Selection of the spreading parameters for the downlink of coded OFDM-CDMA systems

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Abstract—This article presents a new method for the selection of the spreading parameters for the downlink of two dimensional (2D) OFDM-CDMA systems in the presence of channel coding. This new approach allows the determination of the spreading factor values in time (SFt) and in frequency (SFf) directions in order to find the most reliable 2D transmission scheme. The method is based on the Exponential Effective SNR mapping (EESM) technique applied to the asymptotic Signal to Interference plus Noise Ratio (SINR) computed at the output of the equalizer. Thanks to this asymptotic compressed SINR, we are also able to assess the performances of 2D OFDM-CDMA systems in terms of BER (Bit Error Ratio) at the output of the channel decoder.

I. INTRODUCTION

Recently, Orthogonal Frequency and Code Division Multiplexing (OFCDM) access technology has been investigated for the next generation of mobile communication systems [1][2]. The interest of two dimensional (2D) spreading is to combine in a flexible way the benefits of both time (ST) and frequency (SF) spreading in order to optimize the trade-off between diversity and Multiple Access Interference (MAI). The impact of the modulation and coding schemes on the performances of 2D OFDM-CDMA systems has been studied in [1][2]. For a given scheme, the performance depends on the radio link conditions (coherence bandwidth and coherence time of the channel) and the spreading factors ST and SF. The optimum selection of ST and SF is thus a key issue for the design of OFDM-CDMA systems.

The influence of the channel coding scheme must be taken into account in the selection of the optimum spreading parameters ST and SF. For a given convolutional coding scheme, the bit error ratio at the output of the channel decoder can be estimated as detailed in [4]. It depends on the SINR distribution. In [4], an analytic expression of the SINR distribution has been derived for an OFDM-CDMA system with an equal gain combining (EGC) equalizer. This allows selecting the optimum couple of parameters ST and SF according to the mobile speed and the delay spread of the channel for an EGC equalizer. Unfortunately, practical OFDM-CDMA systems are likely to use minimum mean square error (MMSE) equalizer. To the authors’ knowledge, no contribution has been presented on the SINR distribution with a MMSE equalizer. This is still an open issue.

In this article, we propose a new and different approach for selecting ST and SF, which is applicable to any single user equalizer. It is based on the combination of the asymptotic SINR computation and the Effective Exponential SNR mapping (EESM) method. The asymptotic SINR at the output of the equalizer is obtained analytically thanks to the random matrix theory [5][7]. The asymptotic regime is reached when both the spreading factor (SF) and the number of users (K) become large while the system load \( \alpha = K/SF \) remains constant. It has been validated for a practical spreading factor (e.g. SF=32) [8]. In the asymptotic regime, it is demonstrated that the SINR only depends on SF, the system load, the transmitted power per spreading code, the channel frequency response, equalizer coefficients and the noise variance. The main advantage compared to [9] is the dependence on the actual value of the spreading codes that vanishes.

Once they have been computed, the asymptotic SINRs of the coded bits are introduced in the formula proposed in [10] for the EESM method. The result is a unique SINR per coded frame, noted “compressed SINR”. The EESM method is empiric, but has been validated within 3GPP for the OFDM study item [11]. It is currently used to provide a reliable link to system level interface for system simulations. As the compressed SINR can be formulated as a function of SF and ST, our method will then consist in the selection of the parameters SF and ST that maximizes it.

The paper is organized as follows. Section II describes the system model and details the computation of the asymptotic SINR. Section III introduces the new approach for selecting the optimum 2D OFDM-CDMA scheme. Eventually section IV provides simulation results in order to evaluate the proposed method while section V draws some conclusions.

II. SYSTEM MODEL AND SINR ESTIMATION

A. System model

In this section, a downlink two dimensional OFDM-CDMA system is described. Figure 1 shows a conventional 2D OFDM-CDMA transmitter [1].

![2D OFDM-CDMA transmitter](image)

Figure 1: 2D OFDM-CDMA transmitter.

Each bit \( a_k \) of user \( k \) is sent to a channel coding module. Then the coded bits are modulated into QAM complex symbols. Each QAM symbol \( d_k(n) \) is spread with a user specific Walsh-Hadamard sequence \( C_k \) of SF chips \( c_k[u] \). The power assigned to each code is \( P_k \). Then the signals are added

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before being sent to the 2D chip-mapping module. As depicted in [1], the chips are mapped on \( S_T \) consecutive OFDM symbols and \( S_F \) adjacent subcarriers. \( F \) is the size of the FFT and the overall spreading factor is \( SF = S_F S_T \).

