

# Fourier Transform Time Interleaving in OFDM Modulation

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**Abstract** – This work introduces a new transform-based time interleaving algorithm: FTI-OFDM (Fourier Transform Interleaved OFDM) in which binary information is spread over several consecutive symbols that can be further scrambled in the frequency domain. Simulations are used to show its superiority over the usual binary time interleaving used in ordinary OFDM under several impairment scenarios that include impulsive noise and deep fading.

**Keywords** – Digital communications, OFDM, Time interleaving, digital television

## I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) modulation has been widely used in communications systems due to its robustness against multipath distortion and fading. Application examples include the DVB-T and ISDB-T digital terrestrial television systems [1-3]. To achieve acceptable performance the insertion of both reference pilot carriers and intersymbol time guard intervals are necessary. Performance improvement under deep fading and impulsive noise is usually improved by employing long time interleaving (e.g. up to 0,5s in the ISDB-T system). However, time interleaving does not reduce total bit error rate; burst errors are spread in time until the average bit error rate is reduced within the capabilities of an error correcting code.

This work introduces a novel transform-based time interleaving algorithm, herein called FTI-OFDM (*Fourier Transform Interleaved OFDM*) whereby the binary information is spread over several consecutive symbols by applying an inverse Fourier transform to the binary data.

Comparative simulations between conventional OFDM and FTI-OFDM are presented for several channel impairments scenarios that include second-order effects arising from non-ideal channel estimation in mobile communications.

This paper is organized as follows: after a short review of OFDM (Sec. II), the new algorithm is introduced in Sec. III, which is followed by schemes to overcome common impairments using FTI-OFDM (Sec. IV). Simulation results

of several scenarios (Sec V) are followed by conclusions summarized in Sec. VI.

## II. BACKGROUND

Figure 1 portrays a typical ordinary OFDM system whose baseband signal  $S(t)$  is formally described by:

$$S(t) = \sum_{n=0}^{\infty} \sum_{k=0}^{K-1} C(n,k) \Psi(n,k,t)$$

where  $C(n,k)$  is the complex data for the  $n$ -th OFDM symbol associated with by  $k$ -th carrier ( $k=0,\dots,K-1$ ) where

$$\Psi(n,k,t) = \begin{cases} e^{j2\pi \frac{k-K/2}{Tu}(t-Tg-nTs)} & nTs \leq t < (n+1)Ts \\ 0 & t < nTs, \quad (n+1)Ts \leq t \end{cases}$$

represents each carrier, with  $Tg$ ,  $Ts$  and  $Tu$  respectively standing for the guard interval, the total and the effective symbol durations ( $Ts = Tg + Tu$ ).

After data randomization and Forward Error Correction (FEC), binary data are grouped into symbols  $C(n,k)$  (one for each data carrier). Carriers  $\Psi(n,k,t)$  are usually modulated in QPSK, 16- or 64-QAM. An inverse Fourier transform is used to generate the modulated signal  $S(t)$  with symbol duration equal to  $Tu$ . A time guard interval is then generated, where a copy of the last samples of  $S(t)$ , with duration  $Tg$ , are placed in front of the effective symbol data. This insertion works against multipath distortion if  $Tg$  is longer than the associated propagation spread times of the received signal.

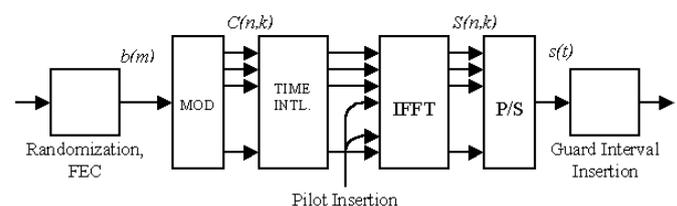


Fig. 1 – OFDM Modulation

Pilot carriers, modulated with known amplitudes and phases, are intermixed with data carriers, and are used by the receiver to estimate and equalize the channel frequency response. This leads to reliable demodulation over fast

changing environments (viz. for mobile and portable receivers).

