A 4^N-QAM ADAPTIVE DECISION DEVICE TO MITIGATE I/Q IMBALANCE AND IMPAIRMENTS CAUSED BY TIME-VARYING FLAT FADING CHANNELS

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Abstract.

In this work, we propose an adaptive decision device based on a Kohonen network that can automatically generate the classes associated with each symbol of a 4^n-QAM in the presence of nonlinearities caused by the I/Q imbalance and additive Gaussian white noise, being also capable of compensating phase and gain variations produced by a time-varying flat-fading channel. Our proposal can achieve optimality in the maximum-likelihood sense with a small computational cost. Furthermore, due to the tracking ability inherent to the devised scheme, there is no need for an automatic gain controller or a phase-locked loop.

INTRODUCTION

In complex envelope modulation signals, the demodulation process may give origin to a distortion known as *I/Q imbalance* [4] may occur in the process of demodulating a signal with complex envelope if:

1) The cosine and the sine used by the demodulator have not the same phase;

2) The in-phase channel and quadrature channel have different gains.

This effect, which may cause considerable performance degradation, can be modelled as a nonlinear distortion imposed to the baseband signal.

Another well known problem that arises in wireless communication system is the presence of fading. Due to the arrival of multiple delayed versions of the

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transmitted signal at the receiver, the signal may be combined in a destructive way, thus originating the phenomenon. The fading may vary in amplitude and phase over time as a result of the movement of the receiver/transmitter or other changes in the propagation environment. Therefore, these changes must be tracked by the receiver to estimate correctly the transmitted signal.

In this paper we propose a self-organizing map [1],[2] specially tailored to compensate the noxious effects introduced by both I/Q imbalance and the fading.

SIGNAL MODEL

The complex baseband signal representation is given by:

$$\widetilde{x}(t) = x_I(t) + jx_Q(t), \tag{1}$$

where $x_I(t)$ and $x_Q(t)$ are respectively the in-phase and quadrature real components of a 4^n-QAM signal.

The signal $\widetilde{x}(t)$ is then upconverted and we obtain the transmitted signal:

$$x(t) = \Re\left\{\widetilde{x}(t)e^{-j\omega_0 t}\right\} = x_I(t)\cos(\omega_0 t) - x_Q(t)\sin(\omega_0 t) \tag{2}$$

where ω_0 is the carrier angular frequency.

Firstly, we assume that the channel does not introduce any distortion. Therefore, the received signal at the antenna can be written as:

$$y(t) = A(t)\{x_I(t)\cos(\omega_0 t + \theta(t)) - x_Q(t)\sin(\omega_0 t + \theta(t))\} \tag{3}$$

The signal $y(t)$ is downconverted by a mixer and a local oscillator (LO) which provides:

$$\begin{aligned} x_{LO,I} &= 2(1+\epsilon)\cos(\omega_0 t - \phi/2) \\ x_{LO,Q} &= -2(1-\epsilon)\sin(\omega_0 t + \phi/2) \end{aligned} \tag{4}$$

being ϵ and ϕ, respectively, the gain and the phase errors. Multiplying (3) by (4) and processing the result by a low-pass filter, we obtain the following baseband signals:

$$\begin{aligned} y_I(t) &= (1+\epsilon)\{x_I(t)\cos(\phi/2) + x_Q(t)\sin(\phi/2)\} \\ y_Q(t) &= (1-\epsilon)\{x_Q(t)\cos(\phi/2) + x_I(t)\sin(\phi/2)\} \end{aligned} \tag{5}$$

In matrix notation, we have:

$$\begin{bmatrix} y_I(t) \\ y_Q(t) \end{bmatrix} = \begin{bmatrix} 1+\epsilon & 0 \\ 0 & 1-\epsilon \end{bmatrix} \begin{bmatrix} \cos(\phi/2) & \sin(\phi/2) \\ \sin(\phi/2) & \cos(\phi/2) \end{bmatrix} \begin{bmatrix} x_I(t) \\ x_Q(t) \end{bmatrix} \tag{6}$$

