Estimation of Nonuniform Invariant Density Chaotic Signals with Applications in Communications ⋆

Marcio Eisencraft * **Marcos Almeida do Amaral** **

* *Escola de Engenharia da Universidade Presbiteriana Mackenzie and Escola Politécnica da Universidade de São Paulo, São Paulo, Brazil (e-mail: marcioft@mackenzie.br; marcio@lcs.poli.usp.br)*
** *Escola de Engenharia da Universidade Presbiteriana Mackenzie, São Paulo, Brazil (e-mail: marcos.amaral@mackenzista.br).*

Abstract: In this paper we compare two techniques applied to the estimation of discrete-time one-dimensional chaotic orbits: the maximum likelihood estimation and the modified Viterbi algorithm. As the second one performs better, it is used in two digital modulation schemes based on identification of chaos generator maps: the Modified Maximum Likelihood Chaos Shift Keying using one and two maps. Both have better symbol error rate characteristics than non-coherent chaos communication schemes.

Keywords: Chaos, Estimation algorithms, Communications systems, White noise, Nonlinear systems

1. INTRODUCTION

Recently, various digital modulations using chaotic carriers have been proposed, e.g. (Kennedy et al., 2000b; Lau and Tse, 2003; Stavroulakis, 2005; Tam et al., 2006) and references therein. Among them, Chaos Shift Keying (CSK) and its variants based on noncoherent or differential demodulation are promising for practical applications (Kennedy et al., 2000a,b; Lau and Tse, 2003; Stavroulakis, 2005; Tam et al., 2006; Kolumbán et al., 1998a,b). However, these systems still have lower performance than conventional equivalent ones when it comes to symbol error rate in Additive White Gaussian Noise (AWGN) channel (Kennedy et al., 2000b; Lau and Tse, 2003; Escribano et al., 2006; Xiaofeng et al., 2004). This happens basically because the receiver does not use information on the chaotic maps used to generate the transmitted signals. The performance of these systems would be basically the same if one uses random instead of chaotic signals.

An alternative for improving these results allowing explicit use of map information involves to employ estimation techniques to counter channel noise before demodulation (Kisel et al., 2001; Escribano et al., 2006; Xiaofeng et al., 2004). This can be accomplished in a variety of forms.

Papadopoulos and Wornell (1993) deduced the Maximum Likelihood Estimator (MLE) for a class of tent maps. This method is asymptotically efficient, which means that it is asymptotically unbiased and attains the Cramer-Rao Lower Bound (Eisencraft and Baccala, 2006, 2008) for large number of samples or large Signal to Noise Ratio (SNR). However, this method is not easily extensible to broader classes of maps.

Another attractive way to obtain a cleaner estimated signal is to use the Viterbi Algorithm (VA). This requires interpreting the chaotic signals generated by any map as Markov processes that, at each instant, achieve one of N_S possible states defined by a partition of the domain U into N_S subintervals. The VA was applied to estimate chaotic orbits in (Kisel et al., 2001; Xiaofeng et al., 2004) who used uniform partitions, which is a suitable choice for maps with uniform invariant density. Recently, we have extended these results for general one-dimensional maps using adequate partitions (Eisencraft et al., 2009). Our proposal is called the *Modified VA* (MVA).

The objectives of this paper are twofold: i) to compare MLE and MVA and ii) to study the application of MVA in communication systems, like the Maximum-Likelihood CSK (ML-CSK) proposed by Kisel et al. (2001).

The paper is organized as follows. In Section 2 we review MLE and MVA for the estimation of chaotic signals and compare their performance. In Section 3 we present the application of estimation methods in identification of chaotic systems and its possible uses in chaotic communications. Finally, Section 4 deals with our conclusions.

2. ESTIMATION OF CHAOTIC SIGNALS

The estimation problem treated here can be stated as follows. An N-point sequence $s'(n)$ is observed. It is modeled as

$$s'(n) = s(n) + w(n),\ 0 \leq n \leq N-1, \tag{1}$$

where $s(n)$ is an orbit of a one-dimensional system

$$s(n) = f(s(n-1)) \tag{2}$$

and $w(n)$ is AWGN with variance σ^2. The map $f(.)$ is defined in an interval U. We seek an estimate $\hat{s}(n)$ of the orbit $s(n)$.

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The Cramer-Rao Lower Bound (CRLB), the minimum mean square error that an estimator of the initial condition $s(0)$ can attain, was derived by Eisencraft and Baccala (2006, 2008).

In this section, we describe two estimation technique for this problem: MLE and MVA.

