Channel Estimation for Wavelet Packet based UWB Transmissions

Hiroki Harada, Marco Hernandez and Ryuji Kohno
Division of Physics, Electrical and Computer Engineering, Yokohama National University
79–5 Tokiwadai, Hodogaya, Yokohama, 240–8501, Japan.
Tel.: +81-45-338-1176 Fax: +81-45-338-1176
E-mail: {hiroki,Marco.Hernandez}@iee.org, kohno@ynu.ac.jp

Abstract—Maximum likelihood (ML) data aided channel estimation for wavelet packets based ultra wideband (WP-UWB) transmissions is investigated. Training pilot symbols are employed for ML channel estimation based on a least square algorithm derived from the optimum ML approach. An optimum training sequences criterion is derived. Based on this, nearly optimum perfect root of unity sequences (PRUS) are tested and compared to Gold sequences and random sequences. In this estimation strategy, a generalized inverse of the training pilot symbols matrix can be pre-computed and stored in memory. Furthermore, PRUS allows to save the inverse matrix operation and so reducing the complexity of the ML estimator significantly. This technique is especially attractive for low data rate transmissions as in sensor networks because of its low complex implementation. Performance analysis results in terms of the MSE are provided. Trade-offs between performance and complexity are described as well. The investigated channel estimation algorithms for WP-UWB channels offer flexibility, low complexity and robustness, making channel estimation fairly simple and effective.

I. INTRODUCTION

Channel estimation for wireless systems is a challenging task. Moreover, channel estimation for UWB systems seems even more challenging because of the new system’s requirements to operate. Originally conceived for military applications, UWB systems has attracted much attention due to the potential transmission of either high data rates or low data rates at relatively low cost and moderate complexity, in a spectrum already occupied by existing radio services. Indeed, by transmitting with a low power spectral density, UWB systems are released of frequency coordination, allowing transmission in a spectrum already in use by other systems. This opens an umbrella of possible application, from consumer electronics, wireless local area networks (fixed or ad-hoc) to radar and massive sensor networks.

Despite the fact that the first generation of UWB systems will employ more likely a carrier (DS-UWB) or set of carriers (MB-UWB), this correspondence deals with wavelet packet based UWB transmissions, denoted as WP-UWB. This novel UWB system is under patent process, so we delineate the signal model only. The detection strategy in WP-UWB is based on the coherent treatment of sufficient statistics obtained with a digital front-end. The research reported in [1] has shown that the creation of arbitrary pulse waveforms in the order of nanosecs and their detection with a digital front-end are doable. The superior performance of coherent detection motivates this analysis, although it requires channel state information (CSI) at the receiver.

In general, channel estimation for wireless communications can be classified into two groups: so-called structured and unstructured. In any case, the radio channel is modeled as a tapped delay line, where the main difference is that in structured approaches path delays and path coefficients are jointly estimated exploiting some a-priori knowledge of the structure of the channel response, like pulse shaping, number of paths, typical multipath intensity profile, etc. Hence, path delays have arbitrary values in the tapped delay model.

In contrast, unstructured approaches do not exploit any a-priori knowledge of the channel response, and the channel tap delays are equally spaced. So, once the first path delay is known (timing synchronization), the rest of the delays are known.

The objective of this paper is to introduce an overall channel estimation algorithm (unstructured approach) for WP-UWB systems in a multiple access scenario. In such a scenario, active users transmit at the same time, sharing the same bandwidth, like in a spread spectrum system. Thus, users’ signals are distinguished at the receiver by a particular set of wavelet packets and scrambling sequences.

Initial timing acquisition is assumed, so that the system is considered block synchronous, but chip asynchronous. Moreover, the transmission of a block of symbols lasts less than the coherence time, so that the channel can be considered time invariant over the block duration, also known as block fading channel. On the other hand, tracking the channel from block to block might be complex, due to the dynamic user allocation, and burst transmission. So, we assume to estimate the channel of every user in a block by block basis without tracking across different blocks.

The rest of the paper is organized as follows: Section II describes the signal model considered for WP-UWB systems. Section III illustrates the maximum likelihood estimator. Section IV delineates the training sequences design and the approximate estimator proposal. Section V sketches the simulation performed. Finally, Section VI conclusions are drawn.