The received signal can also be written in complex baseband:

$$\widetilde{y}(t) = \alpha\widetilde{x}(t) + \beta\widetilde{x}^*(t) \tag{7}$$

where α and β are complex constants given by:

$$\begin{aligned} \alpha &= \cos(\phi/2) - j\epsilon \sin(\phi/2) \\ \beta &= \epsilon \cos(\phi/2) + j \sin(\phi/2) \end{aligned} \tag{8}$$

It is worth noting that there is no linear operation capable of correcting the complex baseband signal in (7). Nevertheless, if we process separately the signals defined in (5), the I/Q imbalance can be easily eliminated. Such correction is achieved by finding the inverse matrix of:

$$\begin{bmatrix} 1+\epsilon & 0 \\ 0 & 1-\epsilon \end{bmatrix} \begin{bmatrix} \cos(\phi/2) & \sin(\phi/2) \\ \sin(\phi/2) & \cos(\phi/2) \end{bmatrix}$$

and applying it to the $y_I(t)$ and $y_Q(t)$ signals

In figures 1 and 2, we show how the I/Q imbalance distorts the signal constellation.

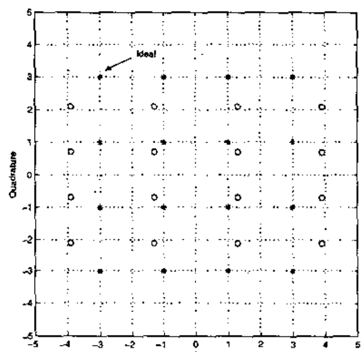

Figure 1: 16-QAM Gain error $\epsilon = 0.3$

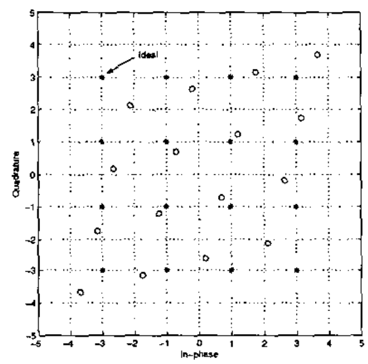

Figure 2: 16-QAM Phase error $\phi = 30^o$

Now, let the channel be:

$$h(t) = \Re\left\{\widetilde{g}(t)e^{-j\omega_0 t}\right\} \tag{9}$$

where $\widetilde{g}(t)$ is a complex fading process with a Jakes spectrum. Therefore, if we represent $\tilde{g}(t)$ in polar coordinates, i.e., $\widetilde{g}(t) = A(t)e^{j\theta(t)}$, $A(t)$ is the time-varying amplitude that follows a Rayleigh distribution and $\theta(t)$ is a time-varying phase uniformly distribution between 0 and 2π. It is straightforward to show that this channel causes no more than a rotation in the phase and a modification in the amplitude of the transmitted symbols. Hence, the received signal becomes, after downconversion:

$$\begin{aligned} y_I(t) &= A(t)(1+\epsilon)\{x_I(t)\cos(\theta(t)+\phi/2) + x_Q(t)\sin(\theta(t)+\phi/2)\} \\ y_Q(t) &= A(t)(1-\epsilon)\{x_Q(t)\cos(\theta(t)-\phi/2) - x_I(t)\sin(\theta(t)-\phi/2)\} \end{aligned} \tag{10}$$

In matrix notation, we have:

$$\begin{bmatrix} y_I(t) \\ y_Q(t) \end{bmatrix} = A(t) \begin{bmatrix} 1+\epsilon & 0 \\ 0 & 1-\epsilon \end{bmatrix} \begin{bmatrix} \cos(\theta(t)+\phi/2) & \sin(\theta(t)+\phi/2) \\ \sin(-\theta(t)+\phi/2) & \cos(\theta(t)-\phi/2) \end{bmatrix} \begin{bmatrix} x_I(t) \\ x_Q(t) \end{bmatrix} \tag{11}$$