We define the estimation gain G_{dB} in decibels as

$$G_{dB} = 10\log\left(\frac{\sigma^2}{e}\right), \tag{3}$$

where $e = \overline{(\hat{s}(n) - s(n))^2}$.

2.1 Maximum Likelihood Estimator

The MLE of a scalar parameter θ is the value that maximizes the likelihood function $p(\mathbf{x};\theta)$ where $\mathbf{x}$ is the observation vector (Kay, 1993). The motivation for this definition is that $p(\mathbf{x};\theta)d\mathbf{x}$ is the probability of observing $\mathbf{x}$ in a small volume $d\mathbf{x}$ for a given θ.

The MLE is asymptotically efficient. This means that it is asymptotically unbiased and attain the CRLB for large N or for high Signal-to-Noise Ratios (SNRs). For the estimation of a chaotic orbit generated by a map with uniform invariant density it was deduced by Papadopoulos and Wornell (1993). For more general maps it may be not easy to obtain the MLE.

In (Papadopoulos and Wornell, 1993), it is shown that the estimation gain for an N-point orbit is limited by

$$G_{dB} \leq 10\log(N+1). \tag{4}$$

2.2 Modified Viterbi algorithm

The MVA here described is based on (Dedieu and Kisel, 1999). We have generalized this algorithm for maps with nonuniform invariant density (Eisencraft et al., 2009).

Consider the domain U as the reunion of disjoint intervals U_j, $j = 1, 2, \ldots, N_S$. In a given instant n, we define that the signal *state* is $q(n) = j$ if $s(n) \in U_j$. A $(k+1)$-length state sequence is represented by

$$\mathbf{q}_k = [q(0), q(1), \ldots, q(k)]^T \tag{5}$$

and the first $k+1$ observed samples by

$$\mathbf{s}'_k = [s'(0), s'(1), \ldots, s'(k)]^T. \tag{6}$$

To simplify notation, we define the N-length sequences $\mathbf{q}_{N-1} \equiv \mathbf{q}$ and $\mathbf{s}'_{N-1} \equiv \mathbf{s}'$. Furthermore, the center of the interval U_j is denoted by $B(j)$.

Given $\mathbf{s}'$, we seek an estimated state sequence $\hat{\mathbf{q}}$ that maximizes the posterior probability

$$P(\hat{\mathbf{q}}|\mathbf{s}') = \max_{\mathbf{q}} P(\mathbf{q}|\mathbf{s}'). \tag{7}$$

Using Bayes' theorem,

$$P(\mathbf{q}|\mathbf{s}') = \frac{p(\mathbf{s}'|\mathbf{q})P(\mathbf{q})}{p(\mathbf{s}')}, \tag{8}$$

where $p(\mathbf{s}')$ and $p(\mathbf{s}'|\mathbf{q})$ are, respectively, the Probability Density Function (PDF) of $\mathbf{s}'$ and the PDF of $\mathbf{s}'$ given that the state sequence of the signal is $\mathbf{q}$. The probability $P(\mathbf{q})$ is the chance of obtaining the state sequence $\mathbf{q}$ when $f(.)$ is iterated.

This way, we must find the argument $\hat{\mathbf{q}}$ so that

$$\hat{\mathbf{q}} = \arg\max_{\mathbf{q}} P(\mathbf{q}|\mathbf{s}') = \arg\max_{\mathbf{q}} p(\mathbf{s}'|\mathbf{q})P(\mathbf{q}). \tag{9}$$

It is important to notice that because of the form as the signals are generated and of the AWGN model, $\mathbf{q}_k$ is a first order Markov process when we consider k as time variable. So,

$$P(\mathbf{q}_k) = P(q(k)|q(k-1))\,P(\mathbf{q}_{k-1}), \tag{10}$$

where $P(q(k)|q(k-1))$ is the transition probability from the state $q(k-1)$ to $q(k)$.

Besides, taking into account the independency of noise samples,

$$p(\mathbf{s}'_k|\mathbf{q}_k) = \prod_{n=0}^{k} p(s'(n)|q(n)) = \prod_{n=0}^{k} p_w(s'(n) - s(n)) \approx \prod_{n=0}^{k} p_w(s'(n) - B(q(n))), \tag{11}$$

where $p_w(.)$ is the PDF of the noise. The approximation in Eq. (11) is valid only if N_S is sufficiently large.

