BLIND MAXIMUM RATIO COMBINING AND CHANNEL SHORTENING FOR CYCLIC PREFIXED SYSTEMS

C. M. Panazio

Escola Politécnica of the University of São Paulo,
Laboratory of Communications and Signals,
São Paulo, SP, Brazil
cpanazio@lcs.poli.usp.br

A. S. Menezes, J. M. T. Romano

State University of Campinas,
Faculty of Electrical and Computer Engineering,
Campinas, SP, Brazil
alamsm@gmail.com,romano@dmo.fee.unicamp.br

ABSTRACT

In this paper we propose a blind maximum ratio combining (MRC) technique along with an initialization method to improve the performance of the blind adaptive channel shortening algorithm called Multicarrier Equalization by Restoration of RedundancY (MERRY) in the 1x2 SIMO channel context. We show through analysis and simulations that the blind MRC technique allow us to take advantage of the spatial diversity improving considerably the performance of the MERRY algorithm in the SIMO context.

1. INTRODUCTION

The cyclic-prefix (CP) technique [1] has become very popular, since it allows the use of computationally efficient and simple frequency-domain equalizer (FEQ). It works by appending, in a block-based transmission, a copy of the end of each block as a prefix, that should be equal to or longer than the channel memory. With adequate synchronization, we apply a fast Fourier transform that allow us to equalize the signal with a bank of the so-called one-tap equalizer. The CP technique is mostly employed in multicarrier modulations, such as orthogonal frequency division multiplexing (OFDM) and discrete multitone modulation (DMT), but also in single-carrier modulations (SCCP) [2].

If the channel memory is longer than the CP, we have interblock interference (IBI), reducing the system performance. This problem can be solved by increasing the CP size along with the block size, in order preserve spectral efficiency. However, this increases latency, complexity and memory needs. Additionally, to avoid intercarrier interference (ICI), the channel should be invariant during the block period. Thus, a larger block size would render the system more sensitive to time-varying channels. A more effective and flexible way of minimizing this problem is to make use of time-domain equalizer (TEQ) before CP removal and the FFT. Such technique aims to provide an effective channel, which is the convolution of the channel with the equalizer, whose memory length is equal or smaller than the CP.

Many techniques have been developed for determining the TEQ coefficients (e.g., [3],[4]). Most of them were made for wireline systems, which, due to slow time-varying characteristics, make use of trained and nonadaptive techniques. However, in wireless channels, adaptive blind techniques are greatly desired. One interesting and efficient technique proposed in the last years is the Multicarrier Equalization by Restoration of RedundancY (MERRY) algorithm [5]. Although it presents good performance in Single-Input Single-Output (SISO) channels, we show in this paper that its version for a Single-Input Multiple-Output (SIMO) channel may not exploit the diversity provided by the channel and, in some cases, it may even cause destructive interference on the desired signal. We then propose a blind adaptive maximum ratio combining (MRC) approach with an initialization method for the MERRY algorithm to improve system performance. We assess such gains through numerical simulations.

This paper is organized as follows. In section II, we describe the system model. Section III recalls the MERRY algorithm and presents its drawbacks in the SIMO context. In section IV we describe and explain the MRC approach for the MERRY algorithm. Simulation results are shown in section VI. Finally, the conclusions and perspectives are stated in VII.

2. SYSTEM MODEL

We base our system model in the one proposed by [6]. For the sake of simplicity, the SIMO system model for OFDM modulation is depicted in figure 1 for only 2 receiver antennas. In the general context, we can assume the use of P receiver antennas. Each of the N sub-carriers modulates a QPSK (Quadrature Phase-Shift Keying) symbol. The OFDM modulation is performed via inverse fast Fourier transform (IFFT) and the demodulation is accomplished via FFT.