Note that the correction is still done the same way as before. We have to estimate the transformation matrix, that now includes the fading, and invert it. The received baseband signal can still be obtained from the original message:

$$\tilde{y}(t) = \underbrace{\alpha A(t) e^{j\theta}}_{\alpha'} \tilde{x}(t) + \underbrace{\beta A(t) e^{j\theta}}_{\beta'} \tilde{x}^*(t) \qquad (12)$$

SELF-ORGANIZING DECISION DEVICE

In simple terms, the Kohonen self-organizing map can be conceived as a linear neural network whose adaptation is performed by means of a competitive learning strategy. The process is simple: the distance between each input stimulus and the weights of all neurons is measured, being the winner the closest element. The parameter vector of this neuron is then updated towards the presented pattern, a mechanism that engenders a division of the set of stimuli into a number of distinct classes. In some cases, it might be desirable to define a neighborhood of neurons that are adapted together with the winner. It is worth noting that the whole process is unsupervised, i.e., the classes are created in a blind fashion.

Since the purpose of a decision-device is exactly to divide the complex plane into classes determined by the transmitted symbols, it is possible to envisage the application of a self-organizing map to its project.

In order to elaborate on this crucial issue, assume that the training phase produced a Kohonen network whose classes correspond with exactitude to the constellation. Thenceforth, only the competition stage will be maintained, being the classification task conducted by its outcomes, which, as shown in [3], is equivalent to the *modus operandi* of a maximum-likelihood (ML)-based decision-device. An interesting feature of the aforementioned training scheme is that it does not depend on *a priori* knowledge of phase and gain.

In the context we have hitherto outlined, there still remains a flaw: the imbalance correlates the in-phase and quadrature components, thus invalidating the ML framework. Fortunately, it is possible to estimate the parameters relevant to modelling this effect, thereby recovering the orthogonality of both noise and signal. However, before we discuss this issue in more detail, let us present a systematic account of our proposal:

Initial Steps:

1. Choose an adequate number of neurons according to characteristics of the transmitted symbols.

2. Transmit N times each symbol of the constellation. In this step, we need to know the transmitted sequence, which makes this step supervisioned. Once received, we make an arithmetic mean among the cor-

responding samples for each symbol, obtaining an initial estimation of the received distorted constellation. Some particularization of this step can be done and they will be commented in next section.

3. Use the estimates obtained in the previous step to initialize the neural network.

Training:

The objective of this phase is to refine the neuron weights before the decision-device starts to operate.

1. Take an ensemble of N_T received signal samples $\mathbf{y}(k)$ for training
2. For each vector $\mathbf{y}(k)$, obtain the index of the winner neuron by:

$$i = \arg\min_j \left(\|\mathbf{w}_j(k) - \mathbf{y}(k)\|^2 \right) \tag{13}$$

   where $\mathbf{w}_j(k)$ is the weight vector of the j-th neuron in the instant k.

3. Having thus obtained the index j, update the weight vector of the corresponding neuron by:

$$\mathbf{w}_j(k+1) = \mathbf{w}_j(k) + \mu \left[\mathbf{y}(k) - \mathbf{w}_j(k)\right] \tag{14}$$

   where μ is the adaptation step-size.

4. If necessary, scramble the N_T samples and return to 1.

Operation:

After these steps, the network should be ready to play the role of a decision-device, which consists in determining the winning neuron and recovering the symbol associated with it. If the channel is time-varying, it might be desirable to continue adaptation according to steps 2 and 3 of the precedent phase.