An important shortcoming of conventional OFDM is its sensitivity to very deep fades and to impulsive noise, in the form of wideband noise bursts, because the latter can affect all demodulated carriers in one or more consecutive symbols. To overcome this, typical OFDM systems employ long time interleaving to dilute such impairments. The interleave length is usually increased until the average error rate gets below the error correction capability [4].

### III. THE PROPOSED METHOD

In the present new proposal, conventional symbol time interleaving is replaced by an inverse Fourier transform performed on subsets of the digital data input. In this way, the individual carriers of an OFDM symbol are no longer modulated by discrete-amplitude QAM or PSK symbols, but rather by a non-quantized complex signal whose distribution is nearly gaussian.

Figure 2 shows a block diagram of the proposed modulator (excluding input randomization and FEC and output guard interval insertion).

The binary input  $b(m)$ , previously randomized and FEC-encoded, is converted,  $p$  bits at a time, into symbols  $Q(n,k)$ , using some complex modulation like QPSK or n-QAM. For instance,  $p=6$  for 64-QAM. A set of  $N$  symbols is then processed via an  $N$ -point inverse discrete Fourier transform, generating  $N$  complex samples  $C(n,k)$ . These samples represent the amplitudes and phases that will modulate the  $k$ -th OFDM carrier, for  $n = 0$  to  $N$  consecutive OFDM symbols.

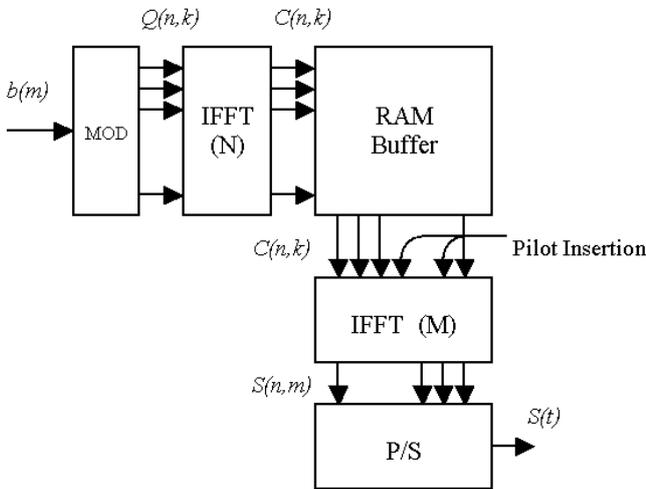


Fig. 2 – FTI-OFDM Modulation

This process is repeated  $K$  times leading to a frame of  $N \times K$  samples  $C(n,k)$  which is then read row-wise, extracting  $K$  samples for some given  $n$ . These are then intermixed with  $M-K$  pilot, auxiliary data and null (guard band) carrier symbols, where  $M$  is the size of the output Inverse Fourier

Transform of the OFDM generator. After IFFT transformation, the resulting  $M$  carrier symbols lead to the  $M$  time samples  $S(n,m)$  of the  $n$ -th transmitted symbol.

Guard time insertion can be applied as usual on the resulting output signal  $S(t)$ .

The rationale behind the proposal is that the energy of impulsive noise occurring in a frame of  $N$  OFDM symbols is distributed among all  $N \times K$  symbols after demodulation, and produces constellation “defocusing” similarly to that produced by additive white gaussian noise of equal energy.

Figure 3 illustrates the relationship among carriers, time and frequency in the FTI-OFDM modulation. In it the highlighted carrier symbols are the  $N$  samples that result from an inverse Fourier transform, performed on an input frame of  $N \times p$  bits. Frame synchronization at the receiver can be extracted from the pilot and auxiliary carriers (not shown).