Using Eqs. (9-11), we can express $P(\mathbf{q}|\mathbf{s}')$ as a product of state transition probabilities by conditional observation probabilities. Thus we conclude that $\hat{\mathbf{q}}$ is the sequence that maximizes

$$\left(\prod_{n=1}^{N-1} P(q(n)|q(n-1))\,p(s'(n)|q(n))\right) P(q(0)). \tag{12}$$

Choosing the partition U_j, $j = 1, 2, \ldots, N_S$ so that the probability of each possible state $q(n) = j$ is the same for all j, the last term in Eq. (12), $P(q(0))$, can be eliminated and we get

$$\hat{\mathbf{q}} = \arg\max_{\mathbf{q}} \prod_{n=1}^{N-1} P(q(n)|q(n-1))\,p(s'(n)|q(n)), \tag{13}$$

as (Kisel et al., 2001) do. Note, however, the importance of the choice of the partition in obtaining this result. This fact was recently pointed by us (Eisencraft et al., 2009).

To find $\mathbf{q}$ that maximizes the product in Eq. (13) is a classic problem for which an efficient solution is given by the VA (Viterbi, 1967; Forney, 1973), applied to the estimation of chaotic signals for the first time by (Marteau and Abarbanel, 1991). Using the VA avoids doing an exhaustive search on the $(N_S)^N$ possible state sequences for an N-point signal.

Let $\gamma(n,j)$ be the probability of the most probable state sequence, in the maximum likelihood sense, that ends in state j, at instant $n \geq 1$, given the observed sequence $\mathbf{s}'$, or

$$\gamma(n,j) = \max_{\mathbf{q}_n} P(\mathbf{q}_{n-1}, q(n) = j|\mathbf{s}'). \tag{14}$$

Using Eqs. (10-11), $\gamma(n,j)$ can be calculated in the recursive form

$$\gamma(n,j) = \max_i \left[\gamma(n-1,i)a_{ij}\right] b_j(s'(n)), \tag{15}$$

for $n > 1$ where

$$a_{ij} = P(q(n) = j|q(n-1) = i) \tag{16}$$

and

$$b_j\left(s'(n)\right) = p\left(s'(n)|q(n) = j\right). \tag{17}$$

The coefficients a_{ij} are the state transition probabilities that depend on the map $f(.)$ and on the partition. We define the transition probabilities matrix as

$$\mathbf{A}_{N_S \times N_S} = a_{ij}, 1 \leq i, j \leq N_S. \tag{18}$$

The coefficients $b_j(.)$ are the observation conditional probabilities that depend only on the PDF of the noise $p_w(.)$.

VA works in two passes, the forward and the backward one:

- **Forward pass**: for each instant $1 \leq n \leq N-1$, Eqs. (14 - 15) are used to calculate $\gamma(n, j)$ for the N_S states. Among the N_S paths that can link states $j = 1, \ldots, N_S$ at instant $n-1$ to state j at instant n, only the most probable one is maintained. The matrix $\varphi(n, j)$, $n = 1, \ldots, N-1$, $j = 1, \ldots, N_S$, stores the state at instant $n-1$ that takes to state j with maximal probability. In the end of this step, at instant $n = N-1$, we select the most probable state as $\hat{q}(N-1)$.
- **Backward pass**: for obtaining the most probable sequence, it is necessary to consider the argument i that maximizes Eq. (15) for each n and j. This is done defining

$$\hat{q}(n) = \varphi\left(n+1, \hat{q}(n+1)\right), \; n = N-2, \ldots, 0. \tag{19}$$

Once obtained $\hat{q}(n)$, the estimated orbit is given by the centers of the subintervals related to the most probable state sequence,

$$\hat{s}(n) = B\left(\hat{q}(n)\right), \; n = 0, \ldots, N-1. \tag{20}$$

To apply the VA it is necessary to choose a partition so that the probability of an orbit point to be in any state is the same, so that $P\left(q(0)\right)$ in Expression (12) can be eliminated. This means that if a given map has invariant density $p(s)$ (Lasota and Mackey, 1985), we should take N_S intervals $U_j = [u_j; u_{j+1}]$ so that, for every $j = 1, \ldots, N_S$,

$$\int_{u_j}^{u_{j+1}} p(s)ds = \frac{1}{N_S}. \tag{21}$$

Using the ergodicity of chaotic orbits (Lasota and Mackey, 1985), it is possible to estimate $p(s)$ for a given $f(.)$ and consequently to find the correct partition.

The maps taken as examples in (Xiaofeng et al., 2004) and (Kisel et al., 2001) have uniform invariant density and the authors have proposed the use of equal length subintervals. However, this choice cannot be generalized for an arbitrary one-dimensional map.

The use of VA with the correct partition is called here *Modified Viterbi Algorithm* (MVA).

2.3 Comparing MVA and MLE

MLE's performance is strongly influenced by the length of the estimated orbit N, as shown by the Inequality (4). MVA is more sensitive to the number of subsets N_S used in the partition.