We define the OFDM symbol as the N output values of the IFFT. The CP is the last ν samples of the OFDM symbol that are appended at the beginning of each OFDM symbol before transmission, originating what we call a complete OFDM symbol. Hence, we have that the k-th complete OFDM sym-

978-1-4244-2046-9/08/$25.00 ©2008 IEEE

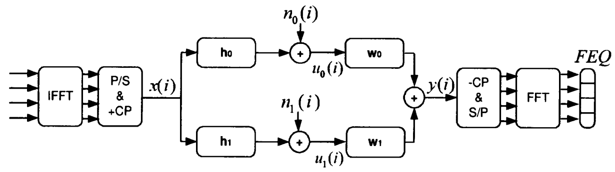

Fig. 1. System Model.

bol signal obeys:

$$x(Mk+i) = x(Mk+i+N) \quad i \in \{0, ..., \nu-1\} \tag{1}$$

where $M = N + \nu$ is the total symbol duration and k is the symbol index. The signal received by the p-th antenna is given by

$$u_p(i) = \mathbf{h}_p^T \mathbf{x}(i) + n_p(i), \tag{2}$$

where $\mathbf{h}_p = [h_p(0) \; h_p(1) \; \dots \; h_p(L_h-1)]^T$ is the p-th sub-channel, $\mathbf{x}(i) = [x(i) \; x(i-1) \; \dots \; x(i-L_h+1)]^T$ is the transmitted signal, $n_p(i)$ is the additive white Gaussian noise with variance σ_n^2 and L_h is the length of the p-th sub-channel.

If the CP is shorter than the channel memory, we can use a TEQ to shorten the channel. The coefficients of the p-th TEQ is represented by $\mathbf{w_p}$ and its output is given by:

$$y_p(i) = \mathbf{w}_p^T \mathbf{u}_p(i) \tag{3}$$

where $\mathbf{w}_p = [w_p(0) \; w_p(1) \; \dots \; w_p(L_w-1)]^T$ is the sub-equalizer, $\mathbf{u}_p(i) = [u_p(i) \; u_p(i-1) \; \dots \; u_p(i-L_w+1)]^T$ is the input of the p-th sub-equalizer and L_w is its length.

The resulting signal used for demodulation is

$$y(i) = \sum_{p=0}^{P-1} y_p(i) = \sum_{p=0}^{P-1} \mathbf{w}_p^T \mathbf{u}_p(i) = \mathbf{w}^T \mathbf{u}(i) \tag{4}$$

where

$$\mathbf{w}^T = \left[\mathbf{w}_0^T \; \mathbf{w}_1^T \; \dots \; \mathbf{w}_{P-1}^T\right] \tag{5}$$

and

$$\mathbf{u}(i) = \left[\mathbf{u}_0^T(i) \; \mathbf{u}_1^T(i) \; \dots \; \mathbf{u}_{P-1}^T(i)\right]^T \tag{6}$$

The effective channel response $\mathbf{c}$ is expressed by:

$$\mathbf{c} = \sum_{\mathbf{p=0}}^{\mathbf{P-1}} \mathbf{h_p} \star \mathbf{w_p} \tag{7}$$

where $\star$ is the linear convolution operation.

Although we do not show the SCCP system, it is worth noting that it only differs from the OFDM by the fact that the IFFT is replaced just after the FEQ in the receiver.

Before presenting the proposed technique, we recall the MERRY algorithm in the next section, showing its drawbacks in the studied context.

3. ANALYSIS OF MERRY ALGORITHM IN THE SIMO CONTEXT

The MERRY algorithm [5] assumes that a CP is used in the transmission and that the source sequence is white before the CP insertion. If the effective channel is no longer than the CP and in the absence of noise, the last sample in the received symbol will be equal to the last sample in the received CP of the symbol. On the other hand, if the CP is shorter than the effective channel response, this will not be true. If we can minimize the difference between these two samples, by restoring the redundancy introduced by the cyclic prefix, we can shorten the effective channel. Mathematically this can be translated into the MERRY cost function [5]:

$$J(\mathbf{w}, \Delta) = E\left\{|y(Mk+\nu+\Delta) - y(Mk+\nu+N+\Delta)|^2\right\} \tag{8}$$

where $\Delta \in \{0, \dots, M-1\}$ is a synchronization parameter used to find the boundaries that separates successive transmitted blocks. The optimum value of Δ is the one that minimizes (8).

