Abstract—The design of random access channel (RACH) preambles for the Evolved Universal Terrestrial Radio Access (E-UTRA) third generation cellular system is considered, with the major goal to minimize the latency of RACH procedure. The E-UTRA RACH preambles are designed starting from a new construction of the sets of Zero Correlation Zone (ZCZ) sequences, derived from the Generalized Chirp-Like (GCL) sequences.

I. INTRODUCTION

The random access channel (RACH) is used on the uplink of a cellular system in order to notify the network that the mobile user equipment (UE) has data to transmit, as well as to allow the base station (Node B) to estimate the timing of the UE. The RACH channel typically consists of a preamble and message bursts [1]. Usually there are multiple orthogonal preambles to allow simultaneous access of multiple UEs to the network. In the frequency division duplex (FDD) mode of the Universal Terrestrial Radio Access (UTRA) third generation cellular system\footnote{UTRA-FDD is often called WCDMA (Wideband Code Division Multiple Access).}, the RACH message bursts are transmitted after the Node B has acknowledged the successful reception of RACH preamble.

In order to achieve low latency in accessing the network it is desirable that the UE transmits at high power at the first transmission and avoids power ramping. It implies that the RACH preambles should allow for small power back-off in the UE’s output power amplifier. The power back-off is usually estimated by the peak-to-average power ratio (PAPR) of the signal. Recently, another measure for power back-off was introduced, in the form of so-called cubic metric (CM) [2].

Another important factor to achieve low latency is the ability of the Node B to correctly detect several simultaneous random access preambles and to correctly estimate their timings. For that purpose, the RACH preambles should have the following properties: a) Good autocorrelation properties to allow for accurate timing estimation; b) Good cross-correlation properties to allow for accurate timing estimation of different simultaneous and asynchronous RACH preambles; and c) Zero cross-correlation for synchronous and simultaneous RACH preambles.

The RACH preambles in the WCDMA system satisfy to large extent all of the desired correlation properties [1]. However, some of these properties, such as mutual cross-correlation, i.e. the detection probability of a single preamble in presence of a number of other simultaneous preambles still could be better at low SIR values.

The range of delays on which both out-of-phase autocorrelation and cross-correlation of RACH preambles should have low values is primarily determined by the maximum round-trip delay from the Node B to the UE, because the UE synchronizes its RACH transmissions to the timing of the broadcasted pilot signals from the Node B. Depending on the length (duration) of RACH preambles, such low-correlation zone (of delays) of interest might be less than the length of the preambles. If that is the case, the sets of RACH preambles can be designed to have improved correlation properties for the delays of interest.

Thus in this paper we deal with the design of the sets of RACH preambles for the Evolved UTRA (E-UTRA) cellular system [3], with the major goal to reduce the latency of RACH procedure compared to the current UTRA system by improving preamble detection probability in presence of other simultaneous preambles.

In Section II the design of RACH preambles based on a new construction of the sets of Zero Correlation Zone (ZCZ) sequences, derived from the Generalized Chirp-Like (GCL) sequences, is described. In Section III the simulation results, both of the evaluation of the probability of preamble detection in presence of other received preambles, as well as of the transmit power back-off, are presented. Finally, Section IV concludes the paper.

II. ZCZ-GCL SEQUENCES

The starting point for the design of the new RACH preambles with improved correlation properties is the application of the sets of ZCZ sequences. Namely, a set of ZCZ sequences consists of equal-length sequences whose periodic out-of-phase autocorrelation is zero over the range of delays $|p| \leq D$, while the periodic cross-correlation between any two sequences from the set is zero in the same range of delays $|p| \leq D$, which is referred to as a ZCZ. For given length of sequences, $N$, and given number of sequences in the set, $M$, the upper bound of the length $D$ of the ZCZ is given by [4]

\[
D \leq N/M - 1. \tag{1}
\]
The E-UTRA RACH preambles should allow for efficient implementations of the corresponding bank of matched filters, of complexity at least similar as for the WCDMA RACH preambles. Hence, the set of ZCZ preambles having the same basic structure as the set of WCDMA preambles, is defined by a new construction of the sets of ZCZ sequences, derived from the GCL sequences [5].

