Improved Throughput over Wirelines with Adaptive MC-DS-CDMA

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Abstract—In this paper, we present a finite-granularity loading algorithm for multicarrier direct sequence code division multiple access (MC-DS-CDMA). The resulting adaptive scheme represents an extension of classical digital multitone (DMT) with spreading in the time domain. The presented algorithm assigns subcarriers, spreading codes, bits and energy per code in order to maximize the data rate in a single user network, or to maximize the minimum data rate in a multiuser network. The optimization is realized at a target symbol error rate and for a given power spectral density (PSD). Optimal allocation is performed through analytical study, and simulation results of the new scheme are presented in single user digital subscriber line (DSL) and multiuser power line communication (PLC) contexts. Compared to DMT, it is shown that the time spreading component of MC-DS-CDMA system yields throughput improvement against a weak increase of complexity.

I. INTRODUCTION

D IGITAL multi-tone (DMT) modulation, also known as orthogonal frequency division multiplexing (OFDM), is a flexible and efficient solution to transmit information over wirelines [1]. With multicarrier modulation, the channel is divided into multiple subchannels that are essentially intersymbol interference-free, and independent additive white Gaussian noise channels. The main property of wireline channels is its static, or quasi-static impulse response. It can then be advantageously assumed that the transmitter has perfect channel state information, and can perform adaptive resource allocation. Under constraints, like power budget, power spectrum density (PSD), peak power, QoS, or throughput, the question is how to find power and bits allotment on each subchannel in order to maximize performance. The optimal solution is known as the waterfilling-based method with adaptive modulations. Many discrete algorithms for allocating subcarriers, power, and bits have been developed. The first proposed [2] gives an optimal solution using a greedy algorithm but leads to computational complexity and would be impractical when the number of bits to be transmitted per DMT symbol is large. Many suboptimal algorithms with less computational complexity have been developed (see, for example, [3] and references inside for an extensive overview), and multiuser extension has been carried out [4]. All these algorithms compute the bit loading subcarrier per subcarrier.

However, for long lines or deep frequency fadings, many available subcarriers cannot bear any bit with DMT system, and only few subcarriers are used to transmit data. Carrier merging [5], [6] has been proposed to reduce the bit loading quantification loss and allows to use more available subcarriers. Moreover, finite order modulations like quadrature amplitude modulations (QAM) and PSD constraint accentuate this quantification loss. To circumvent this problem, trellis coded modulation can be used with variable rates, leading to an important increase of complexity, contrary to carrier merging. In this paper we propose to use the multicarrier direct sequence code division multiple access (MC-DS-CDMA) technique, first introduced in wireless context [7], to perform symbol merging as bit loading quantification loss reduction. This merge is realized in the time domain own to the spreading sequences of the MC-DS-CDMA system, in contrast to carrier merging which realizes merging in the frequency domain. Thus, the bit loading algorithm has to take into account not only the subcarrier but also the spreading component of the system to perform bit, power and code allotment.

This paper is organized as follows. Section II presents the MC-DS-CDMA system model. Section III recalls the DMT system capacity in multiuser context. Section IV gives the mathematical solution of maximization MC-DS-CDMA throughput problem under PSD constraint, and section V gives the corresponding algorithm. The performance of MC-DS-CDMA system is given in section VI and compared to the performance obtained with the DMT system in ADSL (Asymmetric digital subscriber line) single user context, and PLC (Power line communication) multiple user context. Finally, section VII concludes the paper.

II. MC-DS-CDMA SYSTEM

The studied MC-DS-CDMA system results in multicarrier modulation applied to CDMA signal. The MC-DS-CDMA transmitter modulates the data substreams on subcarriers with a carrier spacing proportional to the inverse of the chip rate to guarantee the orthogonality between spectrums of the substreams after spreading [7].

The data stream is first converted onto parallel low rate substreams, before applying the spreading on each substream in the time domain, and modulating onto each subcarrier. Due to code division multiple access, several data of substreams are transmitted at the same time by a given subcarrier. Let \( N_s \) be the number of used subcarriers which is equal to the number of low rate substreams, \( N_{i}^{(s)} \) be the number of simultaneous transmitted symbols by subcarrier \( i \), and \( L_i^{(s)} \) the number of chips per code of subcarrier \( i \), i.e. the length of the code. Note
that, $N_c^{(i)}$ also corresponds to the number of used codes related to subcarrier $i$. To simplify the notation and the presentation of the system we suppose in the sequel that $\forall i$ $N_c^{(i)} = N_c$ and $E_c^{(i)} = E_c$. QAM symbols are transmitted by blocks of size $N_e \times N_c$, denoted $X(n) = \{x_{i,k}(n)\}_{0<i\leq N_e, \ 0<k\leq N_c}$. The same spreading code matrix $C = \{c_{i,k}\}_{0<i\leq N_e, \ 0<k\leq L_c}$ is applied to all subcarriers [8]. These spreading codes are orthogonal codes extracted from the Hadamard matrix of size $L_c \times L_c$. The $n$th spread symbol expressed before multicarrier modulation is