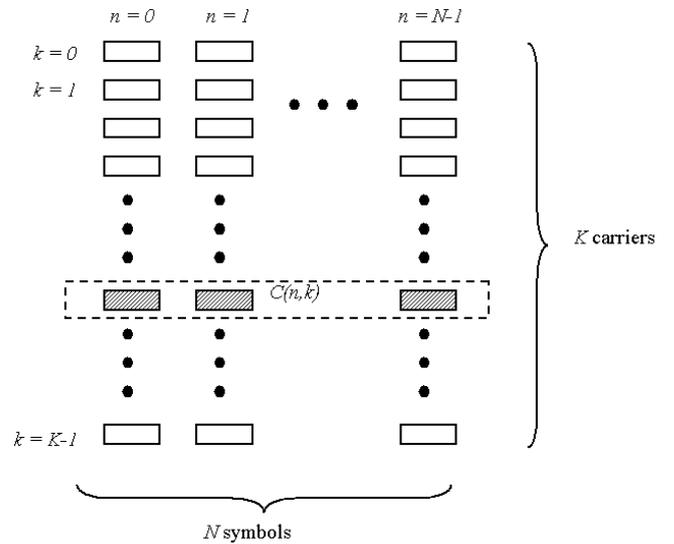


Fig. 3 – Symbols and carriers in FTI-OFDM. For clarity, pilot and extra carriers are not shown.

One should note that the present Fourier transform time interleaving scheme could also be used with single-carrier modulation systems. However, the peak-to-average ratio ( $PAR$ ) of the modulated signal will be degraded in this situation, so its use is advisable only for multiple-carrier systems [5], where the transmitted signals already possess a near-gaussian distribution that is unaffected by FTI.

### IV. FTI-OFDM: AVOIDING COMMON IMPAIRMENTS

Fixed-frequency, narrowband interfering signals hamper all symbols associated with some carriers, leading to high error rates for those symbols. Narrowband energy interference can be spread over to all symbols by assigning different carriers to consecutive samples from the first IFFT (Fig. 2). This spread can be accomplished either by

randomization or via a simple carrier rotation scheme, such as:

$$C'(n,k) = C(n, (k+n) \bmod K)$$

Figure 4 illustrates carrier rotation applied to an FTI-OFDM frame, in the hypothetical case of  $N = K$ .

Data flow and processing can be made more uniform by skewing symbols to the carrier frames obtained from the first IFFT. In this way, as soon as an input frame of  $N \times p$  bits is obtained, one frame of carrier samples  $C(n,k)$  is generated and the first sample is made available for OFDM modulation. Figure 5 illustrates this scheme with and without carrier rotation, again with  $N = K$ .