II. SIGNAL MODEL

For purposes of channel estimation, a WP-UWB signal carrying a training sequence is assumed to be transmitted during a preamble. Hence, the transmitted signal by the $u^{th}$ user is given by

$$s^u(t) = \sum_{n=0}^{N_b-1} a_n^u w^u(t - nT)$$

$$w^u(t) = \sum_{i=0}^{N_c-1} W_{u,i}(t - iT_c)$$

where $a_n^u$ is the $n^{th}$ training or pilot symbol transmitted by the $u^{th}$ user, $T$ is the symbol time, $T_c$ is the wavelet packet duration or chip time, $N_b$ is the size of the preamble block, $W_{u,i}(t)$ is the $i^{th}$ wavelet packet transmitted by the $u^{th}$ user, and $N_c$ is the system’s processing gain.

As mentioned before, the WP-UWB system is under a patent process, so the criteria to select $W_{u,i}(t)$ will be published in a next paper. Nevertheless, Equation 1a is enough to derive and to show the proposed channel estimation techniques.
A. UWB Channel Model

UWB channel measurement campaigns have shown a clustering effect, where a modified Saleh-Valenzuela model is reported to fit well those measurements [3]. Based on those measurements campaigns, a statistical channel model has been proposed by the IEEE 802.15 Working Group [3]. Assuming the pathloss is compensated in the link budget, a conventional wide sense stationary uncorrelated scattering (WSSUS) discrete channel model is supposed. This discrete channel model is the time variant channel impulse response, also known as the input delay-spread function or the first Bello function, assuming a system with finite delay resolution. Such a finite delay resolution is proportional to the inverse of the transmission channel bandwidth, where the received signal can be seen as a linear convolution of a transmitted signal with a band limited input-delay spread function given by [3]

\[ h(t) = \sum_{l=1}^{L} \sum_{k=1}^{K} \alpha_{k,l} e^{j\phi_{k,l}} \delta(t - T_l - \tau_{k,l}) \]  

where \( \alpha_{k,l} \) follows a Nakagami distribution, \( \phi_{k,l} \) follows uniform distribution, \( T_l \) is the time delay of the \( l \)-th cluster, \( \tau_{k,l} \) is the time delay of the \( k \)-th multipath component relative to the \( l \)-th cluster arrival time \( T_l \). Details in the generation of such power delay profiles for different scenarios are provided in [3]. Notice that the block fading channel model has been assumed as the channel is considered constant during the preamble, but varying between preambles.

B. Received Signal

The transmitted signal \( s^u(t) \) undergoes a fading channel, whose impulse response is given in Equation 2. Thus the received signal for the \( u \)-th user is given by

\[ r^u(t) = s^u(t) * h^u(t) + \nu(t) \]  

Or

\[ r^u(t) = \sum_{n} a^n_u g^n(t - nT) + \nu(t) \]  

where \( \nu(t) \sim \mathcal{N}(0, \sigma^2) \) is AWGN and

\[ g^n(t) = \sum_{i=0}^{N_c - 1} W_{u,i}(t - iT) * h^u(t) \]  

The receiver front-end is a bank of an ideal low-pass filters with bandwidth \( W/2 \) and amplitude \( 1/\sqrt{W} \), whose output is sampled at rate \( W = QT \), for \( Q \geq 2 \) without any explicit time reference. Thus, \( T_c = QT_c \), and \( T_c \) is the sampling time. Moreover, let \( D = N_c Q \) be the number of samples contained in a symbol. The discrete time representation of \( g^n(t) \) has a finite support that depends on the channel’s delay spread and processing gain \( N_c \). Thus, after a suitable truncation, discrete time samples of \( g^n(t) \), denoted as \( g^n[iT_c] \), are given by the polyphase representation of the discrete-time low pass filtered overall channel impulse response given by

\[ g^n[i] = \frac{1}{\sqrt{W}} g^n(i/W) \]  

where \( i = 0, 1, ..., N_g - 1 \), and \( N_g \) is the support of \( g^n[i] \) (maximum delay spread has been assumed). The sampling time \( T_c \) is not shown to simplify the notation. Let \( g^n \) be the vector containing the samples of \( g^n[i] \) such that

\[ g^n = [g^n[0], g^n[1], ..., g^n[N_g - 1]]^T \]  

Let \( L = \lceil \frac{N_c}{T} \rceil + 1 \) be the number of pilot symbols that are contained in \( g^n \). Thus, the discrete time representation of the received signal can be expressed as

\[ r[kD + i] = \sum_{u=0}^{U-1} \sum_{m=0}^{L-1} a^{u}_{k-m} g^u[mD + i] + v[kD + i] \]  

where \( i = 0, 1, ..., D - 1 \), for every \( k = 0, 1, ..., N_c - 1 \), and \( v[i] \) represents the discrete-time low pass filtered noise samples, which are i.i.d random variables with distribution \( \mathcal{N}(0, \sigma^2) \). Notice the channel estimator collects \( QN_c N_g \) samples, and Equation 8 allows to study the fractionally-spaced or chip-spaced cases.