APPLICATION TO 4^N-QAM SYSTEMS WITH I/Q IMBALANCE

Although the proposed framework is generic for additive white Gaussian noise, henceforth we will restrict our analysis to the class of 4^n-QAM modulations, for symmetry reasons, and to the case with the presence of I/Q imbalance, which correlates the I and Q components. A common feature of all members of this class is the possibility of dividing the set of symbols into four quadrants. A 16-QAM, for instance, can be decomposed into the following quadrants: 1){[1;1],[1;3],[3;1],[3;3]}, 2){[-1;1],[-1;3],[-3;1],[-3;3]}, 3){[-1;-1],[-1;-3],[-3;-1],[-3;-3]}, 4){[1;-1],[1;-3],[3;-1],[3;-3]}, whose centers of gravity (cg) are 1)[2,2], 2)[-2,2], 3)[-2,-2], 4)[2,-2]. Interestingly, these centers will suffer the same distortions experienced by the transmitted symbols, which opens a perspective: to estimate the parameters of the imbalance model from the position of the centers of gravity. Notice that the organization in quadrants holds for all values of parameters, except for $\epsilon = 1$ or $\phi = n\pi/4$, $n = 1, 3, 5$ and 7. Note also that the I/Q distortion is 180^o symmetrical, i.e., the quadrants 1 and 3 suffers opposite distortions, the same as quadrant 2 and 4. The phase distortion caused by the fading is 90^o symmetrical, i.e., the quadrant 2 suffers the same distortion of quadrant 1 but with 90^o rotation and so on. These properties can be used to simplify the initialization and adaptation steps, as we will show.

During the initial phase, it is necessary to transmit only two different symbols of the constellation, one per non-180^o-symmetrical quadrant. We assume that the highest power symbols are transmitted in order to maximize the symbol-to-noise ratio (SNR). Then, we initialize the neurons assigned to each quadrant and multiply their weights by a gain (defined by (maximal symbol amplitude-minimal symbol amplitude)/maximal symbol amplitude) to estimate the centers of gravity of the received modulation. Note that if the symbols of the quadrants 1 and 2 were transmitted, the neurons of quadrants 3 and 4 are respectively $w_3 = -w_1$ and $w_4 = -w_2$ as a result of the symmetry of the I/Q imbalance. This symmetry can also be used in adaptation by constraining the updating process so that if the m-th neuron is the winner, the (m+1) mod 4 +1 neuron is updated by:

$$\mathbf{w}_{(m+1) \bmod 4+1}(k+1) = \mathbf{w}_{(m+1) \bmod 4+1}(k) - \mu\left[\mathbf{y}(k) - \mathbf{w}_m(k)\right] \qquad (15)$$

where mod means de modulus operator. Due to the symmetry of the neurons and that the constraint keeps it during adaptation, a simpler operation is:

$$\mathbf{w}_{(m+1) \bmod 4+1}(k+1) = -\mathbf{w}_m(k+1) \qquad (16)$$

This constraint has many advantages. First, it limits the possibility that the same neuron could win for more than one quadrant. Secondly, it allows faster convergence or a smaller step-size, minimizing the adaptation noise. Once the neurons have converged to the centers of gravity of the distorted signal, we can estimate the I/Q imbalance and the fading parameters, α' and β'. Using (12), two non 180^o symmetrical neurons (w_m and w_n) and its associated centers of gravity of the original constellation, which we call cg_o, we have:

$$\begin{bmatrix} \alpha'(k) \\ \beta'(k) \end{bmatrix} = \begin{bmatrix} cg_o(m) & cg_o(m) \\ cg_o(n) & cg_o(n) \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{w}_m(k) \\ \mathbf{w}_n(k) \end{bmatrix} \qquad (17)$$

Recalling (11), the transformation matrix:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = A(t) \begin{bmatrix} 1+\varepsilon & 0 \\ 0 & 1-\varepsilon \end{bmatrix} \begin{bmatrix} \cos(\theta(t)+\phi/2) & \sin(\theta(t)+\phi/2) \\ \sin(-\theta(t)+\phi/2) & \cos(\theta(t)-\phi/2) \end{bmatrix}$$

can be obtained directly from the values of α' and β' :

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \Re\{\alpha'+\beta'\} & \Im\{-\alpha'+\beta'\} \\ \Im\{\alpha'+\beta'\} & \Re\{\alpha'-\beta'\} \end{bmatrix} \qquad (18)$$

Therefore, the corrected signal is obtained by:

$$\begin{bmatrix} \hat{x}_I(k) \\ \hat{x}_Q(k) \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} \begin{bmatrix} y_I(k) \\ y_Q(k) \end{bmatrix} \tag{19}$$

Once we have the corrected signal, we apply the traditional ML decision device to estimate the transmitted symbol.