An adaptive algorithm can be obtained by the stochastic gradient descent [7] of (8). However, without a constraint, it would converge to the trivial solution, *i.e.*, $\mathbf{w} = \mathbf{0}$. In order to avoid such solution, we can choose between two constraints [5, 8]: the unit-norm constraint (UNC) and the unit-tap constraint (UTC). Mathematically, the correspondent stochastic gradient descent algorithms are:

• MERRY-UNC

$$\begin{array}{l} \text{set } w_{p_0,i_0}(0) = 1, \\ \widetilde{\mathbf{u}}(k) = \mathbf{u}(Mk+\nu-1+\Delta) - \mathbf{u}(Mk+\nu-1+N+\Delta) \\ e(k) = \mathbf{w}^T(k)\widetilde{\mathbf{u}}(k) \\ \widehat{\mathbf{w}}(k+1) = \mathbf{w}(k) - \mu e(k)\widetilde{\mathbf{u}}^*(k) \\ \mathbf{w}(k+1) = \frac{\widehat{\mathbf{w}}(k+1)}{\|\widehat{\mathbf{w}}(k+1)\|} \end{array} \tag{9}$$

• MERRY-UTC

$$\begin{array}{l} \text{set } w_{p_0,i_0}(0) = 1, \\ \widetilde{\mathbf{u}}(k) = \mathbf{u}(Mk+\nu-1+\Delta) - \mathbf{u}(Mk+\nu-1+N+\Delta) \\ e(k) = \mathbf{w}^T(k)\widetilde{\mathbf{u}}(k) \\ \mathbf{w}(k+1) = \mathbf{w}(k) - \mu e(k)\widetilde{\mathbf{u}}^*(k) \\ w_{p_0,i_0}(k+1) = 1 \end{array} \tag{10}$$

where $\mathbf{w}$ and $\mathbf{u}$ are defined in equations (5) and (6) respectively, μ is the adaptation step-size, $w_{p_0,i_0}(k) = 1$ is the i_0-th coefficient ($0 < i_0 \leq L_h - 1$) of the p_0-th sub-equalizer at instant k, $w_{p,i0}(0) = 1$ is the initial condition and (*) denotes complex conjugate.

Both constraints provide global convergence in the SISO channel context [5]. However, in the SIMO context, the MERRY-UNC presents more than one solution. Each solution is linked to the initialization choice. Therefore, it is not guaranteed that we can exploit the diversity provided by the SIMO channel and, worse, we can end up with destructive interference. The MERRY-UTC will globally converge, but not in the sense that it will exploit the diversity. Let us illustrate these phenomena by the following two simple examples.

3.1. Just one sub-equalizer initialization and AWGN sub-channels

Let us assume each sub-channel to be AWGN, $\nu \geq 1$, and the p_0-th sub-equalizer to be initialized by $w_{p_0,i_0}(0) = 1$, while all other coefficients of $\mathbf{w}$ are set to zero. Since the sub-channels are AWGN, the optimum value of Δ, i.e., the value that minimizes (8), is equal to i_0. Furthermore, when computing the error $e(0) = \mathbf{w}^T(0)\widetilde{\mathbf{u}}(0)$, all terms of $\widetilde{\mathbf{u}}(0)$ will be multiplied by zero, with the exception of that one corresponding to $w_{p_0,i_0}(0)$. This element of $\widetilde{\mathbf{u}}(0)$ is given by:

$$u_{p_0}(Mk+\nu-1)-u_{p_0}(Mk+\nu+N-1)=x(Mk+\nu-1)-x(Mk+\nu+N-1) +n_{p_0}(Mk+\nu-1)-n_{p_0}(Mk+\nu+N-1) \quad (11)$$