Simulations show that the gain obtained via MLE monotonically increases with SNR being bounded by the CRLB. Using MVA, gain attains a maximal value and decays, becoming even negative (in dB) due to quantization error.

The estimation gain for both methods used on orbits of the tent map $s(n+1) = f_T\left(s(n)\right)$, $0 \leq n \leq N-1$, with

$$f_T(s) = 1 - 2|s|, \tag{22}$$

corrupted by AWGN is shown in Figure 1. For the MVA we show the result only for $N = 20$ because simulations show there is no significative change for other values of N.

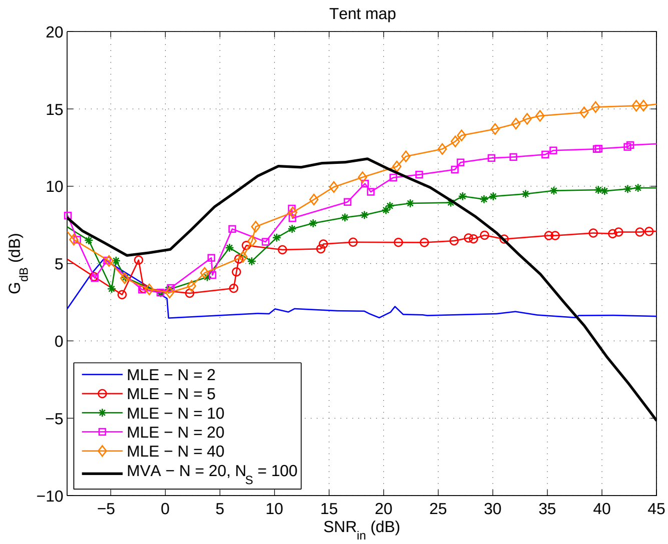

Fig. 1. Estimation gain for MLE and VA for tent map Eq. (22).

From Figure 1 we can see that for SNR $\leq$ 20dB, witch is the usual interesting condition, MVA performance is superior.

These results, allied to the fact that MVA can be applied for a broader class of maps, have leaded to the choice of MVA in the communication applications described next.

3. CHAOTIC SYSTEM IDENTIFICATION: APPLICATIONS IN COMMUNICATION

In this section we propose two binary digital modulation using chaotic system identification. They are the *Modified Maximum Likelihood Chaos Shift Keying* (MMLCSK) using one and two maps. Both are based on the ones proposed by (Kisel et al., 2001). We have modified them using nonuniform partitions for the MVA as discussed in the previous section. This way, it is possible to test the performance of nonuniform invariant density maps.

3.1 MMLCSK using two maps

In this case, each symbol is associated with a different map $f_1(.)$ and $f_2(.)$. To transmit a **0**, the transmitter sends an N-point orbit $s_1(.)$ of $f_1(.)$ and to transmit a **1**, it sends an N-point orbit $s_2(.)$ of $f_2(.)$.

Maps must be chosen so that their state transition probabilities matrix $\mathbf{A}_1$ and $\mathbf{A}_2$ are different. Estimating $s_1(n)$ using MVA with $\mathbf{A}_2$ must produce a small estimation gain or even a negative (in dB) one. The same must happen when we try to estimate $s_2(n)$ using $\mathbf{A}_1$.

The receiver for MMLCSK using two maps is shown in Figure 2. The Viterbi decoders try to estimate the original $s(n)$ using $\mathbf{A}_1$ or $\mathbf{A}_2$. For each symbol, the estimated state sequences are $\hat{\mathbf{q}}_1$ and $\hat{\mathbf{q}}_2$.

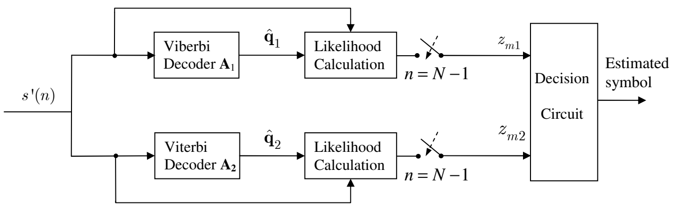

Fig. 2. Receiver for MMLCSK using two maps.

