A robustness and performance comparison between Cyclic Prefixed Single-Carrier and OFDM systems

Amanda Souza de Paula
São Paulo University - USP
São Paulo - SP - Brazil
amanda@lcs.poli.usp.br

Cristiano Magalhães Panazio
São Paulo University - USP
São Paulo - SP - Brazil
cpanazio@lcs.poli.usp.br

***Abstract*— This work aims to establish a comparison between orthogonal frequency division multiplexing (OFDM) and single carrier with cyclic prefix (SCCP) focusing on robustness problems. Differently of most of the papers found in the literature, we analyze, in more practical scenarios, not only the bit error rate (BER) but also the block error rate (BLER) and the system sensitivity to the interleaver configuration for frequency selective block fading channels.**

***Index Terms*— OFDM, single-carrier, channel coding, equalization, interleaver, block fading**

I. INTRODUCTION

The OFDM is a popular block transmission technique which consists in dividing the available bandwidth into several orthogonal subcarriers. One important advantage concerning OFDM is the low complexity of the equalizer. If a cyclic prefix (CP) is appended in the transmission block, the equalization can be easily performed with a single-tap equalizer per subcarrier. However, the OFDM alone does not exploit the channel frequency diversity, which means that in a strongly frequency selective channel a subcarrier may be highly attenuated and the information transmitted in it may be lost. Hence, channel coding is an obligatory technique to recover information in this kind of situation as noted by [1].

On the other hand, the single-carrier (SC) technique transmits the data symbols through just one carrier at much larger symbol rate. The result is that each symbol is spread over the available bandwidth, allowing us to recover the transmitted symbols even in frequency selective channels. This may be accomplished by using a frequency-domain equalizer that can assume the same form used in OFDM if we use a SCCP block transmission technique. The solely difference compared to the OFDM is that the inverse fast Fourier transform (IFFT) used in the transmission is replaced after the equalizer at the receiver [2].

Many articles have compared the performance of SCCP and OFDM techniques in absence of channel state information at the transmission. In [3], it is shown for uncoded systems that linearly equalized SCCP with the minimum mean square criterion always outperforms the uncoded OFDM in frequency selective channels. In [4], it is shown that the coded OFDM performs similarly to a coded linearly equalized SCCP when the code rate is equal or smaller than one-half for QPSK modulation, but it degrades rapidly when we increase the coding rate, in accordance to [3]. In both [5] and [6], it is shown that with coding, both SCCP and OFDM can attain the channel diversity. However, most of these comparisons are restricted to random coding, infinite block length or an asymptotic analysis. Other papers (e.g., [2], [7]) are more practical but they just confirm the results presented in more theoretical papers or just emphasize on the lower peak-to-average-power ratio of the SCCP. Our contribution, presented in this work, is a more critical analysis of the performance of both coded OFDM and SCCP techniques by focusing our attention on the BLER, interleaver configuration choice and their robustness in frequency selective block fading channels.

The paper is organized as follows. In section 2 we present the unified system model used in both transmission systems analysis. In section 3 we show some results obtained under different contexts. Finally, the conclusions are indicated in section 4.

II. SYSTEM MODEL

In the OFDM technique, the symbols are modulated by subcarriers that are generated by an IFFT. A CP is added to the resultant time-domain signal. In this analysis we assume that the CP length is greater than the channel impulse response. Therefore, the linear convolution involving the channel impulse response and the transmitted block is equivalent to a circular convolution and the equalization can be performed with the frequency-domain one-tap equalizer structure.

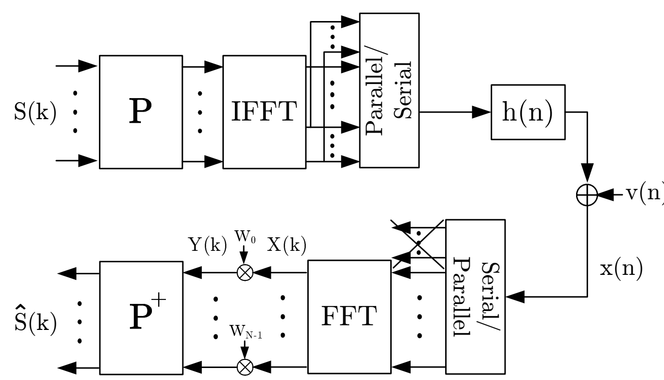

Fig. 1. An unified model for SCCP, OFDM and CDMA

In the SCCP we append to each symbol block a CP that allows us to use the same equalizer structure used in the OFDM technique. However, an IFFT has to be used after the equalizer to allow the symbol detection.