The computational cost is not high, since only the inversion of a 2x2 matrix (18) is needed. The other matrix inversion in equation (17) can be pre-computed.

It is noteworthy that the process of choosing the winner neuron does not take into account the correlation of I and Q noise components. Thereupon, the decision zones for all neurons are not optimal, which may lead to biased or wrong estimation of the centers of gravity in heavily distorted channels with the presence of noise.

PERFORMANCE AND SIMULATION RESULTS

In order to compare the symbol error rate (SER) of the proposed technique with the theoretical SER of an AWGN channel without I/Q imbalance, we use a static channel with $A(t) = 1$ to maintain the same relation of E_b/N_o of an AWGN channel, $\theta = 0, \epsilon = 0.1$ and $\phi = \pi/18$. We used 10 symbols to initialize each pair of neuron and 100 iterations to refine the weights with a step-size of 0.002. We also use a traditional receiver based on a perfect PLL (phase-locked loop) and a perfect AGC (automatic gain control) to benchmark our technique to a more traditional one. The results are shown in figure 3.

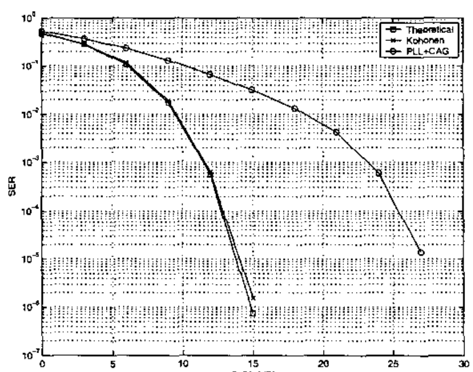

Figure 3: SER vs E_b/N_o

It is noticeable from figure 3 that the proposed technique matches the performance of a system without distortion. The performance of the benchmark with the traditional PLL and AGC is considerably worse. For instance, at 10^{-3} BER, there is a loss of more than 10 dB.

The convergence is analyzed using $A(t) = 1, \theta = 0, \epsilon = 0.1$,$\phi = \pi/12$ and $E_b/N_o = 100$ dB, a scenario wherein the error is mainly due to the adaptation/tracking process. We initialize the neurons at $\pm 4 \pm 4i$ and we use the following error to show the convergence:

$$
\begin{aligned}
Error(k) = |a(k) - \Re\{\alpha' + \beta'\}|^2 + |b(k) - \Im\{-\alpha' + \beta'\}|^2 \\
+ |c(k) - \Im\{\alpha' + \beta'\}|^2 + |d(k) - \Re\{\alpha' - \beta'\}|^2
\end{aligned}
\tag{20}
$$

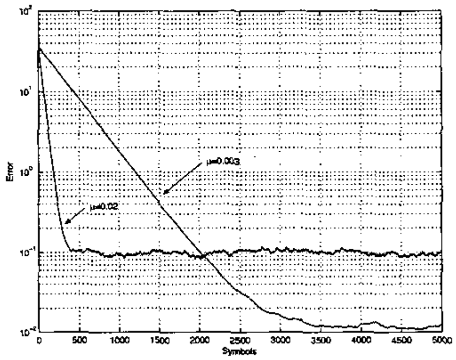

Figure 4: Evolution of Error for static channel and weight initilization at $\pm 4 \pm 4i$.

Evidently, the convergence speed depends greatly on the initialization. However, even with this bad initialization ($\pm 4 \pm 4i$) the algorithm converged considerably fast for a step-size of 0.02 (350 symbols approximately).

The next simulation shows, in figure 5, the error for a time varying channel over 200 independents trials for 16-QAM, $\epsilon = 0.1$, $\phi = \pi/18$ and $E_b/N_o =$ 100 dB, so that the error is mainly due to the adaptation/tracking process. To asses the performance of the algorithm in a time-varying channel, we assumed that there is a phase rotation $e^{j2\pi fTk}$, being $fT = 10^{-4}$, where T is the symbol period. The signal amplitude varies according to the expression $A(k) = 1 + 0.3\cos(2\pi fTk)$. The adaptation step-size is equal to 0.02 and the neurons where initialized with the perfect values of the centers of mass.