However from (1), $x(Mk+\nu-1) = x(Mk+\nu+N-1)$, so that (11) becomes

$$u_{p_0}(Mk+\nu-1)-u_{p_0}(Mk+\nu+N-1)= n_{p_0}(Mk+\nu-1)-n_{p_0}(Mk+\nu+N-1) \quad (12)$$

which is the difference between two uncorrelated zero mean Gaussian samples. Then, the true gradient used in the adaptation of (9) and (10) for the first iteration is given by:

$$E\left\{e(0)\widetilde{\mathbf{u}}^*(0)\right\} = \left[0\ldots,2\sigma_n^2,0,\ \ldots 0\right]^T, \quad (13)$$

where all elements are null except the element corresponding to the i_0 position of the p_0-th sub-equalizer, $2\sigma_n^2$. For the first iteration it holds

$$\begin{aligned}\widehat{\mathbf{w}}(1) &= \mathbf{w}(0) - \mu \mathrm{E}\{e(0)\widetilde{\mathbf{u}}^*(0)\} \\ &= [\mathbf{0}_0 \ldots \mathbf{0}_{p_0-1}\ 0\ \ldots\ 1-2\mu\sigma_n^2\ 0 \ldots 0\ \mathbf{0}_{p_0+1}\ \ldots\ \mathbf{0}_{P-1}]^T\end{aligned} \quad (14)$$

and then, for the MERRY-UNC (9):

$$\begin{aligned}\mathbf{w}(1) &= \frac{\widehat{\mathbf{w}}(1)}{\|\widehat{\mathbf{w}}(1)\|} \\ &= [\mathbf{0}_0 \ldots \mathbf{0}_{p_0-1}\ 0\ \ldots\ 1\ 0 \ldots 0\ \mathbf{0}_{p_0+1}\ \ldots\ \mathbf{0}_{P-1}]^T\end{aligned} \quad (15)$$

and for the MERRY-UTC (10):

$$\begin{aligned}w_{p_0,i0}(1) &= 1 \Rightarrow \\ \mathbf{w}(1) &= [\mathbf{0}_0 \ldots \mathbf{0}_{p_0-1}\ 0\ \ldots\ 1\ 0 \ldots 0\ \mathbf{0}_{p_0+1}\ \ldots\ \mathbf{0}_{P-1}]^T\end{aligned} \quad (16)$$

As a result of the constraints, $\mathbf{w}(1) = \mathbf{w}(0)$, *i.e.*, the equalizer does not move away from the initial conditions. This is not a surprise, since the given initial conditions preserve the redundancy introduced by the CP. As a matter of fact, this will always happen when $L_h \leq \nu$. However, the resulting solution of this initialization does not contemplate the signal power from the other channels and neither the uncorrelated noise in the P antennas, *i.e.*, channel diversity.

One could think of another initialization designed to exploit the channel diversity. For the channel in this example, a possible initialization that would work with the MERRY-UNC is $w_{p,i_0}(0) = 1$ for $p = 0$ *and* $p = 1$. However, if the sub-channels' phases are opposed by π, this initialization will lead to destructive interference, as it is illustrated in the next example.

3.2. Initializing all sub-equalizers and AWGN sub-channels but with opposite phases

Let us consider a 1x2 SIMO channel with AWGN sub-channels. In order to maximize the output SNR of the TEQ, we can initialize both sub-equalizers with $w_{0,i_0}(0) = 1/\sqrt{2}$, $w_{1,i_0}(0) = 1/\sqrt{2}$ (unit-norm initialization) for the MERRY-UNC case and $w_{0,i_0}(0) = 1$, $w_{1,i_0}(0) = 1$ for the MERRY-UTC case. Further, let us consider that the AWGN sub-channels have opposite phases (0 and π) and that we are not aware of that. It is not hard to see that we end up with a result similar to (13), but now we have two non-zero values (σ_n^2 for the MERRY-UNC and $2\sigma_n^2$ for the MERRY-UTC) in i_0 position of both sub-equalizers. After updating the equalizer coefficients and applying the constraint, the MERRY-UNC will remain on the initial conditions for every value of k. For the MERRY-UTC, there is a decrease of $2\sigma_n^2$ on the i_o of the non-constrained tap. When $k \to \infty$, this tap is driven to zero.