The similarities between the SCCP and OFDM allow us to describe both modulation techniques under an unified transceiver, depicted in Fig. 1 [8]. In fact, the difference between them is a linear precoding matrix $\mathbf{P}$.

In accordance to this system model, the transmitter can be regarded as a combined transformation of the linear precoding matrix with the IFFT matrix, resulting in the so-called transmission matrix:

$$\mathbf{T} = \mathbf{P}\mathbf{F}^{-1} \tag{1}$$

where, the matrix $\mathbf{F}$ is the N-dimensional Fourier matrix:

$$[\mathbf{F}]_{n,k} = e^{-j\frac{2\pi}{N}(n-1)(k-1)} \quad n,k = 1,\cdots,N$$

Note that if the matrix $\mathbf{P}$ is a simple identity, the transmission matrix is given by:

$$\mathbf{T}_{OFDM} = \mathbf{F}^{-1} \tag{2}$$

which is exactly the OFDM transmission matrix.

On the other hand, if the transmission matrix is the Fourier matrix, the resulting transmission matrix is

$$\mathbf{T}_{SC} = \mathbf{F}\mathbf{F}^{-1} = \mathbf{I}_N \tag{3}$$

which is equivalent to the SC transmission with cyclic-prefix.

Note that this approach makes clear the idea that each symbol in SCCP is spread all over the bandwidth. Due to this characteristic, the SCCP is also known in the literature as DFT-Spread OFDM.

As well as the OFDM and the SCCP, other transmission techniques can also be implemented based on this same system model.

Such model can also describe synchronous coded division multiple access (CDMA) systems. A very well known example is the Walsh-Hadamard (WH) CDMA, where $\mathbf{P}$ is the WH transformation matrix. For direct sequence (DS) CDMA, the linear precoding matrix is obtained by taking the Discrete Fourier Transform (DFT) of the time domain spreading codes.

The received message is equalized with the single-tap structure, represented by the W_k coefficients in the Fig. 1. These equalizer coefficients can be calculated using one of two criteria: the Zero Forcing (ZF) and the Minimum Mean-Square Error (MMSE). The ZF equalizer cancels the intersymbol interference (ISI), although it is likely to provide a large noise enhancement. Its coefficients are calculated as:

$$W_{ZF}(k) = \frac{1}{H(k)} \tag{4}$$

On the other hand, the MMSE coefficients are obtained solving the following optimization problem:

$$\underset{W}{\operatorname{argmin}}\ E\left\{\sum_{k=0}^{N-1} |D(k) - W(k)\cdot X(k)|^2\right\} \tag{5}$$

where, $D(k)$ and $X(k)$ are the transmitted signal and the received signal respectively, that are defined as:

$$\begin{aligned} \underline{\mathbf{D}} &= \mathbf{F}^{-1}\mathbf{P}\underline{\mathbf{S}} \\ \underline{\mathbf{D}} &= [D(0)\cdots D(N-1)]^T \\ \underline{\mathbf{S}} &= [S(0)\cdots S(N-1)]^T \\ X(k) &= D(k)H(k) + \eta(k) \end{aligned} \tag{6}$$

where the vector $\underline{\mathbf{D}}$ represents the transmitted signal vector, the vector $\underline{\mathbf{S}}$ is composed by the transmitted symbols, $H(k)$ are the coefficients of channel's impulse response in the frequency-domain and $\eta(k)$ is the additive Gaussian noise.

Defining σ_s^2 as the symbol power and σ_n^2 as the noise variance, it can be demonstrated that the coefficients which satisfy the condition imposed by eq.(5) are:

$$W_{MMSE}(k) = \frac{H^*(k)}{|H(k)|^2 + \sigma_\eta^2/\sigma_s^2} \tag{7}$$

From the eq.(4) and eq.(7), we can conclude that when the system is operating in very high SNR conditions, both equalizers are equivalent, i.e,

$$\lim_{\sigma_\eta^2/\sigma_s^2\to\infty} W_{MMSE}(k) = \frac{1}{H(k)} = W_{ZF}(k) \tag{8}$$

It is demonstrated in [3] that the uncoded OFDM performance is the same, regardless of employed criteria. For coded OFDM, if we pass to the cannel decoder the bit likelihood, we can still use both criteria. On the other hand, in channels with spectral nulls, the noise variance in the SCCP tends to infinity when the equalization is accomplished with the ZF structure. For that reason, the one-tap equalizer coefficients for both techniques are obtained using the MMSE criterion.