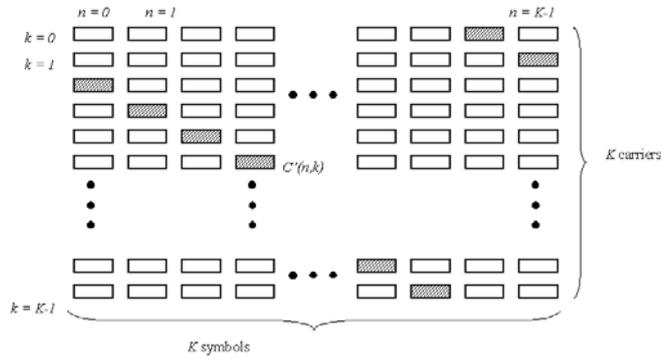


Fig. 4 – Symbol Carrier Rotation

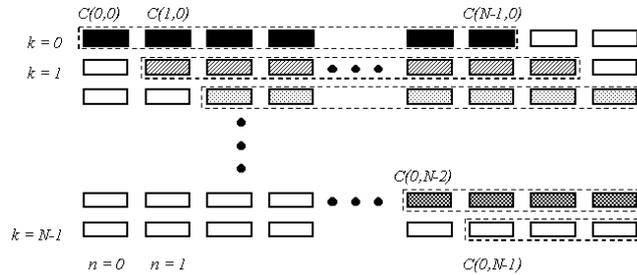


Fig. 5a– Simple Time Shifting

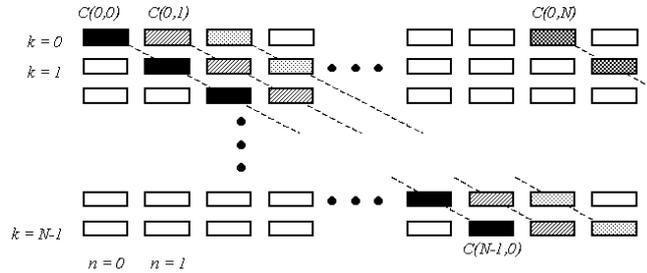


Fig. 5b -Time Shifting with Carrier Rotation

Note that there is no restriction on the relative sizes of  $N$  and  $M$ ; trade-offs are possible depending on transmission requirements (latency, computational resources, channel

statistics etc.). When  $N = K$ , with time shifting, a single computational resource (IFFT engine) can be shared between interleaving and OFDM modulation, resulting in an efficient hardware implementation, as shown in Fig. 6.

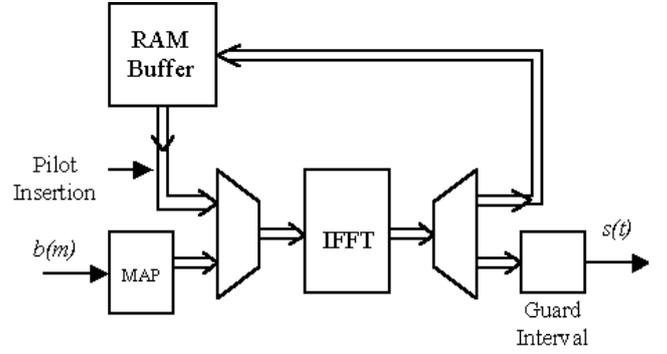


Fig. 6 – Shared IFFT Implementation

## V. SIMULATIONS

The results presented here were obtained from simulating a baseband FTI-OFDM system, with  $N = K = 1024$ ,  $p = 6$  (64-QAM), with carrier rotation and no error correction encoding. Comparisons are made to ordinary OFDM with  $K=M = 1024$  and 64-QAM modulation. The total number of encoded bits in the simulations is  $6 \times 1024 \times 1024$  ( $6.29 \times 10^6$ ).

No time or frequency interleaving was used in ordinary OFDM, since total bit error rate is not affected.

In all plots, except where noted, OFDM values are shown with cross symbols and FTI-OFDM with circles; the horizontal axis contains the SNR per bit ( $\epsilon_b / N_0$ ) while the vertical axis displays the observed average bit error rate.

### A. AWGN Channel

FTI-OFDM performance for an AWGN channel is essentially equal to that of OFDM (Fig. 7). However, when the received signal is quantized to an 8-bit resolution (both real and imaginary axes), with the full signal scale set to +6 dB above the mean signal power level (a typical configuration for digital signal processing), FTI-OFDM slightly outperforms ordinary OFDM.

### B. Random Impulse Noise

Random, high-amplitude impulsive noise is particularly harmful to OFDM modulation, because it affects all carriers in a given symbol. In Fig. 8, randomly selected samples of  $S(t)$  suffered the addition of samples taken from a gaussian noise signal with the same mean power. The resulting BER is plotted against the ratio  $Rb$ , which is equal to the number of “noisy” corrupted samples divided by the total number of samples in a frame (1048576 samples).