The resulting signal at the output of the 2D chip-mapping module can be written as:

\[
x_i[SF + p] = \sum_{t=0}^{ST-1} (\sum_{l=0}^{W-1} h[l] z^{-l}) x_i[pS_T + t]
\]

where \( t = 0, ..., S_T - 1 \); \( i = 0, ..., F - 1 \); \( p = 0, ..., S_F - 1 \). \( p \) is the index of the carrier for the \( i \)th sub-band (block of \( S_F \) adjacent subcarriers) and \( t \) is the time index.

After the frequency/time transposition with the IFFT, a cyclic prefix is inserted as a Guard Interval (GI). It prevents adjacent sub-carriers propagation channel which is modelled by a FIR filter \( h(z) = \sum_{l=0}^{W-1} h[l] z^{-l} \). The resulting signal is then corrupted by a complex additive white Gaussian noise \( w_t[iSF+p] \) with variance \( \sigma_w^2 \).

Figure 2 shows a classical OFDM-CDMA receiver. The Guard Interval (GI) is first removed. The signal is then transposed in the frequency domain thanks to the FFT. Power and channel estimation is performed. Then chip demapping is done, followed by equalization (one coefficient per subcarrier \( g[iS_F + p] \)) and despreading (correlation). Eventually, soft demapping and soft input channel decoding are performed.

After FFT, the received signal \( Y_j(i) \) for the \( i \)th sub-band of the \( j \)th block can be written as:

\[
Y_j(i) = H_j(i) X_j(i) + W_j(i)
\]

where \( i = 0, ..., F - 1 \) and \( j = 0, ..., T-1 \). \( H_j(i) \), \( X_j(i) \) and \( W_j(i) \) are respectively the SFXS matrix diagonal of the channel with entries \( \hat{h}_{jS_F+p} \), the transmitted \( SF \times 1 \) vector with entries \( x_{jS_F+p} \) and the AWGN \( SF \times 1 \) vector with entries \( w_{jS_F+p} \).

Then the soft decision at the output of the desparser for user \( k \) can be written as follows:

\[
\hat{d}_{j,k}(i) = c_i^k \overline{G}_{j,k}(i) Y_j(i)
\]

\( G_{j,k}(i) \) is the SFXS diagonal matrix containing the coefficients of the one tap equalizer \( g_{jS_F+p} \) (\( t = 0, ..., S_T - 1 \); \( p = 0, ..., S_F - 1 \)).

B. Asymptotic SINR

In order to compute the SINR at the output of the equalizer, we use an asymptotic analysis. This technique has been initially applied to the performance analysis of DS-CDMA systems with random spreading [5]. The asymptotic regime is reached when both the spreading factor (SF) and the number of users (K) become large while the system load \( \alpha = K/SF \) remains constant. In the asymptotic regime, it is demonstrated that the SINR only depends on a few set of parameters. The dependence on the actual value of the spreading codes vanishes. This work was then extended to take into account the orthogonality between spreading codes [7]. Based on these previous contributions, we derived the asymptotic SINR for a 2D OFDM-CDMA system (details are given in Appendix). For the \( i \)th sub-band of the \( j \)th block of \( S_T \) OFDM symbols (Figure 3), we obtain:

\[
SINR_j(i) = \frac{\sigma_i^2}{\sigma_i^2 + \sigma_i^2}
\]

with

\[
\sigma_i^2 = \sigma_a^2 \frac{1}{SF} \sum_{p=0}^{S_F-1} \left| g_{jS_F+p} \right|^2 \left| h_{jS_F+p} \right|^2
\]

\[
\sigma_i^2 = \sigma_a^2 \frac{1}{SF} \sum_{p=0}^{S_F-1} \left| g_{jS_F+p} \right|^2 \left| h_{jS_F+p} \right|^2 + \sigma_m^2 \frac{1}{SF} \sum_{p=0}^{S_F-1} \left| g_{jS_F+p} \right|^2 \left| h_{jS_F+p} \right|^2
\]

\[
\sigma_i^2 = \sigma_a^2 \frac{1}{SF} \sum_{p=0}^{S_F-1} \left| g_{jS_F+p} \right|^2 \left| h_{jS_F+p} \right|^2
\]

\( \sigma_i^2 \) is the power of the useful signal after equalization and despreading, \( \sigma_i^2 \) is the variance of the MAI and \( \sigma_i^2 \) is the
variance of the thermal noise filtered by the equalizer and the spreading sequence. \( \bar{p} = \frac{1}{K-1} \sum_{k=1}^{K-1} \bar{p}_k \) is the average power of the interfering codes.