$$Y(n) = [Y_1(n) \ldots Y_{L_c}(n)] = X(n) C.$$  

The multicarrier modulation is applied to the vector $Y_k(n) = [y_{k,1}(n) \ldots y_{k,N_e}(n)]^T$, and the MC-DS-CDMA scheme needs $L_c$ DMT symbols to transmit one full MC-DS-CDMA symbol. To make the notation more compact and without loss of generality, the time variable $n$ is omitted in the following.

The multicarrier component of MC-DS-CDMA signal is assumed to be adapted to the channel, and the channel is constant over $L_c$ DMT symbols, i.e. over one MC-DS-CDMA symbol. In that case, the channel can be modeled by one single complex coefficient per subcarrier [9]. After multicarrier demodulation with guard interval removal and fast Fourier transform, channel correction with zero forcing (ZF), and despreading, the received signal carried by the $i$th subcarrier and the $j$th code is [10]

$$z_{i,j} = L_c x_{i,j} + \sum_{k=1}^{L_c} c_{j,k} \frac{\zeta_{i,k}}{h_i},$$

where $h_i$ is the frequency channel coefficient of subcarrier $i$, and $\zeta_{i,k}$ the sample of complex background noise (assumed to be Gaussian and white, with variance $\sigma^2$) of subcarrier $i$ and $k$th DMT symbol. Because of transmitted PSD constraint, the power of the transmitted signal depends on the length $L_c$ of the code. The matrix of the received symbols over all subcarriers and all codes is $Z = L_c X + G \cdot W \cdot CT$ with $G$ the diagonal equalization matrix such that $g_{i,j} = 1/h_i$, and $W = (w_{i,j})_{0<i\leq N_e, \ 0<j\leq N_c}$.

### III. MULTICARRIER BIT LOADING

DMT or OFDM modulation decomposes the channel into a set of $N_e$ orthogonal subcarriers. The total system capacities is then the sum of the capacity of each subcarrier

$$C^{(\text{DMT})} = \sum_{i=1}^{N_e} \log_2 \left( 1 + \frac{|h_i|^2 e_s}{N_0} \right),$$

where $|h_i|^2 e_s/N_0$ is the signal to noise ratio (SNR) of subchannel $i$ at the receiver side. With PSD constraint, the maximum energy $e_s$ per symbol and per subcarrier is limited to $E$, i.e. $\forall i$, $e_s \leq E$. The throughput in $\mathbb{R}$ is given by

$$R^{(\text{DMT})} = \sum_{i=1}^{N_e} R_i = \sum_{i=1}^{N_e} \log_2 \left( 1 + \frac{|h_i|^2 E}{N_0} \right),$$

where $\Gamma$ is the SNR gap or normalized SNR that is calculated according to the gap-approximation analysis [1].

In single user context with QAM constellations, the reliable throughput per subcarrier is $[R_i]$, where $[\cdot]$ is the floor function. Only subcarriers with $R_i \geq 1$ are used to transmit data. In multiple user context, a simple solution to share the resources is frequency division multiple access (FDMA); the set of subcarriers is then divided between the users. The optimal solution that maximizes the minimum throughput needs lots of subchannel swappings, and a suboptimal solution as the one proposed in [4] already achieves results close to the optimal solution. This algorithm allocates the subcarriers one by one to each user. At each step, the user with the smallest rate is being assigned the subcarrier, among the remainings, that leads to the highest increase of its rate. In this paper, a modified version of this algorithm is applied to the MC-DS-CDMA scheme to perform multiple access.

### IV. MC-DS-CDMA BIT LOADING

In this section, the bit loading strategy applied to MC-DS-CDMA is presented in the single user case. The algorithm has to assign bits and energy per code, and also to find the number of codes that maximize the throughput per subcarrier.

