mixture of chaotic signals if the independence assumption is not violated [36]. Notice that the case of multiple chaotic signals can be particularly relevant for communication systems based on multiple-input and multiple-output channel models [46], which represents an important communication paradigm.

Although ICA, as a method of great generality, is a most useful tool to deal with the problem shown in Fig. 6, the incorporation of more knowledge about the features of the involved signals has the potential of performance improvement and of extending the domain of application of BSE and BSS. For instance, ICA can be safely adopted to separate two independent chaotic signals, but probably is not suitable to separate two state variables of the same dynamical system due to the violation of the independency assumption. Moreover, there are situations in which the mixing process does not fit the "linear and instantaneous" mould or in which it is impossible to assume that the mixing matrix is full-rank (e.g. when the number of available mixtures is lower than the number of sources, the so-called underdetermined case). In these situations, nonetheless, approaches other than ICA may still be effective in performing BSE or BSS, being this a central research topic in modern unsupervised signal processing theory [44].

The incorporation of more specific a priori information to deal with chaotic signal processing is the goal of some recent works in the BSE and BSS area. For example, Soriano et al. [37] propose that the separating system can be adapted by seeking, out of a mixture of a chaotic signal and a noise term, the most deterministic output to a BSE structure. In this case, determinism is objectively quantified by the statistics of a recurrence plot, as defined by Marwan et al. [47], which provides criteria that are more robust to noise than the classical ICA measures, i.e. kurtosis and mutual information. On the other hand, this approach is not so effective when mixtures of several chaotic signals and noise are considered, as recurrence plots per se are not so clear in distinguishing multiple chaotic signals in a blind fashion [37].

In summary, denoising chaotic time series is a practical problem that is the object of study of researchers with different backgrounds, which, to a certain extent, justifies the diversity of approaches found in the literature. Our aim here was to focus on approaches also related to signal separation and extraction, which can play a key role in chaos-based communication systems.

2.3. Delay

In practical implementations of chaos-based communications, it is expected that the chaotic oscillators from the transmitter and receiver systems to be physically apart from each other, such that a distance must be traveled by the transmitted signal, implying time delays.

In fact, chaos communication can be effective in scenarios involving time delays, as it was shown in simultaneous bidirectional communication schemes between delay-coupled oscillators, e.g. [48,49]. As one considers realistic situations, where time delays are almost surely present, chaos-based communication may be exploited as a means to overcome the inconveniences caused by channel delay yet maintaining a conceptually simple framework. Nevertheless, as the concept of simultaneous bidirectional chaos-based communication relies on synchronization, one needs to determine what synchronization means in the context of delay-coupled oscillators. Following, we briefly address the phenomenon of *isochronal synchronization*, which is the backbone of the simultaneous bidirectional chaos-based communication framework.

2.3.1. Isochronal synchronization of mutually delay-coupled oscillators

Recent results have shown that mutually coupled chaotic systems are capable of realizing zero-lag synchronization even in an environment with channel delay. Such form of synchronization is named isochronal synchronization, and it has appeared in numerical simulations [50–52], in experimental setups [49,50,52] and, more recently, through analytical results based on the Lyapunov–Krasovskii stability theory [53].

Consider for instance a pair of delay-coupled systems whose individual dynamics is given by the Rössler equations

$$\begin{aligned} \dot{x}_1 &= -x_2 - x_3, \\ \dot{x}_2 &= x_1 + a x_2, \\ \dot{x}_3 &= b + (x_1 - c)x_3, \end{aligned} \tag{23}$$

with $a = b = 0.2$ and $c = 7$. Consider the coupling scheme presented in Fig. 8. It can be shown both numerically and analytically that isochronal synchronization occurs and it is stable for adequately designed feedback matrix K [53]. Fig. 9 shows the numerical simulation for the synchronization of two Rössler systems, for $\tau = 10^{-3}$ and

$$K = \begin{pmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -10 \end{pmatrix}, \tag{24}$$

where the error vector is defined as $\mathbf{e}(t) = \mathbf{x}(t) - \mathbf{y}(t)$ and t is the continuous-time variable.

Isochronal synchronization allows the conception of simultaneous bidirectional communication schemes despite of the delay time introduced by the channel, by using as little as one oscillator at each end of the communication link. As it has been shown in numerical simulations within the framework of lasers [48] and experimental setups using electronic circuits [49], isochronal synchronization allows not only the realization of simultaneous bidirectional communication, but also the encryption and decryption of information and negotiation of secret keys. Although mutual driving of chaotic systems subject to coupling delay are very sensitive to the magnitude of time delay, parameter mismatches and additive noise, successful realizations of chaos-based communication in simple experimental setups [49] reinforce the possibility of fully exploiting chaotic dynamics to transmit information in such realistic environments presenting delay.

2.3.2. Chaos-based simultaneous bidirectional transmission

The idea of simultaneous bidirectional transmission and reception is based on the fact that the systems can synchronize with zero lag in the presence of channel delay and thus information can be injected and retrieved properly at both ends of the communication link. According to [49], by considering a bidirectional binary system, as simultaneous transmission occurs, there are two possibilities of symbol encoding at a given moment: (i) either both systems are encoding the same symbol or (ii) each system is encoding a different symbol. That being considered, two different circumstances concerning the maintenance of the synchronous state arise. In the first case, as mutually coupled systems are coding the same symbol at a given instant, the synchronization is maintained due to the fact that both systems are subject to the same perturbation caused by the information injection. In this case, an eavesdropper would have no clue of which symbol is being encoded [49]. In the second case, as the systems encode different symbols at a given instant the message is treated as additive noise at the receiving circuit and it is filtered due to chaos-pass filtering of the synchronized systems [54].