To sum up, the resulting equalizer of the MERRY-UNC approach will generate a null effective channel, for any value of k. The equalizer that uses the MERRY-UTC approach will forget the initial condition with a time period whose length is directly proportional to the channel signal-to-noise ratio (SNR). Therefore, in the first iterations, the effective channel will be almost null. At last, none of the initializations can exploit the channel's diversity, which is crucial to improve performance.

In the next section, we propose a technique that can exploit part of this diversity.

4. THE JOINT MRC AND UNC-MERRY APPROACH

In order to better exploit the diversity provided by the channel and to avoid ill convergence solutions of the MERRY-UNC, we came up with an MRC approach in a 1x2 SIMO channel context (figure 2). We consider that we initialize both sub-equalizers with $w_{0,i_0}(0) = 1/\sqrt{2}$ and $w_{1,i_0}(0) = 1/\sqrt{2}$. As shown in the example in subsection (3.2), the sub-channels will be added and we may end up with destructive interference. But we know that this is mostly due to a phase problem that cannot be compensated by the MERRY algorithm. If we take one sub-channel as the phase reference, we can rotate the other one and by maximizing the following criterion we expect to gather most of the channel spatial diversity:

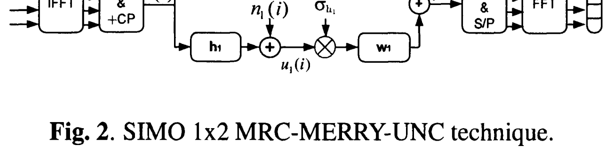

Fig. 2. SIMO 1x2 MRC-MERRY-UNC technique.

$$J(e^{j\theta}) = \sum_{k=0}^{L_h-1} \left| e^{j\theta} h_1(k) + h_2(k) \right|^2 \quad (17)$$

We call the attention to the fact that both channels are added due to the adopted initialization.

The maximum value of (17) is achieved when:

$$e^{j\theta} = \frac{\sum_{k=0}^{L_h-1} h_0^*(k) h_1(k)}{\left| \sum_{k=0}^{L_h-1} h_0^*(k) h_1(k) \right|} = \frac{R_{h_1 h_0}}{|R_{h_1 h_0}|} \quad (18)$$

where $R_{h_1 h_0}$ is the cross-correlation between sub-channels $\mathbf{h_1}$ and $\mathbf{h_0}$. This value can be estimated adaptively by the cross-correlation of the outputs of the sub-channels using, for instance, a retangular sliding window, as follows:

$$\tilde{R}_{h_1 h_0}(k) = \frac{1}{W} \sum_{i=k-W+1}^{k} u_0^*(i) u_1(i) \quad (19)$$

where $\tilde{R}_{h_1 h_0}(k)$ is the estimated value of $R_{h_1 h_0}$ at the instant k and W is the size of the retangular sliding window. A more efficient implementation can be carried out by a moving sum.

It is worth noting that we can estimate θ much faster than the equalizer, since we can iterate at every received sample, while the MERRY can iterate only at symbol-rate.

We also know that the sub-channel may have different gains. Therefore, in order to maximize the SNR, we can ponder each sub-channel output according to the square-root of their output power. The power estimation of the received signal can be easily carried out by an exponential sliding window:

$$\sigma_u^2(i) = \alpha \sigma_u^2(i-1) + (1-\alpha) |u(i)|^2 \quad (20)$$

where α is the forgetting factor. Then, the channel power is:

$$\sigma_h^2 = \begin{cases} 0, & \text{if } \sigma_u^2 - \sigma_n^2 < 0 \\ \sigma_x^{-2}(\sigma_u^2 - \sigma_n^2), & \text{otherwise} \end{cases} \quad (21)$$

where σ_n^2 is the noise power, considered to be previously estimated in the absence of the information signal and σ_x^{-2} is the inverse of the signal transmitted power that is known *a priori*.