Given the observed samples, z_{m1} e z_{m2} are proportional to the probability of obtaining $\hat{\mathbf{q}}_1$ and $\hat{\mathbf{q}}_2$ respectively. More precisely,

$$z_{m1} = \prod_{n=1}^{N-1} P(\hat{q}_1(n)|\hat{q}_1(n-1), \mathbf{A}_1)p(s'(n)|\hat{q}_1(n)), \quad (23)$$

$$z_{m2} = \prod_{n=1}^{N-1} P(\hat{q}_2(n)|\hat{q}_2(n-1), \mathbf{A}_2)p(s'(n)|\hat{q}_2(n)). \quad (24)$$

In this equations we have used the likelihood measure of Eq. (13). The probability $P(\hat{q}(n)|\hat{q}(n-1), \mathbf{A}_i)$ can be read directly from $\mathbf{A}_i$ and $p(x'(n)|\hat{q}(n))$ depends only on the noise and can be approximated as described by (Dedieu and Kisel, 1999).

Choosing the largest between z_{m1} e z_{m2} we can *identify* the map used in the transmitter with maximum likelihood and, this way, decode the transmitted symbol.

Given a map $f_1(.)$, an important problem is to find a map $f_2(.)$ so that its probability transition matrix $\mathbf{A}_2$ permits to discriminate between the likelihood measures of Eqs. (23) and (24). For piecewise linear maps on the interval $U = [-1, 1]$ we can use the following rule adapted from (Kisel et al., 2001):

$$f_2(s) = \begin{cases} f_1(s) + 1, & f_1(s) < 0 \\ f_1(s) - 1, & f_1(s) \geq 0 \end{cases}. \quad (25)$$

Figure 3 shows the construction of map $f_2(.)$ from $f_1(.) = f_T(.)$. This way, $f_1(s)$ and $f_2(s)$ map a point s a unity away.

In this case, using an uniform partition for $N_S = 5$ we have

$$\mathbf{A}_1 = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} 0 & 0 & 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 1/2 & 1/2 & 0 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0 & 1/3 & 1/3 & 1/3 \end{bmatrix}. \quad (26)$$

It can be shown that almost every orbit generated by $f_2(.)$ are in fact chaotic (Kisel et al., 2001). Note however that this method is not necessarily optimal and must be used prudently. There is no guaranty that the orbits of $f_2(.)$ given by Eq. (25) are chaotic in general.

For instance, if we apply the same strategy for the quadratic map

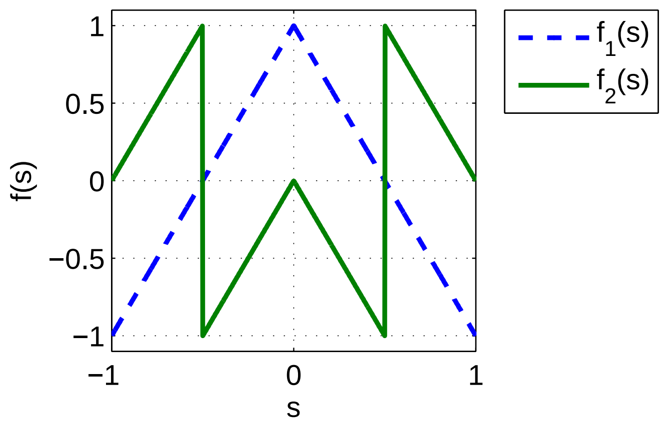

Fig. 3. Construction of map $f_2(.)$ for $f_1(.) = f_T(.)$ using Eq. (25).

$$f_1(s) = f_Q(s) = -2s^2 + 1, \quad (27)$$

we obtain $f_2(s)$ show in Figure 4. All the orbits of this map converge to a stable fixed point, (Devaney, 2003) at $s = 0$ and hence are not chaotic at all.

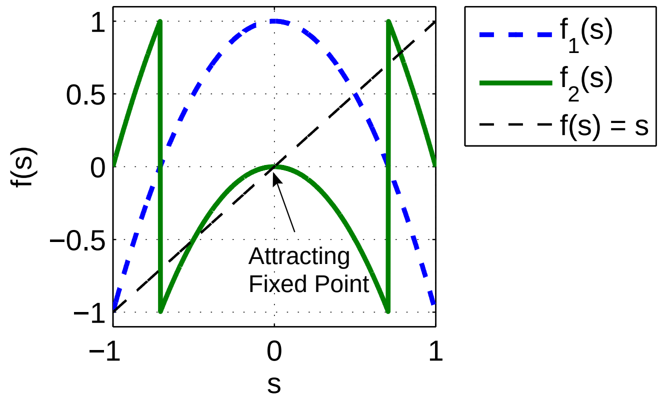

Fig. 4. Construction of $f_2(.)$ for $f_1(.) = f_Q(.)$ using Eq. (25). Note the attracting fixed point.