III. Maximum Likelihood Estimation

A. Training Pilot Signal Structure

The training sequence consists of guard symbols and pilot symbols. The guard symbols are used to cover the time for ISI and timing synchronization errors and they are located before and after the pilot symbols, or equivalently, two times the number of guard symbols before the pilot symbols.

Let \( L \) be the number of guard symbols and \( P \) be the number of pilot symbols such that \( L + P = N_c \). Thus, the training sequence is represented as

\[ m = [m_0, m_1, ..., m_{L-1}, m_L, m_{L+1}, ..., m_{L+P-1}]^T \]  

where \( m_i \) can be real or complex symbols.

Without losing generality, let the estimated pilot symbols \( \hat{g}_v^u \) be replaced by \( m_v^u \). Thus, the received signal for the \( v \)-th user, corresponding to the pilot symbols is given by

\[ r^v[iD + i] = \sum_{j=0}^{L-1} m_{v,j}^u g^u[jD + i] + v[iD + i] \]  

where \( i = 0, 1, ..., D - 1 \) for every \( k = L, L + 1, ..., L + P - 1 \).

Collecting those samples in matrix representation, Equation 10 can be expressed in terms of \( i = 0, 1, ..., D - 1 \) as

\[ r_v = M^u g_v^u + v_i \]  

where

\[ r_v = [r^v[0], r^v[1], ..., r^v[(L + 1)D + i], ..., r^v[(L + P - 1)D + i]]^T \]  

\[ M^u = \begin{bmatrix} m_0 \ m_{-1} & \cdots & m_{-1} \\ m_{L} & m_{L-1} & \cdots & \cdots & m_{2} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ m_{L+P-1} & m_{L+P-2} & \cdots & \cdots & m_{P} \end{bmatrix} \]  

\[ g_v^u = [g^u[0], g^u[D + i], ..., g^u[(L - 1)D + i]]^T \]  

Notice that \( r_v \in \mathbb{C}^{P \times 1} \), \( M^u \in \mathbb{C}^{P \times L} \) or \( \mathbb{R}^{P \times L} \), \( g_v^u \in \mathbb{C}^{L \times 1} \) and \( v_i \in \mathbb{C}^{P \times 1} \)

Then, the received signal corresponding to the superposition of \( U \) users is given by

\[ r_i = \sum_{u=0}^{U-1} r_v^u = M g_v^u + v_i \]  

where

\[ r_i = [r_i[0], r_i[1], ..., r_i[D - 1]]^T \]
Additionally the term \( \mathbf{K} \) a Gaussian noise vector with zero mean and covariance matrix \( \mathbf{K} \). Hence, given a set of observations \( r_i \), their joint conditional probability density function is given by [4]

\[
p(r_i | g_i) = \frac{1}{\sqrt{(2\pi)^D \det(\mathbf{K})}} 
\exp \left[ -\frac{1}{2} (r_i - \mathbf{M} g_i)^H \mathbf{K}^{-1} (r_i - \mathbf{M} g_i) \right]
\]

Thus, the ML estimation of \( g_i \) is given by

\[
\hat{g}_i = \underset{g_i}{\arg \max} p(r_i | g_i)
\]

Notice that \( v_i \) has i.i.d. components, so the ML estimation is equivalent to the least square estimation [4] given by

\[
\hat{g}_i = \arg \min_{g_i} \| r_i - \mathbf{M} g_i \|^2
\]

whose solution is given by the Gauss-Markov theorem

\[
\hat{g}_i = (\mathbf{M}^H \mathbf{K} \mathbf{M})^{-1} \mathbf{M}^H \mathbf{K} r_i
\]

This estimator is a linear minimum variance unbiased estimator (LMVUE) with covariance \( \mathbf{K}_g = [\mathbf{M}^H \mathbf{K}^{-1} \mathbf{M}]^{-1} \) [4].

Because of the noise values \( v_i \) in Equation 13a are AWGN, the LMVUE is simplified as

\[
\hat{g}_i = (\mathbf{M}^H \mathbf{M})^{-1} \mathbf{M}^H r_i
\]

with covariance \( \mathbf{K}_g = [\mathbf{M}^H \mathbf{M}]^{-1} \).

Notice that \( i = 0,1,...,D-1 \) in Equation 18 in order to get \( \hat{g} = [(\hat{g}_0)^T, (\hat{g}_1)^T, ..., (\hat{g}_{D-1})^T]^T \) where \( \mathbf{g}^* \) is similar to Equation 7. Additionally the term \( (\mathbf{M}^H \mathbf{M})^{-1} \mathbf{M}^H \) of the LMVUE can be computed in advanced and stored in memory, making the channel estimation fairly fast at the receiver.

