A LEAST-SQUARE UNCONSTRAINED FREQUENCY-DOMAIN ADAPTIVE FILTER APPROACH FOR CHIP-LEVEL EQUALIZATION OF DS-CDMA SYSTEMS

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ABSTRACT

In this paper, we propose a modified least-square unconstrained frequency-domain block adaptive filter for chip-level equalization of the downlink of a DS-CDMA system. The modification allows us to refine the equalizer coefficients and improve the convergence and tracking by reducing the misadjustment error. We also analyze the impact of the wrap-around effect that is introduced by the circular convolution. Such analysis takes into account the position of the FFT window, the channel's phase, the signal-to-noise ratio per bit and the system load.

1. INTRODUCTION

Linear chip-level equalization for downlink broadband CDMA systems can be a computational demanding task due to the adaptation of a large finite-impulse response (FIR) equalizer and the signal filtering process. This functionalities can be efficiently implemented in the frequency-domain through the Fast Fourier Transform (FFT), resulting in the so-called frequency domain block adaptive filter (FDAF) [1],[2]. Such technique uses the least-mean squares (LMS) algorithm and was extensively analyzed in the literature [3]. Lately, the use of the cyclic prefix (CP) to generate a circular convolution has given a new impulse on this approach, since it can further simplify the signal equalization. Based on the circular FDAF (CFDAF) technique, we propose in this paper a least-square frequency-domain adaptive filtering technique that is similar to the FDAF in complexity and can achieve higher performance in time-varying channels.

This paper is organized as follows. In Section 2, the system model is presented. The proposed frequency domain equalization technique is described in Section 3. Section 4 assesses the performance and, finally, the conclusions are stated in Section 5.

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2. SYSTEM MODEL

We consider a synchronous downlink of a DS-CDMA system. The transmitted signal $x(i)$ is given by:

$$x(i) = \sum_{l=1}^{L} \left\{ a_l \left(\left\lfloor \frac{i}{N} \right\rfloor \right) c_l\left(mod(i,N)\right) s(i) \right\} \qquad (1)$$

where l stands for the l-th user, $a_l(m)$ is the m-th symbol, $c_l(i)$ is the spreading sequence, N is the length of the spreading sequence (spreading factor) and $s(i)$ is a random complex scrambling sequence. For this paper, we assume that $c(i)$ are Walsh-Hadamard sequences, which can assume the values ±1. The scrambling sequence $s(i)$ is an uniformly distributed random sequence that can assume the values $(\pm1\pm j)/\sqrt{2}$ and that $N = 64$ for all simulations.

The received signal $r(i)$ is represented by:

$$r(i) = \mathbf{h}^T\mathbf{x}(i) + n(i) \qquad (2)$$

where $\mathbf{h} = [h(0)\ h(1)\ \cdots\ h(M-1)]^T$ is the channel vector with M coefficients, $\mathbf{x}(i) = [x(i)\ x(i-1)\ \cdots\ x(i-M+1)]^T$ is the signal vector and $n(i)$ is an additive white Gaussian noise with variance σ_n^2.

3. FREQUENCY-DOMAIN EQUALIZATION

Time dispersive channels in CDMA systems break the spreading codes orthogonality generating multiuser interference. One way to mitigate this problem in a synchronous downlink is to use a linear chip-level equalizer. Considering a FIR filter with N coefficients, the complexity involved in the filtering process of each spread symbol in a time-domain implementation is proportional to $\mathcal{O}\{N^2\}$. On the other hand, the complexity of a frequency-domain implementation using the FFT (fast convolution) is proportional to $\mathcal{O}\{2N\log_2 2N\}$. For $N > 8$, the frequency-domain approach is more efficient and, specially for moderate and large values of N, it largely reduces the implementation complexity. Furthermore, the adaptation of the filter coefficients with a least-mean square (LMS) algorithm can have similar complexity savings with the use of the fast correlation technique that also uses the FFT.