In the simulations, we have used $f_2(.) = -f_Q(.)$ shown in Figure 5. This map is possibly not optimum because points next to the roots of $f_1(.)$ and $f_2(.)$ are mapped near to each other by both functions. The transition matrix for these two maps for $N_S = 5$ using the partition obeying Eq. (21) are

$$\mathbf{A}_1 = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \end{bmatrix}, \ \mathbf{A}_2 = \begin{bmatrix} 0 & 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 1/2 \end{bmatrix}. \quad (28)$$

In this case, it can be shown that $f_2(.)$ once more generates chaotic orbits (Devaney, 2003). However, note that a_{23} e a_{43} exhibit nonzero probabilities in both matrices what will probably generate errors in the MMLCSK receptor. This way, we expect a worst performance of this couple of maps when compared to the one with matrices given by Eq. (26).

To find $f_2(.)$ given a map $f_1(.)$ that presents optimal properties when it comes to identification throw the matrices

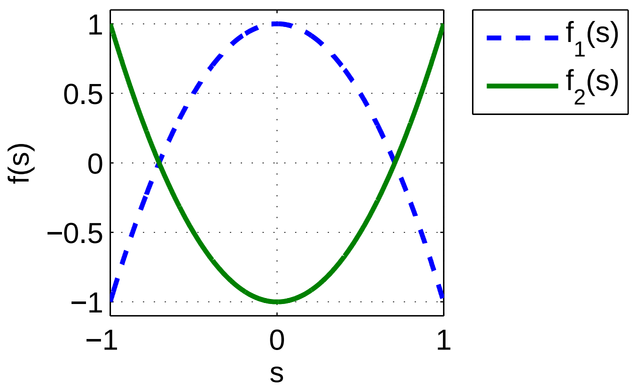

Fig. 5. Construction of $f_2(.)$ for $f_1(.) = f_Q(.)$ used in simulations.

$\mathbf{A}_1$ and $\mathbf{A}_2$ is an open problem. As shown by the last example, it is necessary to impose that $f_2(.)$ generates chaotic orbits.

Figure 6 shows examples of transmitted MMLCSK using two maps for $f_1(.) = f_T(.)$ and $f_2(.) = f_Q(.)$.

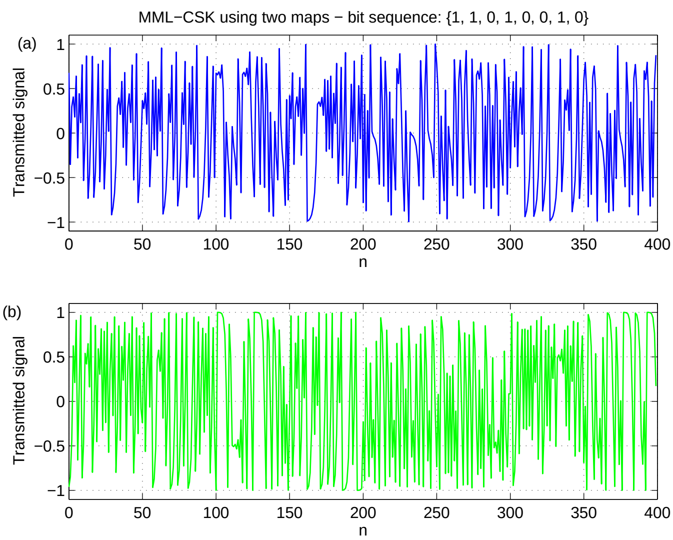

Fig. 6. MML-CSK signals using two maps for the bit sequence $\{1, 1, 0, 1, 0, 0, 1, 0\}$: (a) $f_T(.)$; (b) $f_Q(.)$. In both cases, it is used 50 samples per bit.

3.2 MMLCK using one map

As an alternative, it is possible to construct a communication system based on MVA estimation using just one map. In this case, according to the symbol that is intended to be communicated, the chaotic signal is directly transmitted or an invertible transformation is applied on the sequence. This operation must modify the sequence so that it is no more a valid orbit of the used map. This way, it is no longer necessary to find a map $f_2(.)$.

In the binary case, for maps that are not odd, this transformation can be, for instance, $T(s) = -s$ witch can be undone multiplying again the sequence by -1. To transmit a **0**, we send an N-point orbit $s_1(.)$ of $f_1(.)$. To transmit a **1**, we send $-s_1(.)$.

The receiver for this system is shown in Figure 7. The variables z_{m1} and z_{m2} are calculated by Eq. (23). However, when calculating z_{m2}, $s'(n)$ is substituted by $-s'(n)$. So, when a **0** is received, the likelihood expressed by z_{m1} must be greater than z_{m2} because $-s_1(n)$ is not an orbit of $f_1(.)$. The opposite is true when a **1** is received.