It is quite important to emphasize that the equalization can be performed with the single tap structure regardless the linear precoding matrix. Thus, the low complexity of the equalizer is not a characteristic inherent to the OFDM system, and any system that can be interpreted with the structure depicted in Fig. 1 can be equalized applying the same scheme.

So far we have not introduced the error correcting code in the discussion. However, very often using an error correcting code is required to transmit the message with the demanded reliability. In such cases, an extra block must be appended to the scheme depicted in Fig. 1. This block can be interpreted as a non-linear transformation applied to the symbols just before the linear transformation imposed by $\mathbf{T}$.

The error correcting code is usually implemented with one of these structures: bit-interleaved coded modulation (BICM) or the trellis coded modulation (TCM). For a frequency-selective channel, the received symbols in the OFDM technique can be seen as they were passed through a fast time-varying flat fading channel. Therefore, a bit-interleaved coded modulation (BICM) looks more adequate, since it outperforms trellis coded modulation in this context [9]. For this reason, we will consider the BICM technique in our analysis. One crucial question in the BICM design is the interleaver project. An approach is to arrange the bits in the interleaving matrix filling its lines and reading through its columns. This procedure

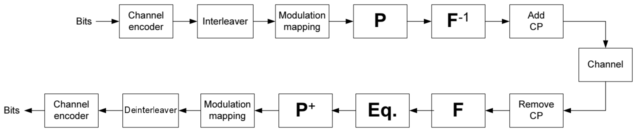

Fig. 2. The overall system model

applied to an interleaver matrix with m rows and $n = N/m$ columns is below illustrated:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_N \end{bmatrix} \xrightarrow{write} \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ x_{n+1} & x_{n+2} & \cdots & x_{2n} \\ \vdots & \vdots & & \vdots \\ x_{(m-1)n+1} & x_{(m-1)n+2} & \cdots & x_{mn} \end{bmatrix} \Big\downarrow read \begin{bmatrix} x_1 \\ x_{n+1} \\ x_{2n+1} \\ \vdots \\ x_N \end{bmatrix} \tag{9}$$

The main goal in interleaving the bits is to avoid error bursts in the decoder. Then, a question arises: which is the more suitable interleaver choice? And how this choice affects the system error correcting capability. The overall system is indicated in Fig. 2.

III. Performance Comparison

We will proceed with a performance comparison, analyzing how the system responds in a variety of contexts. The results depicted in this sections were obtained through Monte Carlo simulations. First of all, we adopt the same system parameters used in [2], i.e., block fading, 512 subcarriers, QPSK modulation, and a convolutional code $[133\ 171]_{\text{octal}}$. We have chosen a three-path Rayleigh channel, given by:

$$H(z) = h_0 + h_1 z^{-2} + h_2 z^{-3} \tag{10}$$

where the coefficients $h_k,\ k = 0, 1, 2$ are Rayleigh variables with zero mean and unitary variance.

Then, we analyze the interleaver configuration importance to the system performance and extend the analysis to other channels and to higher order modulation. Like [2], we have initially chosen a 32 row/32 column configuration. The equalizer coefficients are obtained assuming perfect channel knowledge. In our simulations, we also provide the matched filter bound (MFB) curves.

In Fig. 3, the BER of both OFDM and SCCP appears to be exaclty the same, in accordance to the results obtained in [4]. However, the BLERs, not shown in [2] and [4], are different. This is not completely unexpected since the received signal has different forms of interference: intersymbol interference for the SCCP and subcarriers suffering from flat-fading for the OFDM. In the latter, the recovery of the information transmitted in the faded subcarriers relies exclusively on the redundancy provided by the code. If the interleaver does not distribute the redundancy by taking into account the channel coherence bandwidth, the system may suffer a considerable performance loss. On the other hand, the SCCP may not have such dependency on the interleaver since it will only break the noise correlation caused by the equalizer. In order to assess the role of the interleaver, we change its number of rows/columns. The results shown in Fig. 4 confirm our previous assumption that the OFDM is indeed more affected by the the interleaver than the SCCP with a linear equalizer.