A GCL sequence \( \{c(k)\} \) is defined as
\[
e(k) = a(k)b(k \bmod m), \quad k = 0, 1, \ldots, N - 1,
\]
where \( N = tm \), \( t \) and \( m \) are positive integers, \( \{b(k)\} \) is a "modulation" sequence of \( m \) arbitrary complex numbers of unit magnitude, while \( \{a(k)\} \) is a special "carrier" sequence, which has to be a Zadoff-Chu sequence defined as
\[
a(k) = W_N^{k^2/k + N \bmod 2}/2 + qk, \quad k = 0, 1, \ldots, N - 1,
\]
where \( W_N = e^{-2\pi r/N} \), \( r \) is relatively prime to \( N \), and \( q \) is any integer. Note that the length \( N \) in the original definition of GCL sequences [5] is defined as \( N = sm^2 \) where \( s \) is a positive integer.

If the two GCL sequences \( \{c_x(k)\} \) and \( \{c_y(k)\} \) are defined by using the same Zadoff-Chu sequence \( \{a(k)\} \) but different, arbitrary modulation sequences \( \{b_x(k)\} \) and \( \{b_y(k)\} \), it can be shown (similar as in [6], see the Appendix) that the periodic cross-correlation \( \theta_{xy}(p) \), defined as
\[
\theta_{xy}(p) = \theta'_{xy}(-p) = \sum_{k=0}^{N-1} c_x(k)c_y^*(k + p),
\]
where \( p \) is the delay and """" denotes complex conjugate, is zero for all time shifts \( p \neq lt \), \( l = 0, 1, \ldots, m - 1 \), i.e.
\[
\theta_{xy}(p) = 0, \text{ for } 0 < |p| < t, \quad t < |p| < 2t, \ldots, (m-1)t < |p| < tm.
\]
Thus, if the above two modulation sequences are orthogonal, the resulting GCL sequences will be not just orthogonal, but also will have a ZCZ of length \( D = t - 1 \), i.e. the periodic cross-correlation between any two sequences from the set will be zero for all the delays between \(-t\) and \(+t\). Based on this property, the set of \( m \) RACH preambles is defined by the following new construction of ZCZ sequences:

**New Construction of ZCZ sequences:** The set of ZCZ-GCL sequences is obtained by modulating a common Zadoff-Chu sequence \( \{a(k)\} \) of length \( N = tm \) with \( m \) different orthogonal modulation sequences \( \{b_i(k)\} \), \( i = 0, 1, 2, \ldots, m - 1 \), \( k = 0, 1, 2, \ldots, m - 1 \). The length of ZCZ has the maximum possible value \( D = t - 1 \) for given sequence length \( N = tm \) and the number of sequences in the set \( M = m \), i.e. \( D \) satisfies the upper bound (1).

Although the matched filters for RACH preambles actually calculate the aperiodic auto/cross-correlations, it is expected that the ideal periodic cross-correlation properties will be to large extent preserved. The reason is that for delays between \(-t\) and \(+t\), typically much smaller than the length of the sequences, the summations in the formulas for the aperiodic and periodic cross-correlation values only differ in a small number of terms. This expectation is confirmed by numerical evaluations, as it will be shown later.