Due to orthogonality in frequency and code spaces, each received symbol $z_{i,j}$ is estimated independently without intersymbol interference, as evident from (2). Thus, the total system capacity is the sum of the system capacities associated with each subcarrier $i$ and code $j$. The capacity, expressed in bit per MC-DS-CDMA symbol, of the MC-DS-CDMA system using ZF detection is then

$$C^{(\text{MC-DS-CDMA})} = \sum_{i=1}^{N_c} \sum_{j=1}^{N_c} c_{i,j}^{(\text{MC-DS-CDMA})},$$

$$= \sum_{i=1}^{N_c} \sum_{j=1}^{N_c} \log_2 \left( 1 + \frac{\mathbb{E}[z_{i,j} u_{i,j}]}{\text{var}[z_{i,j} u_{i,j}]} \right),$$

where

$$\mathbb{E}[z_{i,j} u_{i,j}]^2 = L_c^2 e_{i,j},$$

and

$$\text{var}[z_{i,j} u_{i,j}] = \sum_{k=1}^{N_c} \frac{c_{j,k}^2}{|h_i|^2} \text{var}[\zeta_{i,k}] = L_c \frac{N_0 |h_i|^2}{|h_i|^2}.$$
therefore meet the targeted error probability related to margin because each modulated symbol receives the exact number of users, and the number of tranmitted bits over this code of each user, and except that the user which cannot improve its rate (see condition in 3a) is no more taken into account by the allocation procedure in step 3. The user with the smallest \( R^{(u)} \) is being allocated at first, and then are the others in second step. Any subcarriers cannot been dedicated to one user with zero bit transmission in third step. Of course the third step is stopped when no more user can improve its throughput. Without condition in 3a, i.e. without \( 2R^{(u)} > 0 \), the user with the worst channel would impose its throughput on all the others user, which would reduce the total throughput.

VI. SIMULATION RESULTS

MC-DS-CDMA system uses \( L_e \) DMT symbols to transmit one MC-DS-CDMA symbol. To compared DMT and MC-DS-CDMA performance, the throughput must be expressed using the same unit. All the throughputs are expressed per DMT symbol, hence the MC-DS-CDMA throughput writes \( R^{(MD-CDMA)} / L_e \). With \( L_e = 1 \) the MC-DS-CDMA system amounts to the classical DMT. We use the \( 2^q \)-ary QAM constellations specified for DSL. Results are given for a targeted SER of \( 10^{-3} \) without channel coding, corresponding to an SNR gap \( \Gamma = 6 \text{ dB} \).

A. Single user ADSL context

In this context, the generated MC-DS-CDMA signal is composed of \( N_s = 220 \) subcarriers transmitted in the band \([0.146 ; 1.104] \text{ MHz} \). The ADSL transmission line is a standard European line with a diameter of 0.4 mm. The spectrum mask is limited to \(-39 \text{ dBm/Hz} \) and the PSD of the stationary background noise is \(-140 \text{ dBm/Hz} \). The equivalent subcarrier SNR at the receiver taking into account the SNR gap \( \Gamma \) is displayed Fig. 1 for a length of the line equal to 3500 m. Furthermore the rate transmitted by each subcarrier are exhibited for DMT (dashed line) and MC-DS-CDMA (solid line) with \( L_e = 4 \). The step amplitude of the staircase curve is lower for MC-DS-CDMA (\( L_e = 4 \)) than DMT (\( L_e = 1 \)) due to merging different values are calculated using the channel coefficients of user \( u, h_{i,j}^{(u)} \).

Let us now enounce the proposed allocation algorithm in three steps.

1) Initialization
   a) Compute \( \forall u \in B_u \) \( a_u = R^{(u)} \), with \( B_u = [1, N_u] \)
   b) Set \( \forall u \in B_u \) \( a_u = 0 \), \( B_u = \emptyset \)

2) While \( \exists u \in B_u \)
   a) Find \( u = \arg \min \{ a_u | \text{card}(B_u) > 0 \} \)
   b) For the found \( u \), find the best unused subcarrier \( i \), i.e. with the best link budget
   c) Update \( B_u, R^{(u)} \): \( B_u = \{ i \}, R^{(u)} = R^{(u)} + R^{(u)}_{i,j} \)

3) While there exists unused subcarrier
   a) Find \( u = \arg \min \{ R^{(u)} | \exists R^{(u)} > 0 \} \)
   b) For the found \( u \), find the best unused subcarrier \( i \)
   c) Update \( B_u, R^{(u)} \): \( B_u = B_u + \{ i \}, R^{(u)} = R^{(u)} + R^{(u)}_{i,j} \)

To realize multiple access, the principle of the subcarrier allocation algorithm in [4] is used, except that the \( N_u \) first subcarriers are assigned (see step 2) with respect to a priority order among the users based on the achievable throughput \( R^{(u)} \) of each user, and except that the user which cannot improve its rate (see condition in 3a) is no more taken into account by the allocation procedure in step 3. The user with the smallest \( R^{(u)} \) is being allocated at first, and then are the others in second step. Any subcarriers cannot been dedicated to one user with zero bit transmission in third step. Of course the third step is stopped when no more user can improve its throughput. Without condition in 3a, i.e. without \( 2R^{(u)} > 0 \), the user with the worst channel would impose its throughput on all the others user, which would reduce the total throughput.