We next analyze the robustness of the techniques for fixed E_b/N_o values. In order to do so, we return to the original interleaver configuration (32 rows/columns) and, for each channel realization, we normalize its power, since we are only interested in how each system behaves for the ensemble of three-tap channels for a fixed E_b/N_o. The channel's transfer function in this scenario is

$$H(z) = \frac{h_0 + h_1 z^{-2} + h_2 z^{-3}}{\sqrt{|h_0|^2 + |h_1|^2 + |h_2|^2}} \tag{11}$$

The results are indicated in figure 5. In this case, the SCCP presents a better performance than the OFDM for E_b/N_o values larger than 4 dB. This result shows a hidden behavior, not shown in Fig. 3, where both BERs are the same. This can be explained by the fact that, in the non fixed E_b/N_o scenario, the BER performance accounts the E_b/N_o variations around the mean E_b/N_o simulated values, which includes the regions where the OFDM is better than the SCCP and vice-versa.

As can be inferred from Fig. 6, the interleaver also plays an important role in this case. It shows again that an interleaver with 8 rows represents a more favorable configuration to the OFDM system for the proposed channel. Applying this structure, the error rate curves are those presented in Fig. 7. If the OFDM is implemented with a suitable interleaver, its performance is equivalent to the SCCP in this case. But, nonetheless, we still remark a large sensitivity to the interleaver choice.

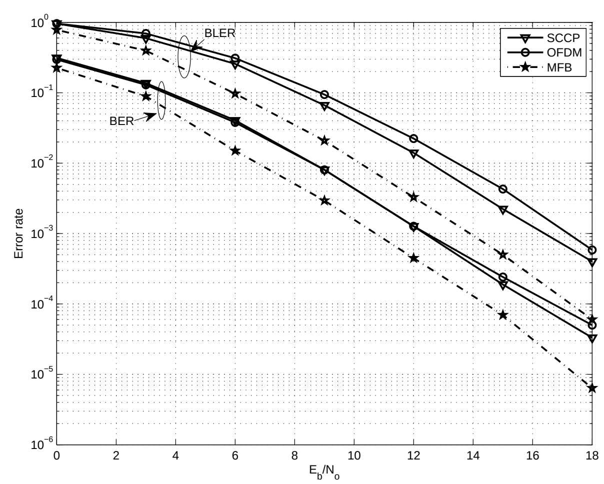

Fig. 3. Performance comparison for a three-path Rayleigh block fading channel with uniform power profile and $[133\ 171]_{\text{octal}}$ convolutional code.

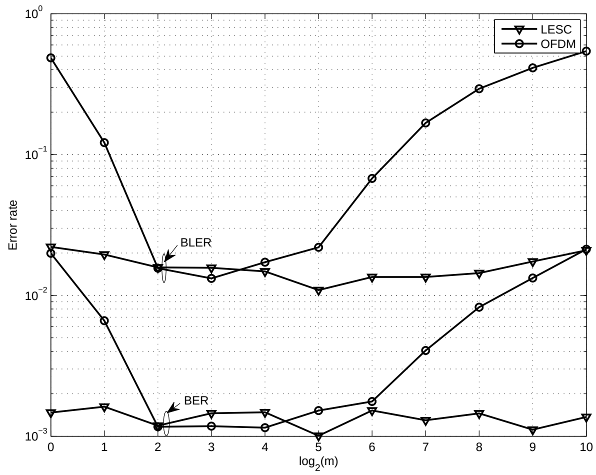

Fig. 4. Impact of the interleaver configuration on the system performance for a three-path Rayleigh fading channel with uniform power profile, $[133\ 171]_{\mathrm{octal}}$ convolutional code, E_b/N_o=12dB. Interleaver matrix with m rows.

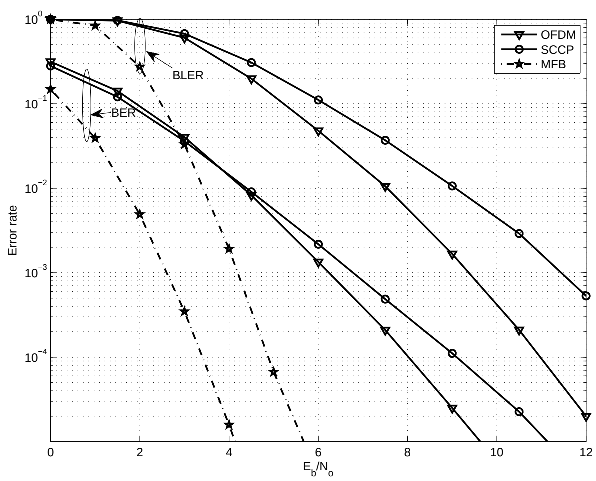

Fig. 5. Performance comparison for a three-path Rayleigh block fading normalized channel with uniform power profile and $[133\ 171]_{\mathrm{octal}}$ convolutional code.