A classical implementation of a FDAF using the minimum mean square error (MMSE) criterion is described in [3]

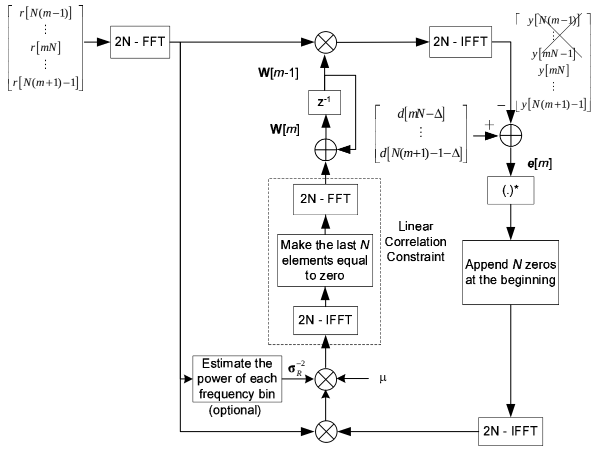

Fig. 1. FDAF equalization technique

and is depicted in figure 1. One interesting characteristic of the frequency-domain approach is that the FFT can separate the adaptation of the filter parameters. In addition, updating with a normalized LMS algorithm improves convergence and tracking [4]. In the unconstrained FDAF (UFDAF), the constraint on the gradient calculation is eliminated which saves some complexity, but it may lead to a biased Wiener solution due to the wrap-around effect [5]. However, if FIR identification is performed with the UFDAF, it is possible to achieve the unbiased Wiener solution if the size of the FIR filter to be identified is smaller or equal than N.

If a CP of appropriate size, i.e., equal or larger than the channel impulse response, is used in the transmission, it is possible to use a circular FDAF (CFDAF) approach. This technique does not need to use any constraint, since the received signal appears to have been circularly convoluted with the channel so that the m-th received symbol can be written as:

$$R(k,m) = H(k)X(k,m) + N(k) \tag{3}$$

where $H(k)$ is the discrete Fourier transform (DFT) of the channel, $X(k,m)$ is the DFT of the m-th spread symbol and $N(k)$ is the DFT of the additive Gaussian noise.

In this context, the mean square error (MSE) of the linear chip-level equalizer for the considered DS-CDMA system can be written as:

$$J_{MSE}(W(k)) = E\{a_l(m)C_l(k,m) - W^*(k)R(k,m)\} \tag{4}$$

where $E\{\cdot\}$ is the expectation operator, $C_l(k,m)$ is the FFT of the spreading code $\{c_l(mod(i+mN,N))s(i+mN)\}$, $(\cdot)^*$ is the conjugate operator and $W(k)$ is the k-th bin equalizer coefficient.

Since $\{c_l(mod(i+mN,N))s(i+mN)$ is independent and has an unitary variance, the solution that minimizes (4) is given by:

$$W(k) = \frac{\sigma_a^2 N H^*(k)}{\sigma_R^2} \tag{5}$$

where $\sigma_R^2 = LN\sigma_a^2 \left|H(k)\right|^2 + N\sigma_n^2$.

Instead of using the stochastic steepest-descent version of (4), we can achieve the same solution by using a least-square approach to achieve (5):

$$\begin{aligned}\hat{H}(k,m) &= \hat{H}(k,m-1) + \\ &\quad \mu(R(k,m) - D(k,m)\hat{H}(k,m-1))D^*(k,m) \\ \alpha &= (1+\lambda)\hat{\sigma}_R^{-2}(n-1,k) - \lambda\hat{\sigma}_R^{-4}(k,n-1)\left|R(k,n)\right|^2 \\ \hat{\sigma}_R^{-2}(k,n) &= \begin{cases} \varepsilon & \text{for } \alpha < \varepsilon \\ \alpha & \text{otherwise} \end{cases}\end{aligned} \tag{6}$$

where $D(k,m)$ is the Fourier transform of $a_l(m)\{c_l(mod(i+mN,N))s(i+mN)$, the estimation of σ_R^{-2} needs a constraint to avoid instability and ε is a small constant.

This least-square approach allow us to refine the channel coefficients estimation through tap-selection and windowing in the time-domain, improving convergence and tracking performance [6]. The refining process is defined by:

$$\begin{aligned}\hat{\mathbf{h}} &= \text{DFT}^{-1}\{\hat{H}\} \\ \hat{h}(i) &= 0, \;\; \text{for } i \geq \hat{M} \\ h_{max} &= \max\{|\hat{\mathbf{h}}|^2\} \\ \check{h}(n) &= \begin{cases} \hat{h}(n), & \text{if } |\hat{h}(n)|^2 > \beta h_{max} \\ 0, & \text{otherwise} \end{cases} \\ \check{\mathbf{H}} &= \text{DFT}\{\check{\mathbf{h}}\}\end{aligned} \tag{7}$$

and we use $\check{\mathbf{H}}$ to obtain $W(k)$.