### A. Some Interesting Special Cases of ZCZ-GCL Sequences

The most obvious choices for the selection of orthogonal modulation sequences would be either the sets of Hadamard sequences or Discrete Fourier Transform (DFT) sequences. The set of DFT sequences is defined as
\[
b_i(k) = W_m^{ik}, \quad i, k = 0, 1, \ldots, m - 1.
\]
The set of Hadamard sequences is defined as the rows in an \( m \times m \) Hadamard matrix, which is defined as follows: A Hadamard matrix \( H_m \) of order \( m \) consists of only 1s and -1s and has the property \( H_mH_m^T = mI \) where \( I \) is the identity matrix and \( T \) denotes transpose. For \( m = 2^n \), \( n \) is a positive integer, Hadamard sequences can be defined as
\[
b_i(k) = (-1)^{\sum_{l=1}^n i_l k_l}, \quad i, k = 0, 1, \ldots, m - 1,
\]
where \( i_l, k_l \) are the bits of the \( m \)-bits long binary representations of integers \( i \) and \( k \).

### B. Design of Numerical Parameters of RACH Preambles

The question is now how to select the actual numbers \( m \) and \( N \) to fit into the requirements of E-UTRA. The access slot, i.e. the time-frequency resource allocated for RACH, has a duration \( T_A \) and can be confined to a sub-band of the total available spectrum. In order to distinguish which cell a transmitted RACH preamble is intended for, the access slots in adjacent cells should as much as possible be separated in time and frequency.

The duration of the preamble is denoted by \( T_S \) and is given as the quotient of the sequence length \( N \) and the bandwidth \( B \) of the RACH preamble: \( T_S = N/B \). The maximum round-trip time, \( \tau_d = 2R/c \), where \( R \) is the cell range (radius) and \( c \) is the speed of light. The performance targets for E-UTRA are required to be met for cell ranges up to 5 km and to be met with slight degradation for cells ranges up to 30 km. The specifications should not preclude cell ranges up to 100 km [7].

The maximum delay spread, \( \tau_s \), depends on the environment. \( T_S \) should be shorter than \( T_A \) to avoid that the received signal extends after the access slot. Let \( \tau_m \) be the sum of the maximum round-trip time and the maximum delay spread: \( \tau_m = \tau_d + \tau_s \). Then \( T_S < T_A - \tau_m \), so it follows
\[
N < (T_A - \tau_m)B.
\]

The duration of the ZCZ is equal to \( D/B \), and should be such that \( D/B > \tau_m \). Since \( D = t - 1 \) and \( N = tm, D = N/m - 1 \), so it follows
\[
N/m - 1 > \tau_m B.
\]
Replacing \( N \) in (9) with the right-hand side of (8), it follows that
\[
m < (T_A - \tau_m)/(\tau_m + 1/B).
\]
The bandwidth $B$ has to be larger than the inverse of the required accuracy of the time-of-arrival estimation, which is much smaller than the shortest duration of the cyclic prefix, 3.69 $\mu$s$^2$. $\tau_m$ is typically of the order of several microseconds and hence is much greater than $1/B$. It can be seen from (10) that the number of sequences in the set is then almost independent of $B$. Therefore, in order to use the spectrum efficiently, $B$ should be as small as possible. The total number of signature sequences in a cell can then be increased by allocating several sub-bands for RACH. We select $B$ to be 1.024 MHz, which gives a time-of-arrival estimation resolution of close to 1 $\mu$s. It also gives a margin to the minimum uplink nominal bandwidth, which is 1.25 MHz. In the numerical design, $T_A$ is first selected to give a large enough number of signatures as given by (10). Then, $m$ and $t$ should be selected such that (8) and (9) are satisfied. In order to fulfill the different requirements for various cell ranges we propose four different parameter sets, as shown in Table I.

### III. SIMULATION RESULTS

Three kinds of evaluations are performed. First, the aperiodic cross-correlation properties of the GCL sequences are investigated, to confirm that the properties are similar to the ideal periodic cross-correlation functions. Then, the detection performance, in particular in the presence of interfering preambles, of the GCL sequences is compared to the performance of truncated WCDMA RACH preambles. Finally, the impact of the proposed sequences on the transmit power back-off is evaluated. The parameters of the GCL sequences in (3) are $N = 400$ ($t = 100$ and $m = 4$), $q = 0$, and $r = 1$ throughout the section.