V. BIT LOADING ALGORITHM

In single user context, the bit loading algorithm applies the proposition 1 to perform the bits, codes and energies allotment across the spectrum.

In multuser context, each user uses its own channel, and we aim at maximizing the smallest throughput of all users, which is equivalent to maximizing the total throughput of the system while ensuring equal rates between users. Let \( N_u \) be the number of users, and \( B_u \) the subset of subcarriers used by user \( u \). Because of frequency division multiple access between user, we have \( \forall u \neq u', B_u \in B_{u'} = \emptyset \). The throughput of the system writes

\[
R^{(MC-DS-CDMA)} = \sum_{u=1}^{N_u} R^{(u)} = \sum_{u=1}^{N_u} \sum_{i \in B_u} R^{(u)}_{i,j} = \sum_{u=1}^{N_u} \sum_{i \in B_u} N_c^{(u)} \left( 2^{R^{(u)}_{i,j}} - 1 \right),
\]

where \( L_{c_{i,j}} \) \( \in \{ \lfloor R^{(u)}_{i,j} / L_e \rfloor, \lfloor R^{(u)}_{i,j} / L_e + 1 \rfloor \} \) according to proposition 1, and where the superscript \( (u) \) indicates that the
effect. The spreading component of MC-DS-CDMA allows to gather the energies of the different chips of the codes, and then provides throughput gain. Contrary to DMT, the proposed system collects an exploits the residual energies, lost on each subcarrier of the DMT system because of the finite granularity of the QAM constellations.

B. Multiple user PLC context

In this context, the generated MC-DS-CDMA signal is composed of $N_u = 1880$ used subcarriers transmitted in the band $[1.6; 20]$ MHz. In simulations, we use power line channel responses that have been measured in an outdoor PLC network by the French power company Électricité de France (EDF). Note that the allocation problem formulation holds in both forward and backward links, since multiple access is realized through an FDMA approach. We applied the proposed algorithm to the case of a 4-user multiple access communication over the above-mentioned PLC channels. The background noise level is $-110$ dBm/Hz, we assume a transmission level of $-40$ dBm/Hz, and a average channel attenuation of 50 dB.

The equivalent subcarrier SNR of the $4$ used channels are displayed Fig. 3. Furthermore the loaded subcarriers are indicated by marker in the DMT and the MC-DS-CDMA cases. As expected, a larger number of subcarriers are exploited by the proposed adaptive MC-DS-CDMA scheme due to the symbol merging effect. With DMT, the minimum SNR needed to transmit some data is roughly 0 dB. Due to spreading gain, this minimum SNR is $10 \log_{10}(L_c)$ lower with MC-DS-CDMA.

As previously presented in the ADSL single user case,
the total throughput is increased in the PLC multiple user case with MC-DS-CDMA, Fig. 4. For example, DMT transmits xxx bit/DMT symb. while MC-DS-CDMA transmits xxx bit/DMT symb. for $L_c = 4$. The throughput gain is higher than 20\% for $L_c \geq 4$. The minimum throughput is also increased of 10\% with $L_c = 4$. For low values of $L_c$, user #4 is the only one who is able to exploit the highest frequencies of the spectrum.

VII. CONCLUSION

In this paper, we proposed an adaptive MC-DS-CDMA system suitable for wireline networks. We introduced a novel loading algorithm that handles the subcarrier, code, bit and energy resource distribution among the active users of the system. We focused on the system throughput maximization constrained to PSD limitations and finite order modulations. We derived the optimal solution in the case of single user transmissions and proposed a simple algorithm in the case of multiple user transmissions. The subcarrier distribution strategy is based on a greedy approach which represents a suboptimal but practical solution to the stated problem.

We analyzed the performance of the new system and compared the results to those obtained with the DMT system. We thereby highlighted that DMT is equivalent to the proposed MC-DS-CDMA system with $L_c = 1$. The spreading component was shown to provide throughput gain especially for low channel gains. This behavior was explained by the energy gathering capability of MC-DS-CDMA within each spread symbol. Contrary to DMT, the proposed system can exploit the residual energy conveyed by each subcarrier because of the finite granularity of the QAM modulations. Furthermore, it raised a trade-off concerning the spreading factor choice. On one hand, it was to be sufficiently large to increase the throughput, and on the other hand, it was to remain relatively small to ensure low process delay. For well-chosen spreading factors, we then concluded that the proposed adaptive system was able to transmit higher rates than DMT.

REFERENCES