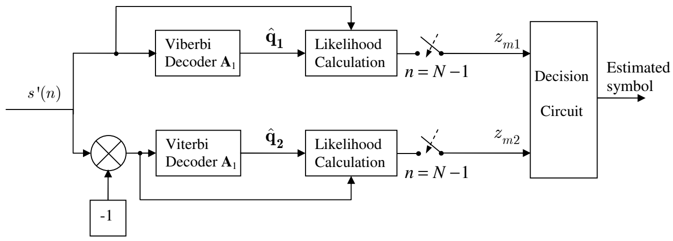

Fig. 7. Receiver for MMLCSK using two maps.

It is relevant to note that this scheme can be easily generalized for an M-ary modulation, $M > 2$. For this it is necessary just to consider other invertible transformations.

Figure 8 shows examples of modulated signals using one map for $f_1(.) = f_T(.)$ and $f_1(.) = f_Q(.)$.

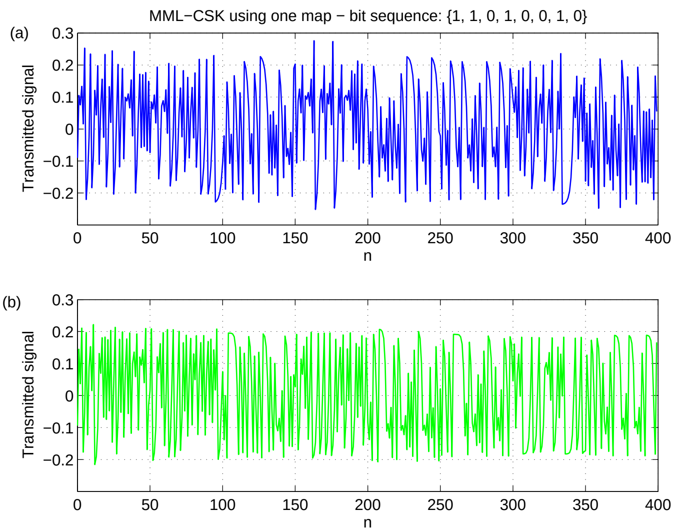

Fig. 8. MML-CSK signals using one map for the bit sequence $\{1, 1, 0, 1, 0, 0, 1, 0\}$: (a) $f_T(.)$; (b) $f_Q(.)$. In both cases, it is used 50 samples per bit.

3.3 Numerical simulations

Figure 9 show the Bit Error Rate (BER) for the MML-CSK using one and two maps as a function of the SNR where the latter is defined as the energy per bit divided by the power spectral density of the noise. In the estimation and identification process it was considered $N_S = 100$ subsets and $N = 50$ samples per bit. For sake of comparison, it is also shown the performance of Chaos On-Off Keying (COOK) (Kolumbán et al., 1998a), the non-coherent chaos communication that does not use estimation with best performance. This system is based only on energy estimation to decode the signal.

Our simulations show that MMLCSK using one map has a slightly better performance than MMLCSK using two maps. Besides $f_T(.)$ performs better than $f_Q(.)$. This last results confirms the importance of the choose of map and transform to be employed.

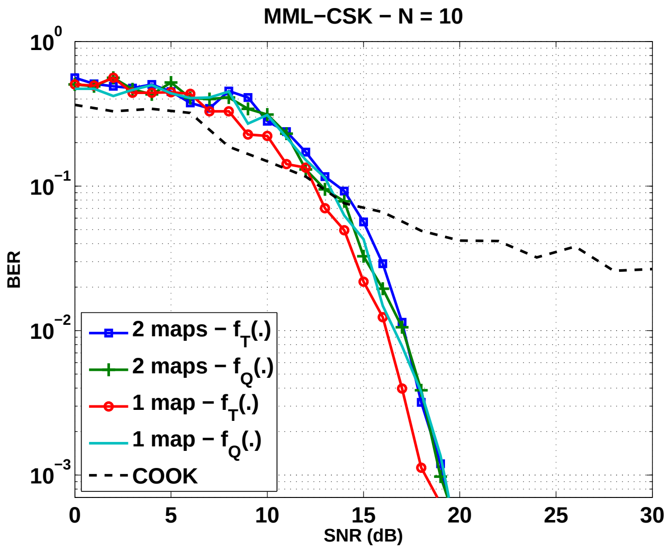

Fig. 9. Bit Error Rate (BER) for the tested MMLCSK modulations. Each bit was represented by $N = 10$ samples.

4. CONCLUSIONS

This paper compares two estimation techniques for chaotic signals immersed in white gaussian noise: MLE and MVA.