#### A. Aperiodic Cross-Correlation Properties

The absolute values of aperiodic cross-correlation function, defined as

$$R_{xy}(p) = R_{yx}^*(-p) = \sum_{k=0}^{N-1-p} c_x(k)c_y^*(k+p), \quad p \geq 0,$$

are shown in Fig. 1, for various pairs of ZCZ-GCL-DFT sequences of length 400. The set of GCL-Hadamard sequences has similar autocorrelation and cross-correlation functions. The peaks of the cross-correlation functions are located near multiples of $t = 100$. The cross-correlation values do not exceed 20 for delays less than 96.

#### B. Detection Probability

The detection performances of the proposed preambles have been evaluated by link-level simulations. The truncated WCDMA RACH preamble has been used as a reference with modulating Hadamard sequences that are 4 bits long, instead of 16 bit long sequences, to keep the same number of signature sequences as for the proposed sequences.

The number of receive antennas is two and correlations from the two antennas at the same delay are combined non-coherently, i.e. the absolute values of the squared matched filter outputs from the two antennas at the same delay are added. The propagation channels simulated are AWGN and Typical Urban (TU) at 3 km/h. The TU channel is widely accepted in the standardization bodies as a realistic channel model in the urban environment. It includes a large number of multipath components, which implies a difficult condition for the receiver.

The detector correlates the received signal with all possible preambles in the search window. A threshold is set to give a false alarm probability of 0.0001 at a single delay. Missed detection is declared if the transmitted preamble is not detected within the true range of delays of received signal replicas. The number of concurrently transmitted preambles ranges form 1 to 4. All preambles are transmitted with independent random delays within the search window, corresponding to randomly distributed mobiles in the cell. If two or more different preambles are transmitted using the same time-frequency resources, the signal-to-noise ratio (SNR) of the observed preamble (whose probability of missed detection is evaluated) is fixed (SNR = 15 dB for the AWGN channel) and the interfering preambles are transmitted with various power offsets to the observed preamble. All the interfering preambles have the same power offset to the observed preamble.

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2The uplink transmission scheme in E-UTRA is based on single-carrier transmissions of blocks of symbols, each block with a cyclic prefix. A number of blocks are transmitted within a 0.5 ms sub-frame and transmissions from different users are time and frequency multiplexed.

### TABLE I

**Numerical parameters of RACH preambles**

<table>
<thead>
<tr>
<th>$R$ (km)</th>
<th>$T_A$ (ms)</th>
<th>$\tau_2$ ($\mu$s)</th>
<th>$\tau_3$ ($\mu$s)</th>
<th>$m$</th>
<th>$N$</th>
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<td>10</td>
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<td>3</td>
<td>2367</td>
</tr>
</tbody>
</table>

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Fig. 1. Auto/cross-correlation functions of GCL-DFT sequences.
The probabilities of missed detection of the observed preamble as functions of SNR are shown in Fig. 2, for the case of a single transmitted preamble, both on AWGN and TU channels. The probabilities of missed detection of the observed preamble as functions of signal-to-interference ratio (SIR), in presence of interferers, are shown in Figs. 3 and 4 for AWGN and TU channels respectively. The SIR is defined as the ratio of the power of the observed preamble to the power of any of the interfering preambles.

From Fig. 2 it is clear that there is no difference in the probability of missed detection in the absence of interfering sequences between WCDMA RACH preambles and the proposed preambles. However, the results shown in Figs. 3 and 4 clearly demonstrate significantly improved detection performance of the proposed preambles in the presence of one or several interfering sequences. The lower probability of missed detection for the proposed set of preambles is due to the good cross-correlation properties of the zero-correlation zone sequences.