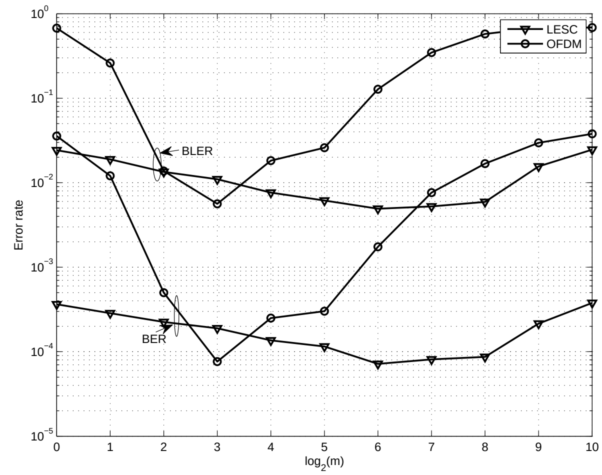

Fig. 6. Impact of the interleaver configuration on the system performance for a three-path Rayleigh block fading normalized channel with uniform power profile, $[133\ 171]_{\mathrm{octal}}$ convolutional code, E_b/N_o=8 dB. Interleaver matrix with m rows.

We also compared the SCCP with the OFDM in four other scenarios: larger delays, richer multipath diversity, higher-order modulation and static channel with spectral null.

A. Larger Delays

In this situation, the channel is still described by a three-path Rayleigh channel with uniform power profile, but the delays among the paths are different. These channels present the following transfer function

$$H_L(z) = h_0 + h_1 z^{-L} + h_2 z^{-2L} \tag{12}$$

The fluctuations in the frequency-domain are more severe with larger L values. In particular, if L divides N, the relation expressed below is satisfied:

$$H_L(k) = H(kL \mod (N)) \tag{13}$$

which means that the channel periodicity has changed in the frequency-domain. Therefore, to achieve an equivalent channel configuration in the receiver, the interleaver parameters should be modified in order to compensate the relation depicted in eq.(13). In accordance to the interleaving rule indicated in eq.(9), the row numbers should satisfy the following relation:

$$m_L = Lm \tag{14}$$

This relation can be confirmed with the simulation results presented in Fig. 8.

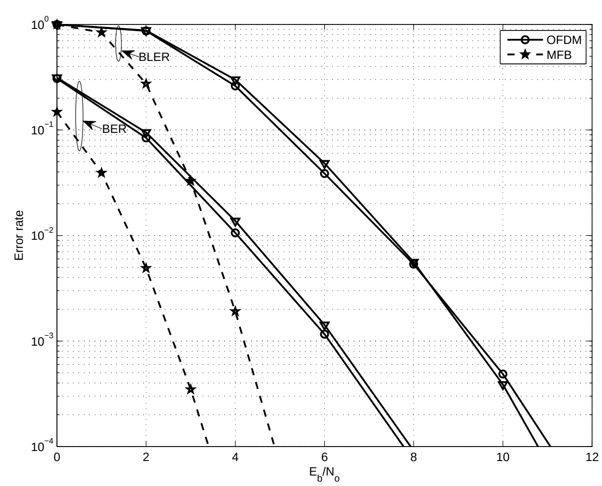

Fig. 7. Performance comparison for a three-path Rayleigh block fading normalized channel with uniform power profile and $[133\ 171]_{\mathrm{octal}}$ convolutional code. For the OFDM, a 8 row/128 column interleaver was used and for the SCCP a 32 row/32 column interleaver was chosen.