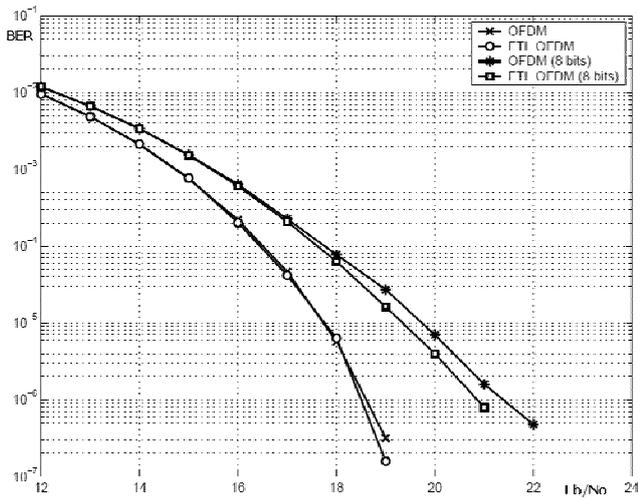


Fig.7 – Comparative Bit Error Rate in an AWGN Channel

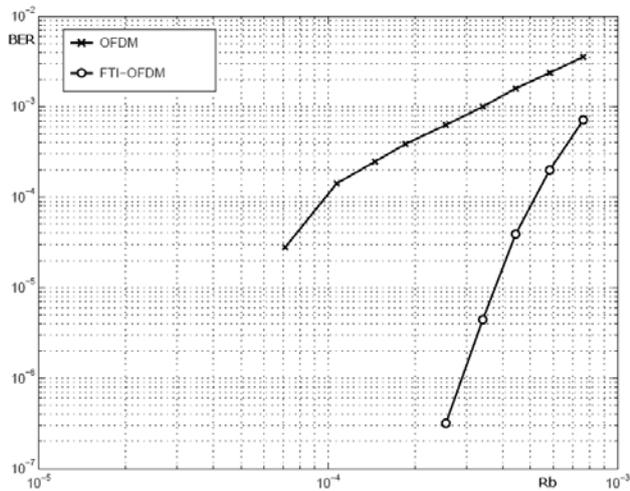


Fig. 8 – Comparative effect of Random Impulsive Noise as a function of the ratio  $R_b$  of noise corrupted samples (see Sec. V-B)

### C. Narrowband Noise

Since the total signal power is shared by many carriers, even low-level, fixed-frequency, narrowband interfering signals can destroy all data in all OFDM symbols within that frequency band. FTI-OFDM with carrier rotation spreads the interfering power over all carriers, reducing the total error rate except for very high interfering signal amplitudes. In Fig. 9, noise occupying a bandwidth equal to 1% of the total signal bandwidth was used in assessing the bit error rate. Fig. 10 compares 64-QAM OFDM constellations and the corresponding FTI-OFDM, both with a bit SNR of approx. 17 dB.

### D. Deep Fading

In this simulation, a block of consecutive samples was deleted (replaced by zero-valued samples) leading to the bit

error rate plotted in Fig. 11. The ratio  $R_f$  refers to the proportion of randomly deleted samples in an FTI frame. In this example, Fourier interleaving becomes advantageous when BER is less than  $2 \times 10^{-3}$ .

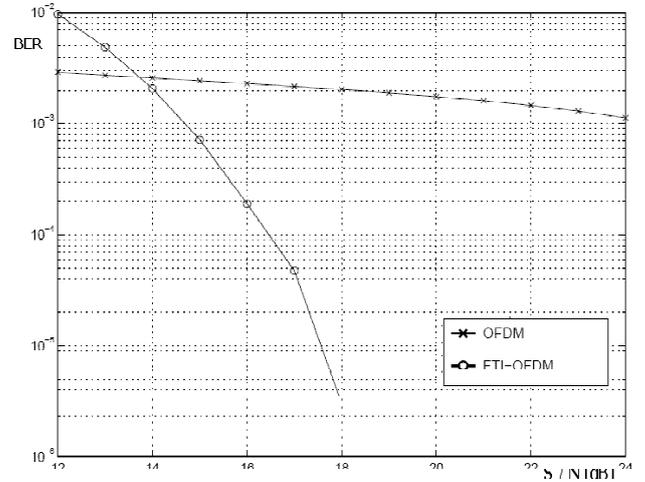


Fig. 9 – Effect of Narrowband Noise (Sec. V-C)