Thus, if we assume, like in subsection (3.2), AWGN sub-channels, it is straightforward to show that, for any channel phase or power, the MRC-MERRY-UNC with the proposed initialization approach will provide optimal performance. The analysis for time-dispersive channels are more complicated, but results from numerical simulation are shown in the next section.

It is worth noting that, in order to provide similar gains to the MERRY-UTC, the unit-tap constraint would also have to be applied in the other sub-equalizer. However, this would limit the equalizer solutions space.

5. SIMULATION RESULTS

The following simulations show the effectiveness of our proposal for time-dispersive channels. The first example considers a 32-tap Rayleigh-fading channel with exponential power-delay profile for both sub-channels, as used in [8]. In the second example, each sub-channel is the result of the convolution of a 5-tap Rayleigh-fading channel with exponential power delay-profile with another Rayleigh-fading channel represented by $\alpha_0 + \alpha_1 z^{-9} + \alpha_2 z^{-18}$, where α_k, $k = 0, 1$ and 2, are uncorrelated unitary variance complex Gaussian random variables.

In all simulations, the (I)FFT size is $N = 64$ and the CP length is $\nu = 16$. The MRC parameters are perfectly estimated from the first transmitted symbol, since we would like to concentrate on the best achievable performance of the proposed technique. Furthermore, the MRC parameters can be estimated very fast and thus, this assumption is not irreal. The equalizer coefficients are updated for 1600 complete OFDM symbols after initialization. Then, the adaptation is stopped and the bit error rate (BER) is measured for another 1600 complete OFDM symbols. This process was repeated for 2000 channel realizations. We have used a QPSK modulation and the frequency-domain equalizer was obtained by assuming perfect channel knowledge. No channel coding was used in the simulations and the indicated SNR is the SNR per sub-channel (or per antenna). The optimum delay Δ was obtained in the same fashion as in [6]. The number of coefficients per sub-equalizer is $L_w = 15$ and center-spike initialization is used. The MERRY-UTC and MERRY-UNC have only one sub-equalizer initialized. The MERRY-UNC double initialization and MRC-MERRY-UNC have both sub-equalizers initialized with center-spike initialization.

Figure 3 shows the BER for the 32-tap Rayleigh-fading exponential power-delay profile channel. The MERRY-UTC

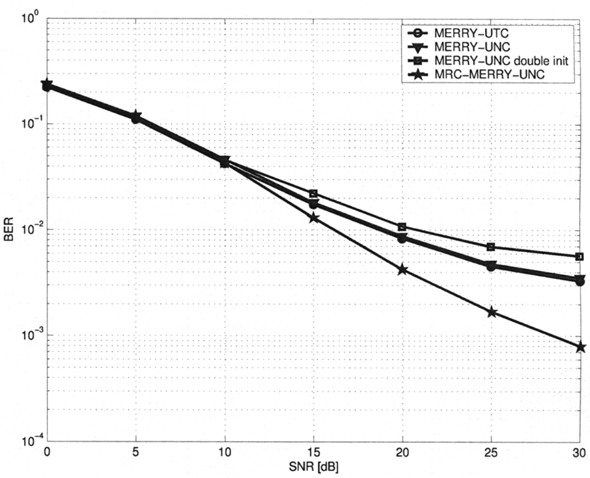

Fig. 3. BER for the 32-tap Rayleigh-fading channel with exponential power-delay profile.

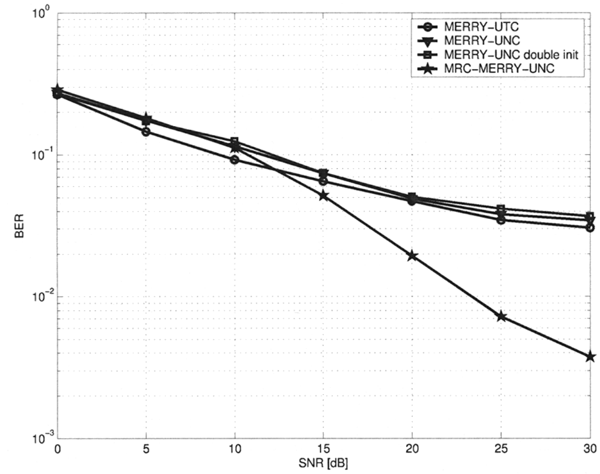

Fig. 4. BER for the 5-tap Rayleigh-fading channel with exponential power delay-profile convolved with a three-path equal power Rayleigh channel.