It is claimed that as the systems encode the same symbol at the same time, an eavesdropper could not possibly infer the symbol being encoded [49]. Thus, according to [49], simultaneous bidirectional communication can be used in practice to negotiate secret keys, as both sides of the link can agree on a secret key consisting on the first N symbols that coincide, in such way that a key with dimension as large as desired can be negotiated in a public channel. This framework is advantageous since it eliminates the need for a private channel to exchange key information and thus greatly simplifies the communication process.

It is worth noting that the modulation of different symbols in a given instant causes bursts in the synchronous state. As resynchronization occurs, the next symbol in the sequence can be encoded at both ends and, as a consequence, the encoding rate is given by the inverse of the resynchronization time. For instance, in the case of semiconductor lasers, resynchronization takes around 0.3 ns, what allows a maximum encoding rate of 3Gbps per system [48].

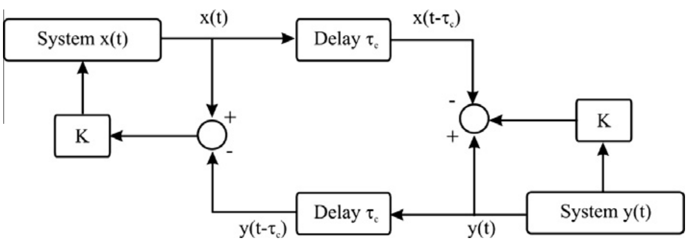

Fig. 8. Scheme of mutually driven Rössler systems.

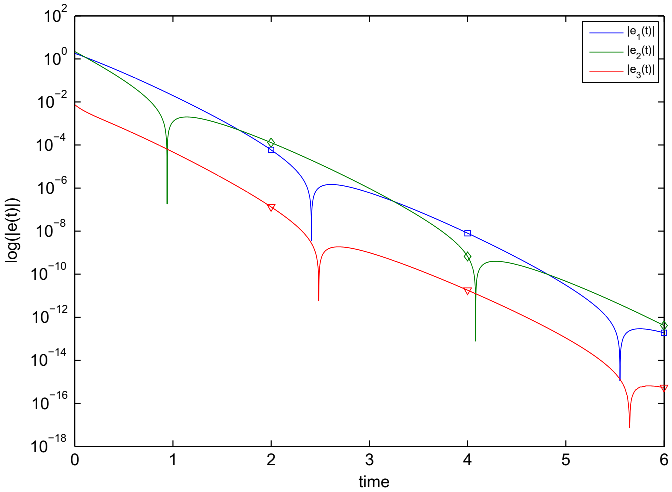

Fig. 9. Synchronization error.

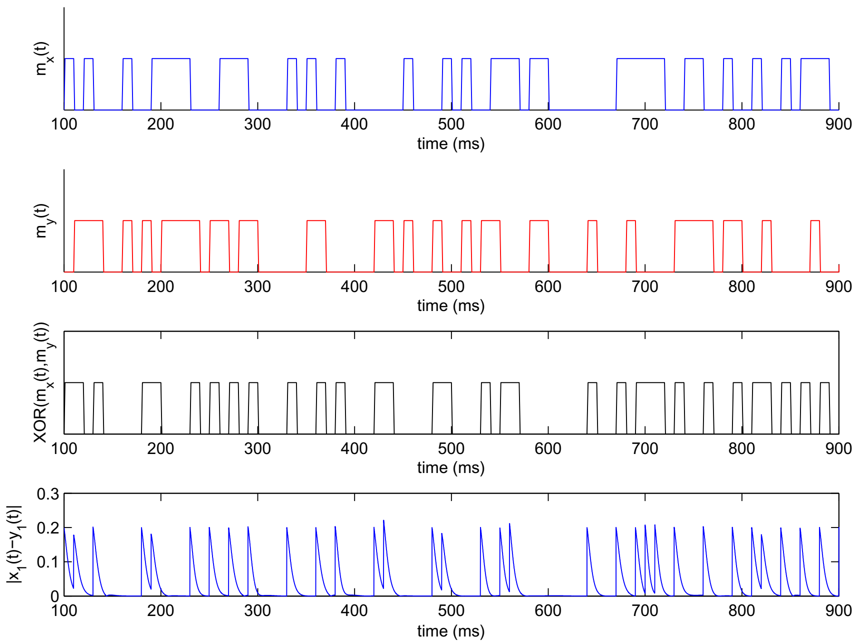

Fig. 10. Example of simultaneous bidirectional chaos-based communication based on isochronal synchronization.

By considering that source coding efficiency and data compression rates using chaotic dynamics can potentially overpass conventional source coding and data compressing techniques [55], chaos-based simultaneous bidirectional communication based on isochronal synchronization tends to be of increasing research interest due to its great information bearing capacity yet maintaining simple framework and cost-effective implementation.

Fig. 10 illustrates the outcome of simultaneous data transmission: $m_x(t)$ and $m_y(t)$ are bit streams transmitted by systems $\mathbf{x}$ and $\mathbf{y}$, respectively. The dynamics of the synchronization error allows the identification of spikes that correspond to each system encoding a different bit. It follows that the received bit streams can be recovered after a simple XOR operation, as it is shown in the figure.

3. Conclusions and perspectives

In this review paper, we succinctly described innovative and up-to-date techniques that can be applied to allows for using chaos-based communication systems in more realistic channel conditions.

We provide a glimpse of new techniques for deal with bandwidth limitation, lack of robustness of chaos-synchronization in respect to additive noise and delay in bidirectional communications.

All these ideas can be taken as works in progress and, certainly, there is a long path before chaos-based communication can be considered as a technically viable alternative in commercial applications. However, the fact that there is a large number of researchers of different areas working together on this subject suggests that chaos-based communications will have a promising future.

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