From figure 5, we can see that the algorithm is able to track the channel variations, but the error is considerably high. As a matter of fact, the simulations show that it is not possible to track channels much faster than $fT = 10^{-4}$ for 16-QAM modulation.

In order to show the blind estimation of the centers of gravity, we will analyze its convergence initializing the neurons at $\pm 4 \pm 4i$ for 1)$A(t) = 1, \theta =$ $\pi/4$, which represents the worst case of phase rotation, $\epsilon = 0.1$ and $\phi = \pi/18$ and 2)$A(t) = 1, \theta = \pi/4$, $\epsilon = 0.1$ and $\phi = \pi/9$. For both simulations (figures 6 and 7), $E_b/N_o = 10$ dB and adaption step-size of 0.02.

In figure 6, we have indicated the regions that are local minima of the algorithm. Note that with a sufficient large step-size, the neurons were able

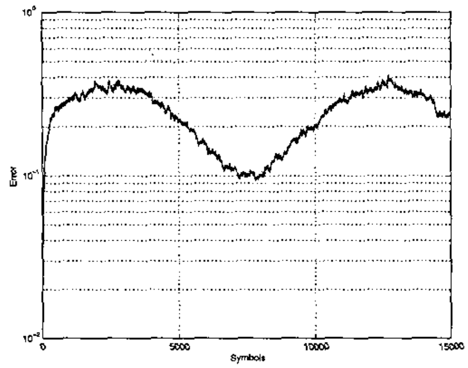

Figure 5: Error for tracking a time varying channel.

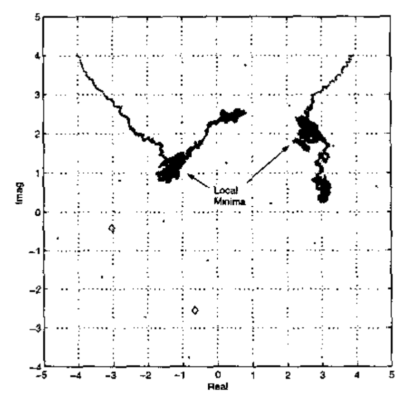

Figure 6: Blind estimation of centers of gravity for condition 1.

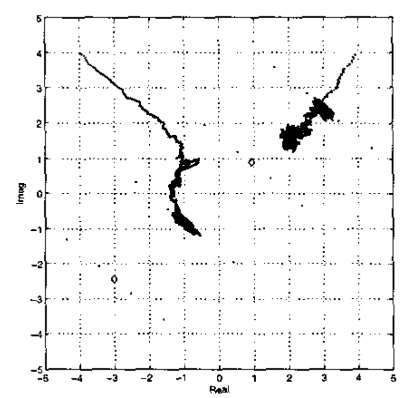

Figure 7: Blind estimation of centers of gravity for condition 2.

to escape from that region. However, in figure 7, which represents a more aggressive distortion, the neurons were not able to converge to the good solution. Firstly, they converged to local minima and thereafter jumped to another sub-optimal solution. This situation shows that blind convergence may not be very reliable in heavily-distorted systems, which renders imperative an initialization procedure with known symbols.

CONCLUSION

In this paper, we have proposed an adaptive decision device based in self-organizing maps to compensate I/Q imbalance and flat fading for 4^n-QAM modulation. Using the symmetry of the transmitted modulation and some specific characteristics of the channel and receiver impairments, we were able to optimize the technique for such impairment and achieve a relatively low computational cost algorithm. Moreover, we have shown by simulations that our technique achieves optimal performance and can track the channel without the aid of a training sequence. Blind initialization is also possible but, in this case, optimal convergence is not guaranteed.

As a future work, we envisage to extend this technique to selective fading channels, using an equalizer to mitigate intersymbol interference and our technique to estimate the I/Q imbalance.

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