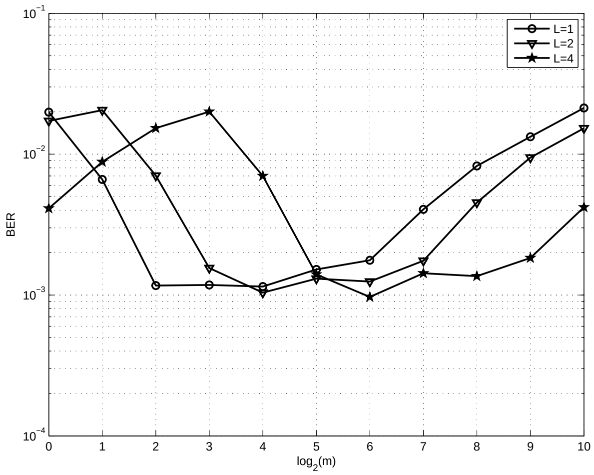

Fig. 8. Interleaver impact on system performance as a function of the channel delay for a three-path Rayleigh channel with uniform power profile, E_b/N_o=12dB.

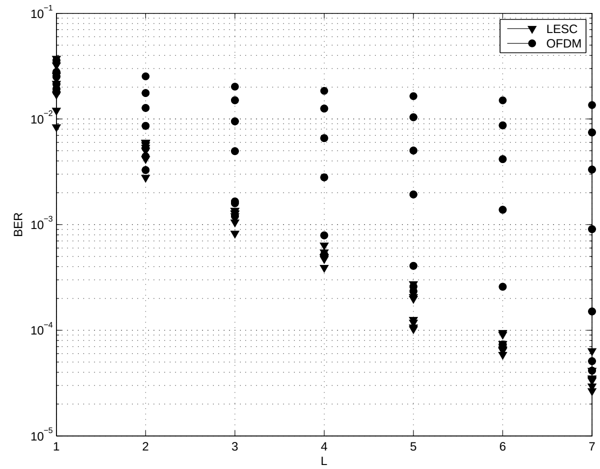

Fig. 9. Interleaver impact on system performance as a function of the channel diversity for a three-path Rayleigh channel with uniform power profile, E_b/N_o=12dB.

From this results, we emphasize that even if an interleaver is suitable for a channel configuration, changes in its delay lead to considerable changes in that interleaver response.

B. Richer Multipath Diversity

In this case, we analyzed the system robustness when the channel's transfer function can be expressed as

$$H(z) = \sum_{k=0}^{L-1} h_k z^{-k} \tag{15}$$

As in the other cases, the coefficients h_k are Rayleigh variables with zero mean and unitary variance.

The results depicted in Fig. 9 indicate the BER for all possible interleaver configuration as a function of the channel's diversity. It is possible to confirm the SCCP robustness almost regardless of the interleaver configuration. On the other hand, the performance of the OFDM varies widely, with some interleaver setups providing unacceptable BER values.

C. Higher-Order Modulation

We now obtain the performance of OFDM and SCCP for different interleaver configurations, using again the channel described by eq.(10), but now for 16-QAM modulation. Such results are presented in Fig. 10.

The sensitiveness to the interleaver parameters of both OFDM and SCCP are similar to the case with QPSK modulation. However, the OFDM modulation can achieve a far superior performance when compared to the SCCP with linear equalization. This performance difference is also shown in [2]. The same reference shows that the Decision-Feedback Equalizer (DFE) with perfect feedback can bridge the performance gap between the SCCP and OFDM. In order to show this, we present in Fig. 11 the BER and BLER of the OFDM and SCCP with DFE and linear equalization as a function of the E_b/N_o.

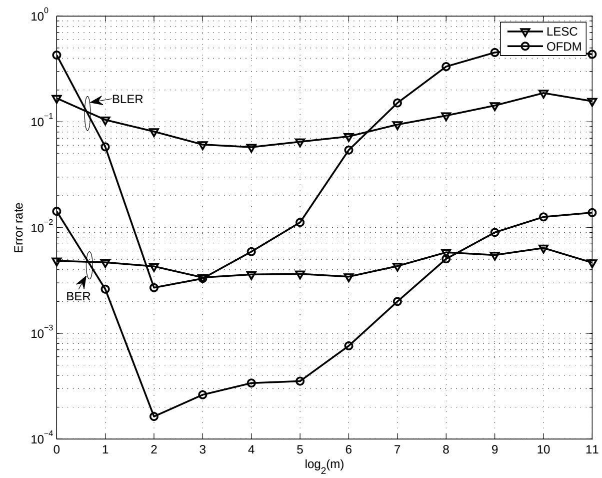

Fig. 10. Impact of the interleaver configuration on the system performance for a three-path Rayleigh , $[133\ 171]_{\mathrm{octal}}$ convolutional code, E_b/N_o=16dB and 16-QAM modulation. Interleaver matrix with m rows.