It can be seen from (5) that the asymptotic SINR can be expressed as a function of the spreading parameters \( S_F \) and \( S_T \).

III. SELECTION OF THE SPREADING PARAMETERS

A. Compression of the asymptotic SINR

A technique called Effective Exponential SNR Mapping (EESM) was proposed in [10] for an OFDM system. First, a unique ("compressed") SINR is computed for each coded block of \( N_c \) QAM symbols:

\[
SINR_{\text{comp}} = -\beta \ln \left( \frac{1}{N_c} \sum_{i=1}^{N_c} \exp \left( -\frac{SINR(i)}{\beta} \right) \right)
\]  

(6)

\( SINR(i) \) is the SINR of the \( i \)th QAM symbol of the coded block. \( SINR_{\text{comp}} \) is then used to estimate the BER at the output of the channel decoder with a simple look-up table (LUT). This is depicted in Figure 4. This LUT gives the BER at the output of the channel decoder as a function of the input SINR for a gaussian channel. This LUT is obtained either analytically or by simulations. The crucial point is that the parameter \( \beta \) is unique. Indeed it is independent of the propagation channel: it is uniquely defined for a given modulation and coding scheme (MCS). As in [11], \( \beta \) is obtained through Monte-Carlo simulations in our study.

![Figure 4: EESM.](image)

We propose to adapt this compressed SINR technique to our OFDM-CDMA system:

\[
SINR_{\text{comp}} = -\beta \ln \left( \frac{1}{N_c} \sum_{i=1}^{N_c} \exp \left( -\frac{SINR(i)}{\beta} \right) \right)
\]  

(7)

In addition, we use the asymptotic SINR formula of \( SINR(i) \), given by (4). One important thing is that \( \beta \) is also independent of the system load and the spreading parameters \( (S_F, S_T) \). This will be checked in section IV.

B. Optimization of the compressed SINR

The selection of the spreading parameters is now possible thanks to the optimization of the compressed SINR given by (7). First, the receiver computes the channel estimates. From these estimates, it also computes the equalization coefficients.

We assume that the set of MCS and the corresponding parameters \( \beta \) are known at the receiver. Eventually, the compressed SINR is determined for various combinations of \( (S_F, S_T) \) according to (4) and (7). For instance if \( SF=32 \), the compressed SINR is computed for the following configurations (1,32), (2,16), (4,8), (8,4), (16,2) and (32,1). The largest value will give the optimal configuration for the considered MCS.

IV. SIMULATION RESULTS

The system parameters of the considered 2D OFDM-CDMA system are given in Table 1. Perfect synchronization and channel estimation are assumed. The convolutional coding (CC) scheme under consideration is the conventional 802.11a code [13]. Bit interleaving is performed over all the entire slot length.

<table>
<thead>
<tr>
<th>Bandwidth</th>
<th>20 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nb of subcarriers</td>
<td>( N_c ) 1024</td>
</tr>
<tr>
<td>FFT length</td>
<td>( N_c ) 1024</td>
</tr>
<tr>
<td>Guard Interval length</td>
<td>216</td>
</tr>
<tr>
<td>Slot Length</td>
<td>( N_s ) 32</td>
</tr>
<tr>
<td>Spreading length</td>
<td>( SF=S_F, S_T ) 32</td>
</tr>
<tr>
<td>Modulation</td>
<td>QPSK/16-QAM</td>
</tr>
<tr>
<td>Channel coding</td>
<td>CC</td>
</tr>
<tr>
<td>Channel coding rate</td>
<td>( R ) 1/3, 1/2, 2/3</td>
</tr>
<tr>
<td>Channel coding memory</td>
<td>6 (CC)</td>
</tr>
<tr>
<td>Block length</td>
<td>( B_{\text{code}} ) ( N_c \times N_s \times R/SF )</td>
</tr>
<tr>
<td>BRAN A and E channels [13], Rayleigh 2 paths</td>
<td></td>
</tr>
<tr>
<td>Mobile speed</td>
<td>( v_m ) 3.6 / 72 / 300 km.h(^{-1})</td>
</tr>
<tr>
<td>Equalizer</td>
<td>SU MMSE [3]</td>
</tr>
</tbody>
</table>

Table 1: OFDM-CDMA parameters

In section IV.A, we will check that the proposed method provides an accurate estimation of the BER at the output of the channel decoder. Then, we will evaluate the sensitivity of our proposed method to various parameters, namely the system load in section IV.B and the channel models in section IV.C.