It is worth noting that refining $\sigma_R^{-2}(k)$ does not yield good results, since the time-domain counterpart is usually composed by many small components. On the other hand, the estimation of $\sigma_R^2(k)$ may be improved by the refining technique. However, we would have to use $2N$ divisions to calculate $W(k)$. Since divisions are more complex than products, we prefer to directly obtain $\sigma_R^{-2}(k)$.

This least-square approach can also be employed with the UFDAF [7]. Thus, we propose to use the least-square approach with the above coefficients refining technique in the synchronous downlink of a non-CP DS-CDMA system.

3.1. The proposed technique

If implemented with an FFT of size N, we would not capture part of the dispersed symbol and the wrap-around effect in the filtering would have a large impact in the performance. However, with an $2N$-point FFT, we are able to capture all the dispersed symbol and the wrap-around effect is greatly reduced as we are going to see later.

The proposed technique is depicted in figure 2. Note the addition of delay Δ that controls the position of the FFT window is limited by $0 \leq \Delta \leq N$. The larger Δ, the larger will be the number of past samples in the FFT window with respect to the training sequence. For a smaller Δ, we have more future samples. The role of this delay Δ as we are going to see is equivalent to the delay used in the training sequence of a conventional adaptive equalizer.

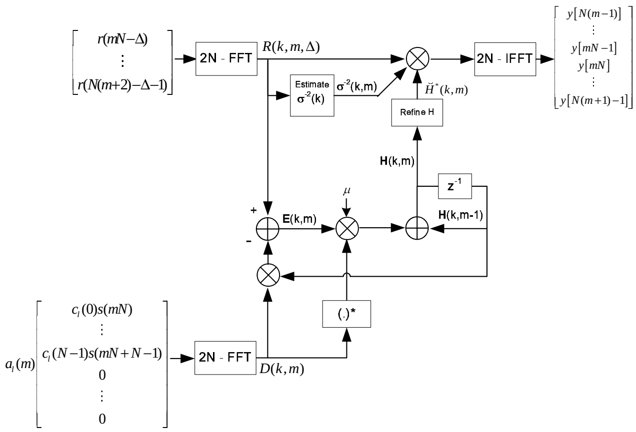

Fig. 2. Enhanced UFDAF equalization technique

The first N samples of the equalizer's output are kept for despreading and the rest is discarded.

Due to the presence of the delay Δ, the important samples to keep in $\hat{\mathbf{h}}$ are the ones going from Δ to $\Delta + \hat{M}$ and the rest can be set to zero.

4. PERFORMANCE ASSESSMENT

In the next subsections, we show how the position of the FFT window with respect to the training sequence impacts on the mean square error (MSE) of the equalized signal. We also compare the performance of the UFDAF with the traditional FDAF technique with respect to signal-to-noise ratio per bit (E_b/N_o), number of active users, convergence time and tracking capability.

4.1. Position of the FFT window and the MSE

The solution to the channel estimator in the proposed technique is given by:

$$\mathbf{H} = \sigma_a^2 \mathrm{diag}\left\{ \mathbf{FJ}\boldsymbol{\mathcal{H}} \begin{bmatrix} \mathbf{0}_{2N\times N} & \begin{matrix} \mathbf{0}_{N-\Delta\times N} \\ \mathbf{I}_{N\times N} \\ \mathbf{0}_{\Delta\times N} \end{matrix} \end{bmatrix} \mathbf{J}^H \mathbf{F}^H \right\} \tag{8}$$

where $\mathbf{F}$ is the discrete Fourier transform matrix of dimension $2N \times 2N$, $\mathbf{J} = \begin{bmatrix} 0 & & 1 \\ & \iddots & \\ 1 & & 0 \end{bmatrix}$, and $\mathcal{H}$ is the convolution matrix:

$$\begin{bmatrix} h(0) & \cdots & h(M-1) & 0 & \cdots & 0 \\ 0 & h(0) & \cdots & h(M-1) & \ddots & \vdots \\ 0 & 0 & h(0) & \ddots & \ddots & 0 \\ 0 & \vdots & 0 & h(0) & \ddots & h(M-1) \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & h(0) \end{bmatrix}$$