D. Static Channel with Spectral Null

We will analyze the system behavior when submitted to a static highly frequency selective channel. We have chosen a channel with three taps and a spectral null:

$$H(z) = 0.415 + 0.807z^{-1} + 0.415z^{-2} \tag{16}$$

In this kind of situation, linear equalization does not provide good results. For this reason, we have also analyzed the system performing the equalization with a DFE, which is notorious for its good performance under highly frequency selective channels.

In order to avoid the error propagation phenomenon, we use the joint DFE with channel decoding described in [10]. The code used in this simulation is the $[15\ 17]_{\mathrm{octal}}$, but we also show the performance of the OFDM with the $[133\ 171]_{\mathrm{octal}}$ for comparison reasons. The interleaver configuration was optimized for each technique.

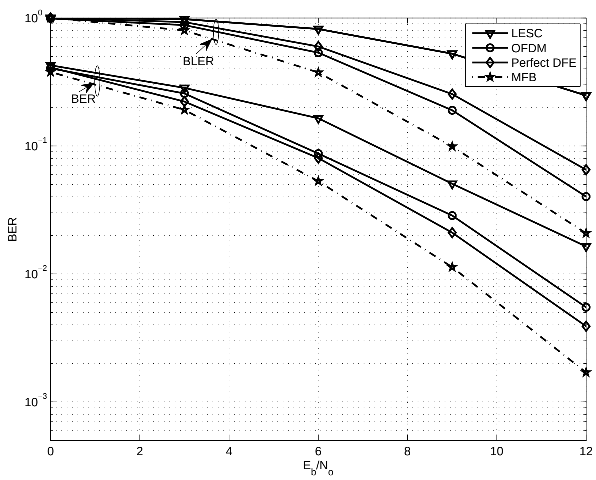

Fig. 11. Performance comparison, three-path Rayleigh channel with uniform power profile, 16-QAM modulation, convutional code $[133\ 171]_{\mathrm{octal}}$.

The results depicted in Fig.12 show that the OFDM, as well as the SCCP with linear equalization, leads to a BER much worse than the bound provided by the matched filter. The SCCP implemented with the joint DFE and decoder has a much superior performance. Even if we take into account the feedback error propagation inherent to DFE equalizers. Comparing the DFE with the OFDM, the performance gain is close to 3dB. When compared to the perfect DFE, we have a performance gain of about 4dB.

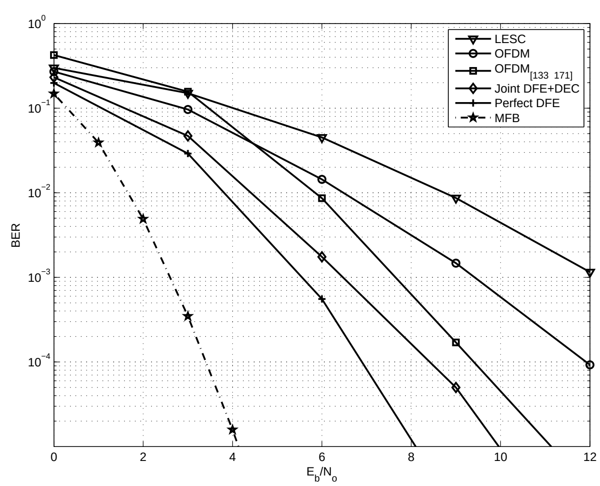

Fig. 12. Performance comparison, fixed channel with spectral null, convolutional code $[15\ 17]_{\mathrm{octal}}$. Best interleaver configuration 8 row/128 columns for the OFDM and 32 row/32 columns for SCCP.

IV. Conclusion and Perspectives

In this paper we compare the OFDM and the SCCP performance in different contexts. We show that the OFDM is highly sensitive to the interleaver configuration, which can be translated as a lack of robustness. However, if well configured it can exploit the channel diversity and provide an equal or superior performance when compared to the SCCP with linear equalization. We also show that the SCCP drawbacks for higher order modulation can be compensated by the use of DFE equalization. It is worth noting that this approach can provide huge performance gains in comparison to OFDM when the channel presents spectral nulls.

Due to such results and the performance gap of the OFDM with regard to the MFB, we are confident that a SCCP with turbo-equalization can lead to even superior performance. This case will be studied in future works.

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