A. The parameter \( \beta \)

Performances obtained for MCS\(_1\)=\{QPSK, CC, \( R=2/3 \)\} and MCS\(_2\)=\{16-QAM, CC, \( R=1/3 \)\} are respectively depicted in Figure 5 and Figure 6. For MCS\(_1\), the considered configuration is MC-CDMA at full load \((S_F=1, \alpha=1)\). Two different propagation channels are considered, namely the BRAN E \((v_m=72 \text{ km/h})\) and BRAN A \((v_m=3.6 \text{ km/h})\) channels. For MCS\(_2\), the considered configuration is the BRAN E channel \((v_m=72 \text{ km/h})\) and \(\alpha=1\). Two different \((S_F \times S_T)\) combinations are plotted, MC-CDMA \((32,1)\) and MC-DS-CDMA \((1,32)\). The asymptotic BER ("Asymp" curves) is averaged over channel realizations whereas the Monte-Carlo one ("MC" curves) is averaged over channel and noise realizations. In both figures, the Monte-Carlo curves match the theoretical ones for BER \(< 10^{-2}\). This shows that \( \beta \) is independent of the propagation channel and the \((S_F, S_T)\) combination. This also shows that the BER at the output of the channel decoder can be predicted accurately. For the two considered MCS, \( \beta \) has been evaluated by simulations and is equal to 2.2 for MCS\(_1\) and 10 for MCS\(_2\).

![Figure 5: MCS\(_1\), MC-CDMA, \( \beta=2.2 \).](image)
B. Modulation schemes and system loads

In the remaining of the document, the BER obtained through Monte-Carlo simulations and the compressed asymptotic SINR are obtained as follows: we draw a channel realization, whose response is kept constant for the simulation of the BER point, the BER is then only averaged over the noise realizations whereas the compressed SINR is obtained thanks to (7).

The compressed SINR and the measured BER versus $S_T$ are plotted in Figure 7 for a QPSK constellation with a 1/4 load system ($E_b/N_0=5$ dB) and in Figure 8 for a 16-QAM constellation with a fully loaded system ($E_b/N_0=8.5$ dB). The considered channel is the BRAN E one with $v_m=72$ km/h. The coherence bandwidth $B_{coh}$ of that channel normalized with respect to the sub-carrier spacing is equal to 33, which means that the channel frequency response is more or less flat over 33 sub-carriers. The coherence time $T_{coh}$ of the channel normalized with respect to the OFDM symbol duration is equal to 28. In both cases, the ranking of the compressed SINRs fully agrees with the ranking of the obtained BER. When the SINR increases, then the BER decreases and vice versa.

The computation of the compressed SINR at the receiver allows deciding the best 2D spreading configuration for a given MCS. For QPSK with 1/4 load, the highest compressed asymptotic SINR is obtained for $S_T=32$. Indeed, for such a low order constellation associated with a small load, the diversity gain due to the mobile speed overcomes the degradation induced by MAI for large $S_T$. For a large constellation such as 16-QAM, which is more sensitive to MAI, the best performances are obtained for a smaller $S_T$. Here, $S_T=4$ gives the best performance.

C. Channel models

In this section, we evaluate the sensitivity of our proposed method to the channel modeling. We consider two kinds of channel. The first one is a BRAN E channel with $v_m=3.6$ km/h. The second multipath channel model contains two paths (delayed by 200 ns) with equal average power and their amplitude follows a Rayleigh distribution.

The compressed SINR and the measured BER are plotted in Figure 9 for the BRAN E channel ($E_b/N_0=7.5$ dB) and in Figure 10 for the second model with $v_m=300$ km/h ($E_b/N_0=9$ dB). The considered constellation is 16-QAM and the system load is equal to 1. Again, the ranking of the compressed SINRs fully agrees with the ranking of the BER. For the BRAN E channel, MC-DS-CDMA is equivalent to COFDM because the channel is almost static during one slot ($T_{coh}=920$). As a consequence, we recover the conclusions that COFDM offers better performance than MC-CDMA when considering high order modulations [3]. In the second case, the channel is fast fading ($T_{coh}=11$) and its frequency response is flat ($B_{coh}=81$). As a consequence the best results will be obtained with $S_T=1$ (pure MC-CDMA), as observed in Figure 10.
V. CONCLUSION

We have shown in this paper that the optimization of the compressed asymptotic SINR at the receiver allows selecting the spreading parameters $S_F$ and $S_T$ for a given modulation/coding scheme. The method is based on the Exponential Effective SINR mapping (EESM) technique applied to the asymptotic Signal to Interference plus Noise Ratio (SINR) computed at the output of the equalizer.