Similarly, the power of each frequency bin is given by:

$$\begin{aligned} \sigma_R^2(k) &= \mathrm{E}\{R(k,m,\Delta)R^*(k,m,\Delta)\} \\ &= \mathrm{E}\{\mathbf{f}(k)\mathbf{Jr}(m,\Delta)\mathbf{r}^H(m,\Delta)\mathbf{J}^H\mathbf{f}^H(k)\} \\ &= \mathbf{f}(k)\mathbf{J}\mathrm{E}\{\mathbf{r}(m,\Delta)\mathbf{r}^H(m,\Delta)\}\mathbf{J}^H\mathbf{f}^H(k) \\ &= \mathbf{f}(k)\mathbf{J}\mathbf{R}_{rr}\mathbf{J}^H\mathbf{f}^H(k) \end{aligned} \tag{9}$$

where $\mathbf{f}(k)$ is the k-th row of the matrix $\mathbf{F}$ and $\mathbf{R}_{rr}$ is the autocorrelation matrix of the received signal.

Finally, the equalizer coefficients are:

$$W(k) = \frac{H^*(k)}{\sigma_R^2(k)} \tag{10}$$

In order to show how the position of the FFT window affects the equalizer performance, we measure the MSE:

$$J_{MSE}(\mathbf{h},\Delta) = \sum_{i=\Delta}^{\Delta+N-1} E\left\{ |x(mN+i-\Delta) - \gamma\hat{x}(m,i,\Delta)|^2 \right\} \tag{11}$$

where $\hat{x}(m,i,\Delta) = \frac{1}{2N}\sum_{k=0}^{2N-1} R(k,m,\Delta)W^*(k)e^{j2\pi ki/2N}$. Let us define $\mathbf{W} = diag\,(W(0), W(1), \cdots, W(2N-1))$, $\mathbf{v}(k)$ as the k-th line of $\mathbf{F}^{-1}$ and expanding (11), we get:

$$\begin{aligned} &J_{MSE}(\mathbf{h},\Delta) = \\ &\sum_{k=\Delta}^{\Delta+N-1} \Big\{ \sigma_x^2 - \gamma\mathbf{q}^H(k)\mathbf{J}^H\mathbf{F}^H\mathbf{W}\mathbf{g}_k^H - \gamma\mathbf{g}_k\mathbf{W}^\mathbf{H}\mathbf{FJq}(k) \\ &\qquad + \gamma^2\mathbf{g}_k\mathbf{W}^H\mathbf{FJR}_{rr}\mathbf{J}^H\mathbf{F}^H\mathbf{W}\mathbf{g}_k^H \Big\} \end{aligned} \tag{12}$$

where $\mathbf{q}^H(k) = \mathrm{E}\left\{x(nN+k)\mathbf{r}^H(n,k)\right\} = \sigma_x^2 \left[\underbrace{0\ \cdots 0}_{k}\ \mathbf{h}^H\ 0\ \cdots 0 \right]$, $\mathbf{g}_k = \mathbf{f}^*/2N$ and γ is a gain compensation to avoid any bias in the analysis and that optimal value is given by:

$$\gamma = \frac{\sum_{k=d}^{d+N-1} \mathbf{q}^H(k)\mathbf{F}^H\mathbf{W}^H\mathbf{g}_k^H + \mathbf{g}_k\mathbf{WFq}(k)}{2\sum_{k=d}^{d+N-1} \mathbf{g}_k\mathbf{W}^\mathbf{H}\mathbf{F}\mathbf{R}_{rr}\mathbf{F}^H\mathbf{W}\mathbf{g}_k^H} \tag{13}$$

Figure 3 illustrates the MSE (12) as a function of the delay Δ and three different channels, all having the same frequency amplitude responses, but different phase responses. The channels used are: a minimum phase channel $H(z) = 0.8944 + 0.4472z^{-4}$, a linear phase response $H(z) = 0.6325 + j0.4472z^{-2} + 0.6325z^{-4}$ and a maximum phase response $H(z) = 0.4472 + 0.8944z^{-4}$. For a linear equalizer that tries to invert a minimum phase channel, the more past samples available (large value of Δ), the lower is the MSE. The

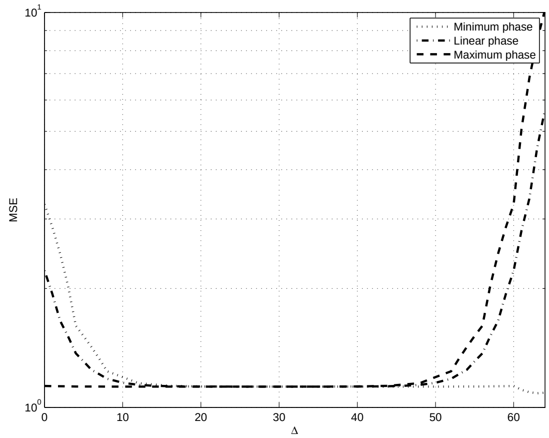

Fig. 3. MSE as a function of the position of the FFT window of the UFDAF and σ_x^2/σ_n^2=20dB.