Following, MVA was applied in the identification of chaotic maps from its orbits and its application in communications. It was proposed the MMLCSK, which is the MLCSK (Kisel et al., 2001) using MVA. Compared with methods that do not use estimation, these systems performs better in terms of BER in AWGN. The cost of this improvement is the complexity of the receiver. However, the information is coded on the dynamic of the chaotic system and not in easily measured properties like energy, what makes it more difficult to be detected without authorization. It is necessary to know the state transitions matrices to demodulate.

More research is necessary in order to find optimal couple maps and transforms to optimize the discrimination between maps in the receiver.

REFERENCES

Dedieu, H. and Kisel, A. (1999). Communications with chaotic time series: probabilistic methods for noise reduction. *International Journal of Circuit Theory and Applications*, 27(6), 577–587.

Devaney, R.L. (2003). *An Introduction to Chaotic Dynamical Systems, 2nd Edition*. Westview Press.

Eisencraft, M. and Baccala, L.A. (2006). Estimating the initial conditions of chaotic orbits: performance bounds. In *Proc. 1st IFAC Conference on Analysis and Control of Chaotic Systems*, 291–295. Reims, France.

Eisencraft, M. and Baccala, L.A. (2008). The Cramer-Rao Bound for initial conditions estimation of chaotic orbits. *Chaos Solitons & Fractals*, 38(1), 132–139.

Eisencraft, M., do Amaral, M.A., and Lima, C.A.M. (2009). Estimation of chaotic signals with applications in communications. In *Proc. 15th IFAC Symposium on System Identification*. Saint-Malo, France. To be published.

Escribano, F., Lopez, L., and Sanjuan, M. (2006). Exploiting symbolic dynamics in chaos coded communications with maximum a posteriori algorithm. *Electronics Letters*, 42(17), 984–985.

Forney, G.D. (1973). The viterbi algorithm. *Proc. IEEE*, 61(3), 268–278.

Kay, S.M. (1993). *Fundamentals of statistical signal processing: estimation theory*. Prentice-Hall, Inc., Upper Saddle River, NJ, USA.

Kennedy, M.P., Kolumbán, G., Kis, G., and Jákó, Z. (2000a). Performance evaluation of fm-dcsk modulation in multipath enviromnments. *IEEE Trans. Circuits Syst. I*, 47(12), 1702–1711.

Kennedy, M.P., Setti, G., and Rovatti, R. (eds.) (2000b). *Chaotic Electronics in Telecommunications*. CRC Press, Inc., Boca Raton, FL, USA.

Kisel, A., Dedieu, H., and Schimming, T. (2001). Maximum likelihood approaches for noncoherent communications with chaotic carriers. *IEEE Trans. Circuits Syst. I*, 48(5), 533–542.

Kolumbán, G., Kennedy, M.P., and Chua, L.O. (1998a). The role of synchronization in digital communications using chaos - part ii: chaotic modulation and chaotic synchronization. *IEEE Trans. Circuits Syst. I*, 45(11), 1129–1140.

Kolumbán, G., P.Kennedy, M., Kis, G., and Jákó, Z. (1998b). Fm-dcsk: a novel method for chaotic communications. In *Proc. ISCAS'98*, volume 4, 477–480. Monterey, USA.

Lasota, A. and Mackey, M. (1985). *Probabilistic Properties of Deterministic Systems*. Cambridge University Press, Cambridge.

Lau, F.C.M. and Tse, C.K. (2003). *Chaos-based digital communication systems*. Springer, Berlin.

Marteau, F.F. and Abarbanel, H.D.I. (1991). Noise reduction in chaotic time series using scaled probabilistic method. *Jornal of Nonlinear Science*, 1(3), 313–343.

Papadopoulos, H. and Wornell, G. (1993). Optimal detection of a class of chaotic signals. *Acoustics, Speech, and Signal Processing, 1993. ICASSP-93., 1993 IEEE International Conference on*, 3, 117–120 vol.3.

Stavroulakis, P. (2005). *Chaos Applications in Telecommunications*. CRC Press, Inc., Boca Raton, FL, USA.

Tam, W.M., Lau, F.C.M., and Tse, C.K. (2006). *Digital Communications with Chaos: Multiple Access Techniques and Performance*. Elsevier Science Inc., New York, NY, USA.

Viterbi, A.J. (1967). Error bounds for convolutional codes and an asymptotically optimum decoding algorithm. *IEEE Trans. Inf. Theory*, 13(2), 260–269.

Xiaofeng, G., Xingang, W., and Meng, Z. (2004). Chaotic digital communication by encoding initial conditions. *Chaos*, 14(2), 358–363.