Nevertheless, two conditions have to be satisfied for the existence of the generalized inverse. Namely,

1) The number of rows of \( \mathbf{M} \) have to be higher than its number of columns

\[
P > LU
\]

2) \( \mathbf{M} \) is full column rank.

IV. TRAINING SEQUENCE DESIGN

The estimation error vector is given by

\[
e_i = \hat{g}_i - g_i = (\mathbf{M}^H \mathbf{M})^{-1} \mathbf{M}^H v_i \sim \mathcal{N}(0, \mathbf{P})
\]

where \( \mathbf{P} = \mathbf{E}[e_i e_i^H] = \sigma^2 (\mathbf{M}^H \mathbf{M})^{-1} \) is the error covariance matrix.

Thus, the normalized MSE of the unstructured channel estimation is given by

\[
\text{MSE} = \frac{1}{ULD} \sum_{i=0}^{D-1} \mathbf{E}[|e_i|^2] = \frac{\sigma^2}{UL} \text{Trace}[ (\mathbf{M}^H \mathbf{M})^{-1} ]
\]

Let \( \lambda_i \) be the eigenvalues of \( \mathbf{M}^H \mathbf{M} \). Thus, optimal training sequences minimize the MSE, or equivalently, minimize the trace of the error covariance matrix subject to the total training energy constrain \( \xi \). Namely,

- Minimize: \( \text{Trace} [(\mathbf{M}^H \mathbf{M})^{-1}] = \sum_j \frac{1}{\lambda_j^2} \)
- Subject to: \( \text{Trace} [\mathbf{M}^H \mathbf{M}] = \sum_j \lambda_j^2 = \xi \)

By taking the eigenvalue decomposition of \( \mathbf{M} \) as \( \mathbf{M} = \Phi \Lambda \mathbf{Y}^T \) such that

\[
(\mathbf{M}^H \mathbf{M})^{-1} = \text{Trace}[ \mathbf{T} \mathbf{A}^{-2} \mathbf{Y}^T ] = \sum_j \frac{1}{\lambda_j^2}
\]

On the other hand, due to the total energy constrain \( \xi = \sum_j \lambda_j^2 \), the trace of Equation 22 is minimized when all eigenvalues are equal.

Namely, \( \lambda_j = \frac{1}{\sqrt{UL}} \), such that

\[
\mathbf{M} = \sqrt{\frac{\xi}{UL}} \Phi \mathbf{Y}^T
\]

The construction of optimal training sequences has been investigated extensively, see for example [5] [6]. Such constructions are based on an exhaustive search of training sequences that minimize the off-diagonal elements of \( \mathbf{M}^H \mathbf{M} \) for small \( N_o \), and heuristic constructions for large \( N_o \).

We propose to employ modified perfect root of unity sequences (PRUS), Gold code sequences and random sequences generate from a Gaussian distribution.

A. Modified PRUS

Elements of perfect root of unity sequences belong to an \( N \)-root of unity alphabet: \( \mathcal{A}_N = \{ e^{2\pi i/N} ; i = 0,1,...,N-1 \} \) where \( N \in \mathbb{Z} [7] \). Let \( x_k \) be the PRUS elements, which are complex with equal magnitude. Thus, their phase difference is only relevant. Namely,

\[
\frac{x_{k+1}}{x_k} = d_k
\]

where \( k = 0,1,...,P-2 \). As mentioned above the training sequences are said to be optimal if the autocorrelation satisfies Equation 24. A solution is given by

\[
d_k = A e^{2\pi k/\gamma}
\]

where \( A \in \mathbb{C} \) such that \( |A| = 1, \gamma \in \mathbb{Z} \) such that \( \gamma \) and \( P \) are relative primes (no common factors except 1).

Thus, given \( x_0 \) as initial condition, the rest of PRUS elements are computed from Equation 25 and Equation 26. Notice that \( A, \gamma \) can be optimized to produce PRUS with sharp autocorrelation function.

B. Gold code

Due to the good correlation properties of Gold codes, training sequences based on Gold codes of length \( P \) are studied.

C. Random sequences

Let \( x_k \) be the elements of a random sequence obtained from a Gaussian distribution as

\[
x_k = \text{Sgn}\{b_k\}
\]
where \( b_k \sim \mathcal{N}(0, \sigma^2) \), and \( \text{Sgn}(z) = \begin{cases} 1 & z \geq 0 \\ -1 & z < 0 \end{cases} \)

Remarkably, these simple random sequences have sharp autocorrelation values.