opposite is seen for the maximum phase channel and the non-minimum phase channel is better equalized with an intermediate delay. Therefore, the best delay can be chosen if we know the channel phase. Note that, due to the construction of the input of the FFT, a large delay Δ means a small delay in the training sequence used for adaptation of an equalizer. It is worth noting that, for the MSE calculation, the equalizer coefficients were obtained with perfect channel and noise power knowledge.

A similar analysis of an equalizer that uses a larger FFT to equalize a system without CP was made in [8]. However, the equalizer coefficients are different from the ones utilized here and it uses only random Rayleigh channels, which shows that an intermediate delay gives the lowest error. Nonetheless, it does not give any indication of the relationship between the channel's phase and the FFT's window position with respect to the training sequence.

4.2. Sensitivity to the E_b/N_o and number of active users

In these simulations, we use a DS-CDMA system with QPSK symbols. We assumed a Rayleigh channel with relative delays 0, $2T_c$, $6T_c$, $9T_c$ and $12T_c$, with T_c being the chip period, and respective powers of 0dB, -1dB, -3dB, -6dB, -9dB. The choice of Δ was based on the following assumptions: 1) we do not know which is the channel's phase, due to its random nature; 2) we assume that the behavior of the BER versus the choice of Δ for a given channel will be very similar to the MSE behavior shown in previous subsection. Therefore, we make a conservative choice, setting the delay to an intermediate value of $\Delta = 32$ for the UFDAF. Since the FBAF has a similar behavior under these assumptions, we have also chosen an equivalent training delay equal to 33. The coefficients for both techniques were obtained with perfect channel knowledge. We show the bit error rate (BER) versus E_b/N_o in figure 4.

The degradation of the proposed technique becomes slightly

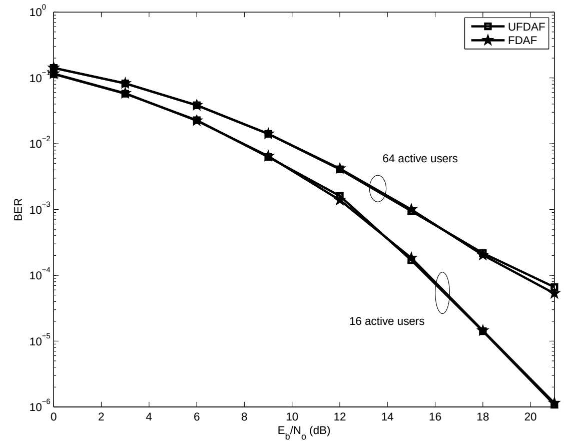

Fig. 4. Bit error rate for 64 and 16 users for a block Rayleigh fading channel with relative delay 0, $2T_c$, $6T_c$, $9T_c$ and $12T_c$ and respective powers of 0dB, -1dB, -3dB, -6dB, -9dB.

apparent only with higher E_b/N_o and full system load. In such a case, the equalizer will tend to invert the channel, producing a larger equalizer. Hence, the distortion generated by the circular convolution becomes more important, degrading the BER. For lower system loads, the performance is equal to the FDAF technique for the whole E_b/N_o simulated range.

4.3. Convergence and tracking assessment

The main advantage of the proposed technique is that it enables the use of a tap-selection and windowing of the estimated channel coefficients, keeping the overall complexity close to the FDAF technique. This procedure allows us to improve the quality of the estimator and obtain faster convergence and superior tracking in comparison to the FDAF technique.

In order to illustrate these characteristics, figure 5 shows the BER as a function of the number of iterations of each technique for a three-path Rayleigh block fading channel with relative delays 0, $4T_c$ and $12T_c$ and respective relative powers of 0dB, -3dB and -6dB. In figure 6, we show the BER for a time-varying Rayleigh channel with $f_d N T_c = 2 \times 10^{-4}$, and paths with relative delays equal to 0, $2T_c$, $6T_c$, $9T_c$ and $12T_c$ and respective relative powers of 0dB, -1dB, -3dB, -6dB and -9dB. The adaptation was done considering 32 active users and one pilot code with the same power of a single user. The spread symbols are BPSK. The BER for both simulations were obtained for 3000 channel realizations. The BER for the time-varying channel was measured after 250 iterations to acquire the channel and each block is composed of 700 symbols. The FDAF estimates the inverse of the power of each frequency bin to achieve faster convergence. The inverse of the power of each frequency bin was estimated using the algorithm described in (6). The adaptation step-sizes for each technique were chosen through simulations in order to obtain the best performance. Finally, a delay of 32 was chosen for both UFDAF and a training delay of 33 for the FDAF.