and the MERRY-UNC present almost the same performance, as noted in [8] in a similar case. However, the MRC technique together with the proposed algorithm initialization achieves a much higher performance for higher SNR regimes. We can also note that the double initialization in the pure MERRY-UNC causes some loss of performance when compared to the single initialization approach.

For the second example, the results are shown in figure 4. The difference between the MERRY without and with MRC approach is still more impressive when compared to the first example. At 30dB, the proposed approach has a tenfold BER advantage over the other MERRY configurations. We may explain this behavior based on the channel characteristics. First, the channel has a maximum length of 23, which leaves only 6 samples outside the cyclic prefix. This reduces the impact of the TEQ channel shortening and emphasizes the use of TEQ in channel diversity exploitation. Additionally, the channel components associated to the exponential power delay-profile decays in just 5 samples, which leaves only some really important taps. Therefore, it is more crucial in this case to coherently add both channels. Finally, the non-MRC techniques present again almost the same performance, with a minor advantage of the MERRY-UTC.

Although we ideally estimate the MRC parameters, there is plenty of performance headroom to absorb some estimation errors.

6. CONCLUSION

In this paper, we show that the MERRY algorithm cannot fully exploit the diversity present in the SIMO channel and sometimes it may even cause destructive interference on the desired signal, with serious impact on the system performance. We then propose a blind MRC approach for a 1x2 SIMO channel in order to combat this problem and to better exploit the diversity. We show through simulations the effectiveness of our technique.

7. REFERENCES

[1] A. Peled and A. Ruiz, "Frequency domain data transmission using reduced computational complexity algorithms," in *Acoustics, Speech, and Signal Processing, IEEE International Conference on ICASSP '80.*, Apr. 1980, vol. 5, pp. 964–967.

[2] D. Falconer, S. Ariyavisitakul, A. Benyamin-Seeyar, and B. Eldson, "Frequency Domain Equalization for Single-Carrier Broadband Wireless Systems," 2002.

[3] N. Al-Dhahir and J. M. Cioffi, "Optimum finite-length equalization for multicarrier transceivers," *IEEE Transactions on Communications*, vol. 44, no. 1, pp. 56–64, Jan. 1996.

[4] G. Arslan, B. L. Evans, and S. Kiaei, "Equalization for discrete multitone transceivers to maximize bit rate," *Signal Processing, IEEE Transactions on [see also Acoustics, Speech, and Signal Processing, IEEE Transactions on]*, vol. 49, no. 12, pp. 3123–3135, Dec. 2001.

[5] R. K. Martin, J. Balakrishnan, W. A. Sethares, and C. R. Jr. Johnson, "A blind adaptive TEQ for multicarrier systems," *IEEE Signal Processing Letters*, vol. 9, no. 11, pp. 341–343, Nov. 2002.

[6] R. K. Martin, J. M. Walsh, and C. R. Jr. Johnson, "Low-complexity MIMO blind, adaptive channel shortening," *Signal Processing, IEEE Transactions on [see also Acoustics, Speech, and Signal Processing, IEEE Transactions on]*, vol. 53, no. 4, pp. 1324–1334, Apr. 2005.

[7] S. Haykin, *Adaptive Filter Theory*, Prentice-Hall, 3rd edition, 1996.

[8] R. K. Martin, "Fast-converging blind adaptive channel-shortening and frequency-domain equalization," *Signal Processing, IEEE Transactions on [see also Acoustics, Speech, and Signal Processing, IEEE Transactions on]*, vol. 55, no. 1, pp. 102–110, Jan. 2007.