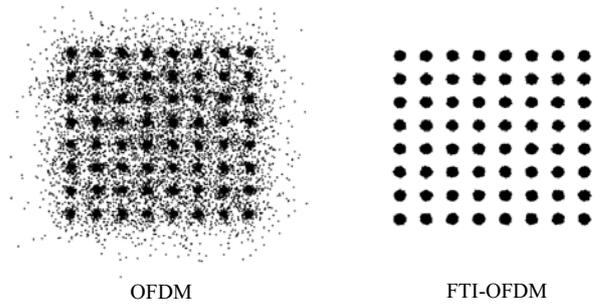


Fig. 10 – QAM Constellation With Narrowband Noise at the Receiver

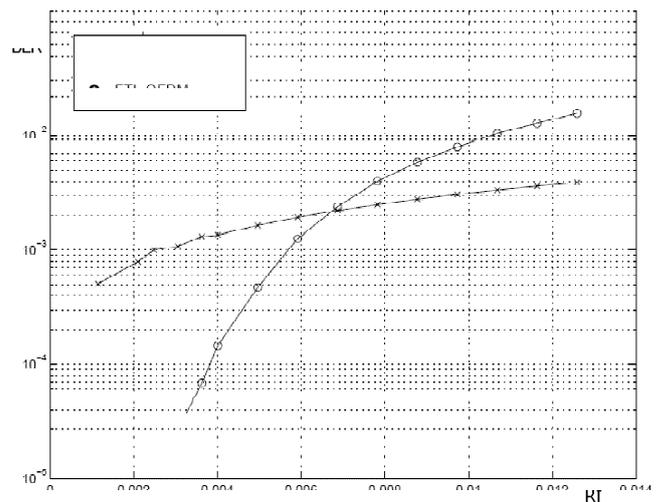


Fig.11– Bit Error Rate for Deep Fading

### E. Signal Clipping

FTI-OFDM is slightly more robust to signal clipping (before demodulation), as shown in Fig. 12, where the resultant BER is plotted against the clipping ratio (in reference to the RMS signal amplitude). In the absence of noise, clipping at a peak level of 6 dB above mean power (Clipping Ratio = 2) leads to a BER that is 5 times lower than conventional OFDM.

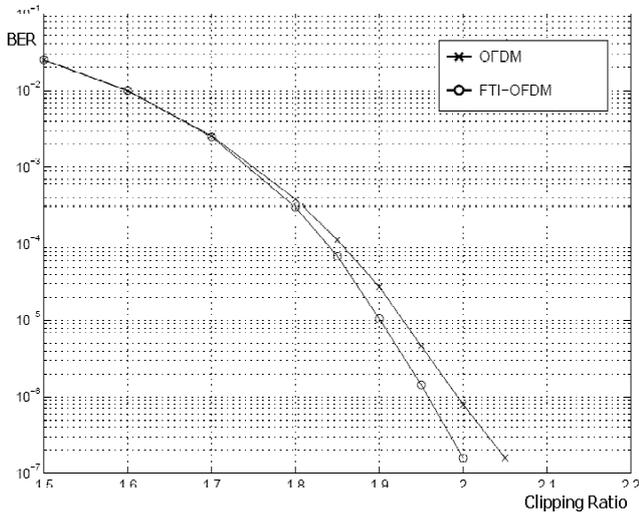


Fig. 12 – BER x Signal Clipping Level

## VI. CONCLUSIONS

This paper presented a simple, robust and computationally efficient method for interleaving digital data in connection to OFDM modulation. This system can provide lower bit error rates for many channel impairments of interest when compared to conventional OFDM modulation. Current research focuses on the theoretical characterization of the new method. An FPGA prototype is being implemented for testing in connection with the Brazilian Digital TV System now under development.

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