C. Transmit Power Back-Off

Two measures related to the transmit power back-off are the peak-to-average power ratio (PAPR) and the cubic metric (CM). Let \( z(t) \) be the normalized baseband signal, such that its expectation value \( E(|z(t)|^2) = 1 \). The PAPR at the 99.9th percentile is defined as the value \( x \) such that the probability that \( 10 \log_{10}(|z(t)|^2) < x \) equals 0.999. The CM is defined as [2]:

\[
CM = \frac{20 \times \log_{10}((v_{\text{norm}}^3)_{\text{rms}}) - 20 \times \log_{10}((v_{\text{norm ref}}^3)_{\text{rms}}) / 1.85,}
\]

where \( v_{\text{norm}} \) is the normalized voltage waveform of the input signal, and \( v_{\text{norm ref}} \) is the normalized voltage waveform of a WCDMA reference signal (12.2 kbps AMR Speech). The CM is devised to model the impact of non-linearities in the power amplifier on the adjacent channel leakage ratio. A small value of the CM signifies a small required power amplifier back-off. Table II lists the PAPR values at the 99.9th percentile for a reference WCDMA RACH preamble truncated to 400 samples with 4-bit Hadamard modulating sequences, and for the GCL sequences with DFT and Hadamard modulating sequences. In all cases the maximum PAPR value is given over all modulating sequences. The range of values given for the WCDMA RACH preamble is over all scrambling codes. For the GCL sequences, the Zadoff-Chu sequence with \( r = 1 \) has been used. Two different pulse-shaping filters are applied, a simple sinc filter and a root-raised cosine filter with roll-off factor 0.15. Table III lists the corresponding CM values.

From the Tables II and III it is clear that the DFT-modulated sequence has both lower PAPR and lower cubic metric than the Hadamard-modulated GCL sequence. Furthermore, applying a root-raised cosine filter improves neither PAPR nor the cubic metric of the DFT-modulated sequence. Finally, the PAPR
of the DFT-modulated GCL sequence is as good as for the UTRA sequences with a root-raised cosine filter, while the cubic metric is somewhat better than for the UTRA RACH preambles.

IV. CONCLUSION

The proposed RACH preambles based on the sets of ZCZ-GCL sequences allow for detection of several simultaneous preambles with a high detection probability even when the powers of the received signals are very different. The probability of detection of each such preamble is independent of the number of interfering signals, quite opposite to the RACH preambles in UTRA system. This property, along with a reduced power amplifier back-off in the case of DFT modulated GCL sequences, can help to reduce the latency in the future evolved UTRA system.

The E-UTRA RACH throughput can be further increased by combination of ZCZ-GCL preambles with random selection of RACH transmission sub-bands and/or by using multiple "carrier" (Zadoff-Chu) sequences.

APPENDIX

PERIODIC CORRELATION OF GCL SEQUENCES OBTAINED FROM A COMMON CARRIER SEQUENCE

The periodic cross-correlation (4) of two GCL sequences $c_x(k) = a(k)b_x(k \mod m)$ and $c_y(k) = a(k)b_y(k \mod m)$, is given by

$$\theta_{xy}(p) = C(p) \sum_{k=0}^{N-1} W^{-pk} b_x(k \mod m) b_y^*(k + p \mod m),$$

where $C(p) = W^{-p(N+2q)/2}$. By introducing a change of variables

$$k = im + j, \ i = 0, 1, \ldots, t - 1, \ j = 0, 1, \ldots, m - 1,$$

into (11), it follows that

$$\theta_{xy}(p) = C(p) \sum_{i=0}^{t-1} W^{-pi} \cdot \sum_{j=0}^{m-1} W^{-pj} b_x(j \mod m) b_y^*(j + p \mod m).$$

If $p \neq lt, l = 0, 1, \ldots, m - 1$, the first summation in (13) is zero, hence $\theta_{xy}(p) = 0$ for arbitrary modulation sequences $\{b_x(j)\}$ and $\{b_y(j)\}$. If the two modulation sequences are the same, it immediately follows that the periodic autocorrelation function is zero within the zero-correlation zone. In this way we have completed the proof of (5).

REFERENCES