**D. Training sequences based on a single code**

In the unstructured ML channel estimation proposal, UWB terminals must have knowledge of all users’ training sequences in the system. In order to overcome this problem and relaxing the complexity at the receiver, training sequences derived from a single periodic basic code is employed.

Let \( x = [x_0, x_1, \ldots, x_{P-1}]^T \) be the basic periodic training sequence obtained from a PRUS, Gold code or random sequence of period \( P \). Thus, \( U \) different training sequences are computed as

\[
x_k^n = x[k+(U-u)L] \mod P
\]

where \( k = 0, 1, \ldots, P-1 \), \( u = 0, 1, \ldots, U-1 \). Notice that \( P > UL \) to satisfy the restriction of Equation 19.

Finally, the preamble is formed by taking the pilot symbols from the training sequence in Equation 28, and repeating the last \( L \) elements as the guard symbols to improve the cyclic correlation, such that

\[
m_k^n = \begin{cases} x_{P-L+k}^n & k = 0, 1, \ldots, L-1 \\ x_{-L}^n & k = L, L+1, \ldots, L+P-1 \end{cases}
\]

**E. Approximated ML channel estimation**

By constructing \( M^n \) and \( M \) from Equation 29, the columns of \( M \) are distinct cyclic shifts of the basic periodic training sequence \( x \) with sharp autocorrelation. Thus, \( M^{UL} \approx PI \), and the LMVUE in Equation 18 can be approximated as

\[
\hat{g}_i \approx \frac{1}{p} M^{UL} r_i
\]

**V. SIMULATIONS**

**A. Simulation parameters**

The simulation parameters are as follows:

- The processing gain is \( N_c = 8 \), so that 8 wavelet packets generated from a Daubechies mother wavelet are employed. Those are computed such that the transmitted signal fulfills the FCC spectral mask.
- The WP duration is \( T_c = 5 \) nsecs. So that the symbol time \( T = N_c T_c = 40 \) nsecs.
- The UWB channel CM1 (residential LOS) with typical delay spread of \( T_M = 78 \) nsecs is employed.
- The training sequence length \( P = 31,63 \).
- The number of users \( U = 2,4,6,8 \).

**B. Results**

Figure 1 illustrates the MSE performance of the ML channel estimator in a scenario with \( U = 2, 4, 6, 8 \) users. The UWB channel CM1 is considered and training sequences of length \( P = 31 \). Figure 2 gives a closer look of Figure 1, where the Gold code and Gaussian random sequences have a similar performance, but the modified PRUS observes better performance. Figure 3 shows the improvement of the MSE performance in the previous scenario for training sequences of length \( P = 63 \). Finally, Figure 4 illustrates that the modified PRUS provides the best performance as the MSE remains practically invariant to the number of users (simulations with larger number of users will be provided). As the PRUS sequence are of length \( P = 31 \) for comparison with Gold codes, those can be shorten and so become good candidates for training sequences for the proposed joint ML channel estimator in multiruser UWB scenarios for either high data rate or low data rate applications.

**C. Test bed experiments**

In order to test the proposed scheme in more practical settings, experiments were conducted using a UWB test bed developed by the UWB Group at the National Institute of Information and Communications (NICT) in Yokosuka, Japan. The signal waveform introduced in the UWB signal generator is WP-UWB based on a Daubechies mother wavelet that fulfill the UWB test bed frequency range of 3 GHz to 5.5 GHz. Such a signal is designed for 1 user employing PRUS of length 7 as training sequences. We present preliminary results of these experiments: Figure 6 illustrates the estimated \( \hat{g} \) from the received signal. This will be used to compute BER performances in a next phase of research.

The intention is to show some experimental evidence that the proposal works in realistic UWB environments. More experimental results will be published later.

**VI. CONCLUSIONS**

The following conclusions can be described from this study:

- Modified PRUS training sequences show the best performance in terms of MSE. Remarkably, the Gaussian random sequences
provide good performance as well. Both approaches are easy to generate, and additionally their sequence length is unconstrained contrary to Gold codes whose sequence length is fixed.

- The joint ML channel estimator offers a good performance in UWB scenarios with multiple users. Moreover, the different estimator matrices in Equation 18 can be pre-computed and stored in memory at the receiver, making the channel estimation fairly simple and fast, which is suitable for high date rate systems. Furthermore, the approximation introduced in Equation 30 with PRUS training sequences is suited for low complex sensor networks applications.

- The proposed algorithms are independent of the pulse shape. Nevertheless, we focus on WP-UWB signals that fulfill the FCC spectral mask. Moreover, the proposed algorithms estimate the overall channel impulse response, where any signal distortion is embedded.

REFERENCES