The compressed asymptotic SINR also allows assessing the BER performances of any 2D OFDM-CDMA system when considering channel coding. Thus it can be used in a system level simulator to predict the BER given the asymptotic SINR input.

APPENDIX

From (3), the decided symbol for user k for the $i^{th}$ sub-band of the $j^{th}$ block at the output of the equalizer can be written as:

$$d_{j,k}(i) = C_k^H \hat{G}_k^H (\gamma_k(i)) I_1 + I_2 + I_3$$

(8)

where $I_1$, $I_2$ and $I_3$ are respectively the useful data term, the intra-cell interference term and the AWGN filtered by the equalizer:

$$I_1 = \sqrt{P_k} C_k^H \hat{G}_k^H (\gamma_k(i)) \xi_k d_{j,k}(i)$$

$$I_2 = C_k^H \hat{G}_k^H (\gamma_k(i)) W_i^2 Q A_j(i)$$

$$I_3 = C_k^H \hat{G}_k^H (\gamma_k(i)) W_i^2 (\bar{W}_i(i))$$

(9)

$Q = \text{diag}(P_0, P_1, ..., P_{K-1})$ is the diagonal matrix containing the power of all the interfering codes. $U = (C_0, ..., C_{k-1}, C_{k+1}, ..., C_{K-1})$ is the SFx(K-1) matrix containing the interfering spreading sequences. $A_j(i)$ is a (K-1)x1 vector containing the data of the other K-1 users.

Assuming that the data symbols are iid with zero-mean and unit variance, the variance of $I_1$, $I_2$ and $I_3$ are:

$$\sigma_{I_1}^2 = \sigma^2 \left( \sum_{k=0}^{K-1} C_k^H \hat{G}_k^H (\gamma_k(i)) W_i^2 \xi_k d_{j,k}(i) \right)$$

$$\sigma_{I_2}^2 = \sigma^2 \left( \sum_{k=0}^{K-1} C_k^H \hat{G}_k^H (\gamma_k(i)) W_i^2 Q A_j(i) \right)$$

$$\sigma_{I_3}^2 = \sigma^2 \left( \sum_{k=0}^{K-1} C_k^H \hat{G}_k^H (\gamma_k(i)) W_i^2 (\bar{W}_i(i)) \right)$$

(10)

Then, the following properties are used for the computation of (10):

- P1: if B is a SFxSF uniformly bounded deterministic matrix and $C_k = \xi_k(0), ..., \xi_k(SF-1)$ where the $\xi_k$’s are iid complex random variables with zero mean, unit variance and finite eighth order moment, then [6] gives:

$$C_k^H B C_k \xrightarrow{SF \to \infty} \frac{1}{SF} tr(B)$$

- P2: Let C be a Haar distributed unitary matrix of size SFxSF. C can be decomposed into a vector $C_k$ of size SFx1 and another matrix $U$ of size SFx(K-1) so that C can be written as $C = (C_k, U)$. Then [7] gives:

$$UQU^H \xrightarrow{SF \to \infty} \alpha P \left(I - C_k C_k^H\right)$$

The computation of $\sigma_{I_1}^2$ and $\sigma_{I_2}^2$ uses directly property P1. Concerning $\sigma_{I_3}^2$, we apply property P2 first and then P1. Based on these assumptions, (10) becomes:

$$\sigma_{I_1}^2 = \frac{1}{SF} v \left( G_k^H (\gamma_k(i)) W_i^2 \xi_k d_{j,k}(i) \right)$$

$$\sigma_{I_2}^2 = \frac{1}{SF} v \left( G_k^H (\gamma_k(i)) W_i^2 Q A_j(i) \right)$$

$$\sigma_{I_3}^2 = \sigma^2 \frac{1}{SF} v \left( C_k^H \hat{G}_k^H (\gamma_k(i)) \right)$$

(11)

This latter equation allows recovering (5).

REFERENCES