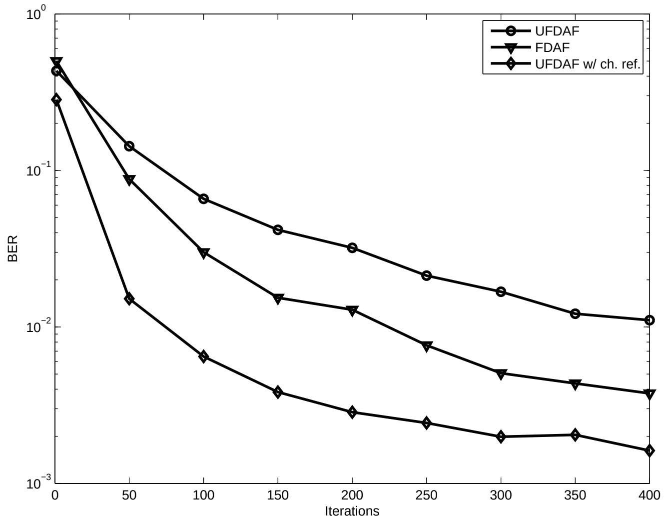

Fig. 5. Time to convergence measured in terms of BER for a three-path Rayleigh block fading channel with relative delay 0, $4T_c$ and $12T_c$ and respective relative powers of 0dB, -3dB and -6dB.

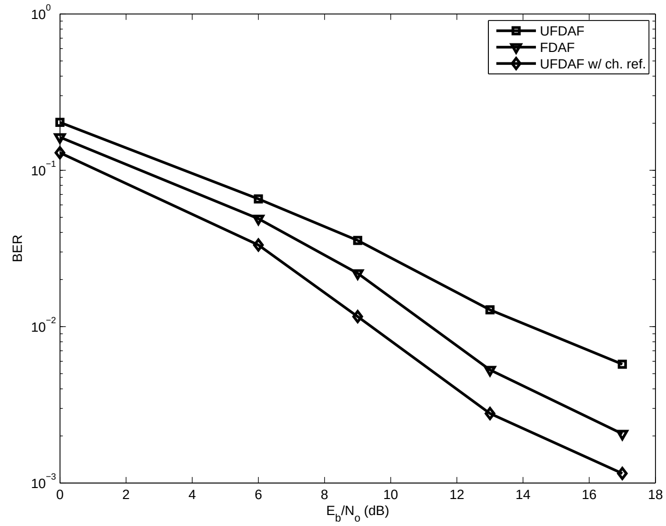

Fig. 6. BER for time-varying channel with $fdNT_c = 0.0002$, paths with relative delay equal to 0, $2T_c$, $6T_c$, $9T_c$ and $12T_c$ and respective relative powers of 0dB, -1dB, -3dB, -6dB and -9dB.

From both simulations, it can be seen that the UFDAF has lower performance than the FDAF. However, the UFDAF using the channel refinement method can achieve the lowest BER with approximately the same computational cost due to better tracking and lower misadjustment of the equalizer coefficients.

5. CONCLUSIONS

In this paper, we have proposed a modified unconstrained FDAF equalizer based on a least-square criterion for chip-level equalization of the synchronous downlink of DS-CDMA systems. The proposed technique can adjust the position of the FFT window in order to minimize the distortion introduced by the wrap-around effect in each filtered block. The results indicate that the adjustment depends on the channel's phase and is similar to the training sequence delay used in conventional adaptive filtering. We also show through simulations in Rayleigh frequency-selective channels that the distortion introduced by the wrap-around effect only becomes important in high E_b/N_o values and high system loads. In term of convergence and tracking, the unconstrained FDAF suffers from higher misadjustment. However, the least-square approach allows us to refine the coefficients. This method improves the convergence and tracking with a overall complexity equivalent to the conventional FDAF. The simulations show that it can outperform the latter technique.

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